Exam 2 # TMath 124 Spring 2012

Key

1. [8] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be functions and x and y be positive numbers.

T F $-2^2 = 4$. $(-2)^2 = (-2)(-2) = 4$ PEMDAS

T F 5 inches is equal to 0.5 feet. 0.5ft is 6 inches

T F The volume of a cylinder of radius R and height h is $R^2\pi h$.

T F If $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$. Thm from class
differentiable continuous

T F $\lim_{\theta\to 0} \frac{\cos(\theta)}{\theta} = 1$ $\frac{1}{\text{tiny}} = \text{Big} \neq 1$

T F $\frac{d}{dx}(2^x) = x2^{x-1}$ $\frac{d}{dx}(2^x) = 2^x(\ln(2))$

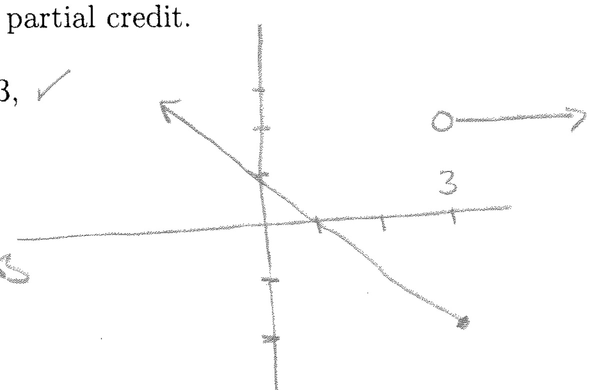
T F $x^2 = 2x$ $\frac{d}{dx}(x^2) = 2x$

T F $\frac{d}{dt}(x^2) = 2x$ $\frac{d}{dt}(x^2) = 2x\frac{dx}{dt}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] (Practice Exam #2) Find the formula for a function f that satisfies the following conditions. Note: drawing a graph will earn some partial credit.

(a) f is differentiable everywhere but when $x = 3$, ✓
(b) $\lim_{x\to 3} f(x)$ does not exist, and ✓
(c) $f'(x) < 0$ when $x < 1$. ✓



note: there are many correct answers

graph (+1.5)
function (1.5)
function (+1)

$$f(x) = \begin{cases} -x+1 & \text{if } x \le 3 \\ 2 & \text{if } 3 < x \end{cases}$$

notation (+.5)
limits prop (+.5)
trig (+.5)
algebra (+.5)

3. [3] (quiz 3 #3) Determine the following, if it exists. Be sure to justify your work.

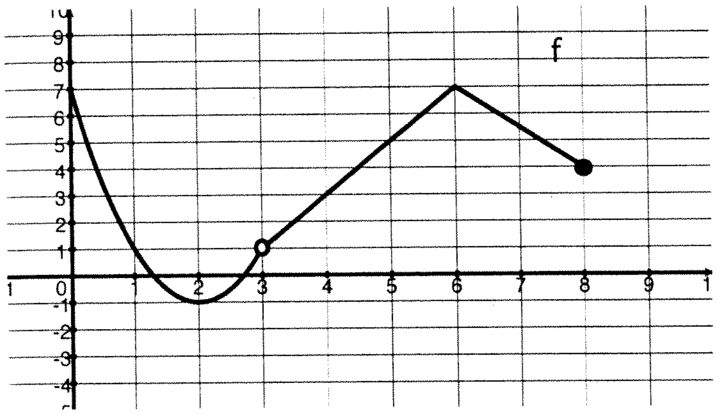
$$\lim_{x\to 0} \frac{x\cos(x+\frac{\pi}{4})}{\sin(x\sqrt{2})} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \lim_{x\to 0}\frac{x\sqrt{2}}{\sin(x\sqrt{2})}\cdot\frac{\cos(x+\pi/4)}{\sqrt{2}} = \underbrace{\lim_{x\to 0}\frac{x\sqrt{2}}{\sin(x\sqrt{2})}}_{1}\cdot\lim_{x\to 0}\frac{\cos(x+\pi/4)}{\sqrt{2}}$$

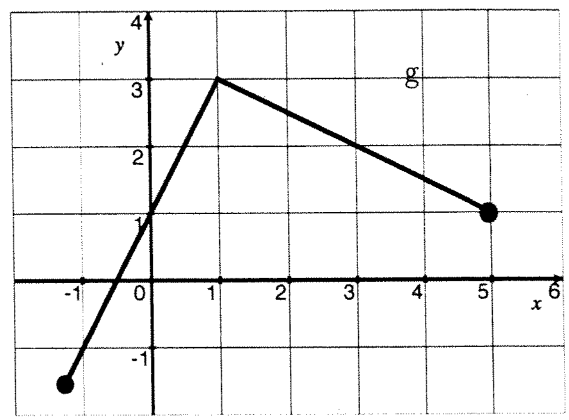
factor (+.5)

