TMath 124

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be functions and x and y be positive numbers.

   T   F $-2^2 = 4$.

   T   F $(x+1)^{\frac{3}{2}} = \left(\sqrt{x+1}\right)^3$.

   T   F $\dfrac{x^2 - 1}{x^2} = 1 - x^{-2}$.

   T   F   If $\lim\limits_{x \to 1} f(x) = 0$ and $\lim\limits_{x \to 1} g(x) = 0$, then $\lim\limits_{x \to 1} \dfrac{f(x)}{g(x)}$ does not exist.

   T   F $\lim\limits_{x \to a}(g(x)) = 3$, then $\lim\limits_{x \to a} 5 \lim\limits_{x \to a} g(x) = 15$.

   T   F   If $\lim\limits_{x \to a} f(x) = f(a)$, then $f'(a)$ exists.

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [3] (§2.2 #16 & §2.7 #22) Sketch the graph of an example function f that satisfies the following conditions:
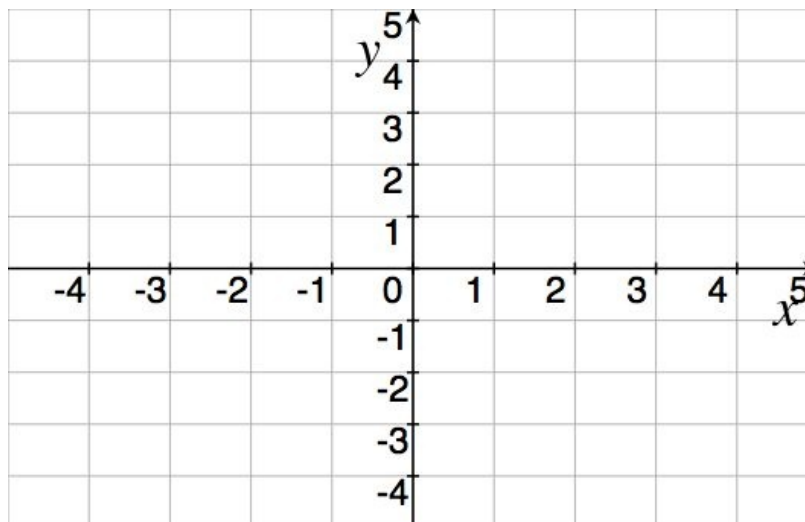
   (a) f is continuous everywhere but when $x = 3$

   (b) $\lim\limits_{x \to 3+} f(x) = 2$

   (c) $\lim\limits_{x \to 3} f(x)$ does not exist

   (d) $f(0) = 1$