$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ (+.5)

$$= \lim_{x\to 0}\frac{\cos(x+\pi/4)}{\sqrt{2}} = \frac{\cos(0+\pi/4)}{\sqrt{2}} = \frac{\cos\pi/4}{\sqrt{2}} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}$$

4. Let the graph of f and g be those shown below.





Estimate the following (if they exist):

[2] (WebHW8 #8)

$(f\cdot g)'(2) = f'(2)g(2)+f(2)g'(2)$

product (+1)

$= \underbrace{0\cdot 2.5}_{+.5} + \underbrace{(-1)(-1/2)}_{+.5}$

$= 1/2$

[2] (product wks #2)

$\frac{d}{dx}(x^3 f)\,|_{x=2}$

(+.5) product $(x^3f)' = 3x^2f(x)+x^3f'(x)$

$(x^3f)'(2) = 3(2)^2\cdot f(2) + 2^3 f'(2)$

$= 12\cdot(-1) + 8\cdot 0$ (+.5) (+.5)

$= -12$

[2] (quiz 3 #3)

$\frac{d}{dx}(g\circ g)\,|_{x=3} = g'(g(3))g'(3)$

Chain (+.5)

$= g'(2)(-1/2)$ (+.5) (+.5)

$= (-1/2)(-1/2)$ (+.5)

$= 1/4$

[2] (§3.2 #44)

$\left(\frac{f(x)}{2+g(x)}\right)'(4)$

$$\frac{[2+g(x)]f'(x)-f(x)[2+g(x)]'}{(2+g(x))^2}$$

quotient (+1)

$$\frac{[2+g(4)]f'(4)-f(4)\cdot g'(4)}{(2+g(4))^2}$$

(+.5) (+.5)

$$\frac{(2+1.5)(2)-3(-1/2)}{(2+1.5)^2} =$$

5. Find $\frac{dy}{dx}$ for each of the following. Do *not* simplify.

[2] (trig wks #1) Chain rule (+.5)

$y = (x^3 - 1)^{100}$

$g(x) = x^3 - 1 \quad g'(x) = 3x^2$
$f(u) = u^{100} \quad f'(u) = 100u^{99}$ (+1)

$f'(g(x))g'(x) = f'(x^3-1) \cdot 3x^2$
$= 100(x^3-1)^{99} \cdot 3x^2$ (+.5)

[3] (§3.3 #10) Implicit diff

$xe^y = x - y$

$\frac{d}{dx}[xe^y] = \frac{d}{dx}[x] - \frac{d}{dx}[y]$

product rule (+1)

$x\frac{d}{dx}[e^y] + \frac{d}{dx}[x]e^y = 1 - \frac{dy}{dx}$

$xe^y\frac{dy}{dx} + e^y = 1 - \frac{dy}{dx}$ (+.5)

(+.5) $+\frac{dy}{dx} - e^y \quad +\frac{dy}{dx} - e^y$

$xe^y\frac{dy}{dx} + \frac{dy}{dx} = 1 - e^y$

$\frac{dy}{dx}[xe^y + 1] = 1 - e^y$

$\frac{dy}{dx} = \frac{1-e^y}{xe^y+1}$ (+.5) alg (+.5) solved for $\frac{dy}{dx}$

[2][3] (WebHW9 #8) Chain rule (+.5)

$y = 2^{\sin(x)}$

$g(x) = \sin(x) \quad g'(x) = \cos(x)$
$f(u) = 2^u \quad f'(u) = 2^u \ln 2$ (+1)

$f'(g(x))g'(x) = f'(\sin(x)) \cdot \cos(x)$
$= 2^{\sin x}(\ln 2)\cos(x)$ (+.5)

[3][2] (§3.2 #28) Product rule (+1) notation (+.5)

$y = e^x\sqrt{x^5}$

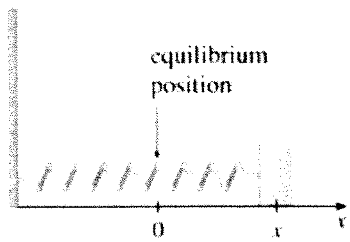
$\frac{dy}{dx} = \frac{d}{dx}[e^x x^{5/2}]$ alg (+.5)

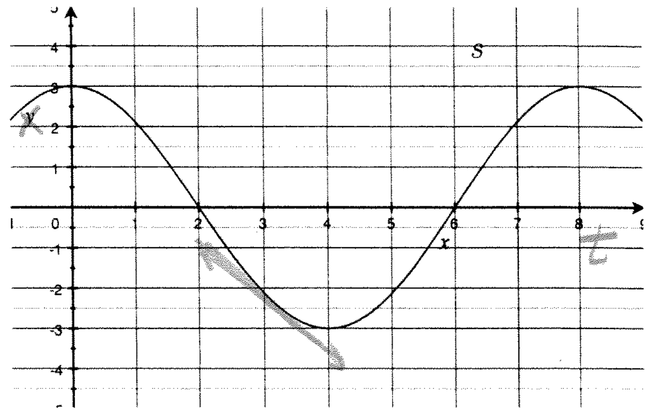
$= e^x \frac{d}{dx}[x^{5/2}] + \frac{d}{dx}[e^x]x^{5/2}$
(+.5) (+.5)

$= e^x \frac{5}{2}x^{3/2} + e^x x^{5/2}$

6. (WebHW9 #6) A mass on a spring vibrates horizontally on a smooth level surface (figure below). The equation of motion and the corresponding graph is given below, where s is measured in centimeters and t in seconds.

$$s(t) = 3\cos\left(\frac{\pi}{4}t\right)$$





(a) [1] Estimate the maximum distance the mass is from equilibrium.

3 cm

(b) [3] Find the velocity of the mass at time t after 3 seconds.

$g(t) = \frac{\pi}{4}t$ $\quad g'(t) = \frac{\pi}{4}$

$f(u) = \cos u$ $\quad f'(u) = -\sin u$ (+1)

velocity $= s'(t)$ (+.5)

$= [3\cos(\frac{\pi}{4}t)]'$

$= 3[\cos(\frac{\pi}{4}t)]'$

(Chain Rule) (+.5)

$= 3 f'(g(t))g'(t)$

$= 3 f'(\frac{\pi}{4}t)\cdot\frac{\pi}{4}$

$= 3(-\sin(\frac{\pi}{4}t))\frac{\pi}{4}$

$= 3\cdot -\frac{\pi}{4}\sin(\frac{\pi}{4}t)$

(+.5) So velocity after 3 sec is $-\frac{3\pi}{4}\sin(\frac{\pi}{4}\cdot 3)$

(+.5) $-\frac{3\pi}{4}\cdot\frac{\sqrt{2}}{2} = -\frac{3\pi\sqrt{2}}{8}$ cm/sec

≈ -1.67

(c) [2] During the first eight seconds, when is the spring expanding?

sees connection with moving right (+.5)

between 4 (+.5) and 8 (+.5) seconds

sees derivative (+.5)

(d) [3] Find the acceleration at time t.

$g(t) = \frac{\pi}{4}t$ $\quad g'(t) = \frac{\pi}{4}$

$f(u) = \sin u$ $\quad f'(u) = \cos u$ (+.5)

(+.5) acceleration = velocity$'(t)$

$= [-\frac{3\pi}{4}\sin(\frac{\pi}{4}t)]'$

$= -\frac{3\pi}{4}[\sin(\frac{\pi}{4}t)]'$

Chain Rule (+1)

$= -\frac{3\pi}{4} f'(g(t))g'(t)$

$= -\frac{3\pi}{4} f'(\frac{\pi}{4}t)\cdot\frac{\pi}{4}$

$= -\frac{3\pi}{4}(\cos(\frac{\pi}{4}t))\frac{\pi}{4} = \frac{3\pi^2}{16}\cos(\frac{\pi}{4}t)$

notation (+.5)

algebra (+.5)

(e) [3] Find the equation of the line tangent to the position function s when $t = 3$.

(+.5) { Looking for $y = mx + b$

(+.5) { m = slope of line tangent to s when $x = 3$ $= s'(3)$

(+.5) { $= -\frac{3\pi\sqrt{2}}{8}$

(+.5) { passes thru $(3, 3\cos(\frac{\pi}{4}\cdot 3))$ or $(3, 3(-\frac{\sqrt{2}}{2})) = (3, \frac{-3\sqrt{2}}{2})$

2.12

(+.5) { So $\frac{-3\sqrt{2}}{2} = \frac{-3\pi\sqrt{2}}{8}(3) + b$

$\Rightarrow b = \frac{-3\sqrt{2}}{2} + \frac{9\pi\sqrt{2}}{8} = \frac{9\pi\sqrt{2} - 12\sqrt{2}}{8}$

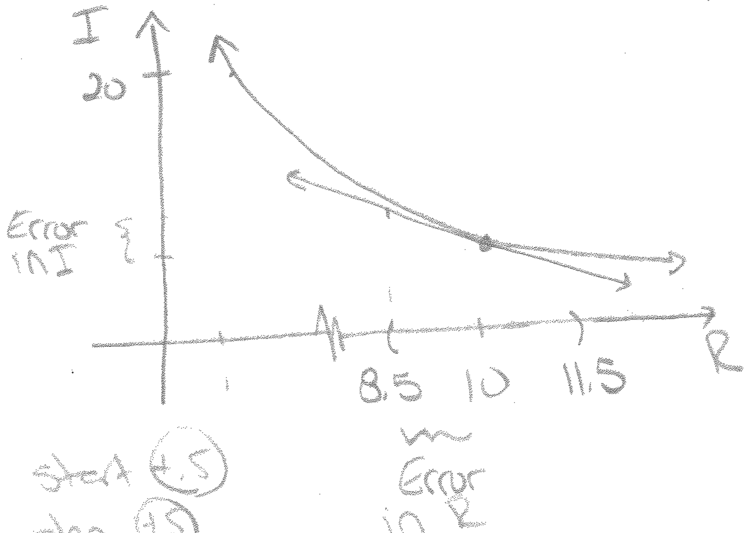
(+.5) { So $y = \frac{-3\pi\sqrt{2}}{8}x + \frac{9\pi\sqrt{2} - 12\sqrt{2}}{8}$

7. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

   (a) (Story Wks #2 & similar to §3.10 #39) If a current i passes through a resistor with resistance r, Ohm's Law states that the voltage drop is $v = ri$. Assume that voltage remains a constant 20 volts. An unreliable resistor claims a resistance of 10 ohms but may be off by up to 1.5 ohms. Use linear approximation to estimate the error in i.

   (b) (§3.9 #31) The top of a ladder slides own a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m from the wall, it slides away from the all at a rate of 0.2 m/s. How long is the ladder?

(a) $V = RI \Rightarrow I = \frac{V}{R} = \frac{20}{R}$



start (.5)
plan (.5)

equation of linear approx
$y = mx + b$

(.5) m = slope of the tangent to I when $R = 10$
$= I'(10) = \frac{-20}{10^2} = -.2$ (.5)

(.5) $\left[I'(R) = \left(\frac{20}{R}\right)' = (20R^{-1})' = -20R^{-2} \right]$

(.5) line passes thru $(10, \frac{20}{10})$

(.5) so $2 = -.2(10) + b$

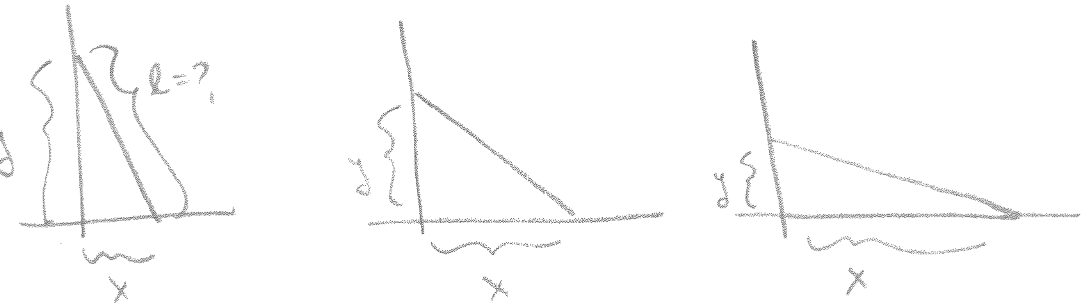
$\Rightarrow b = 2 + 2 = 4$

(.5) line equation: $I = -.2R + 4$

If $R = 8.5$
(1) then $I \approx -.2(8.5) + 4 = 2.3$
If $R = 11.5$
then $I \approx -.2(11.5) + 4 = 1.7$

(.5) $\Rightarrow$ Error would be approximately $2.3 - 2$ or $2 - 1.7$
so $.3$

(b) time passes →



defined variables (+.5)

(+.5) $\left\{ \left.\frac{dx}{dt}\right|_{x=3} = \frac{\text{change in } x}{\text{change in time}} \right.$ when x=3 is $0.2\ m/s$

(+.5) $\left\{ \frac{dy}{dt} = \frac{\text{change in } y}{\text{change in time}} = -0.15\ m/s \right.$

↳ b/c decreasing

Notice $x^2+y^2=l^2$ (+1)

$\Rightarrow \frac{d}{dt}[x^2+y^2] = \frac{d}{dt}[l^2]$ ↳ b/c length of ladder isn't changing

(+1) $\{ = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

(+1) when $x=3$ we have

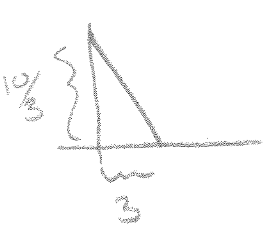
$2\cdot 3m \cdot .2 m/s + 2y(-.15\ m/s) = 0$

$\Rightarrow \quad -.3y = -1$

$\Rightarrow y = \frac{1}{.3} = \frac{10}{3}$

5
.2
1.0

(+.5) recall $x^2+y^2=l^2$



$\Rightarrow l = \sqrt{3^2+\left(\frac{10}{3}\right)^2}$

$= \sqrt{9+\frac{100}{9}}$

$= \sqrt{\frac{81+100}{9}} = \sqrt{\frac{181}{9}}$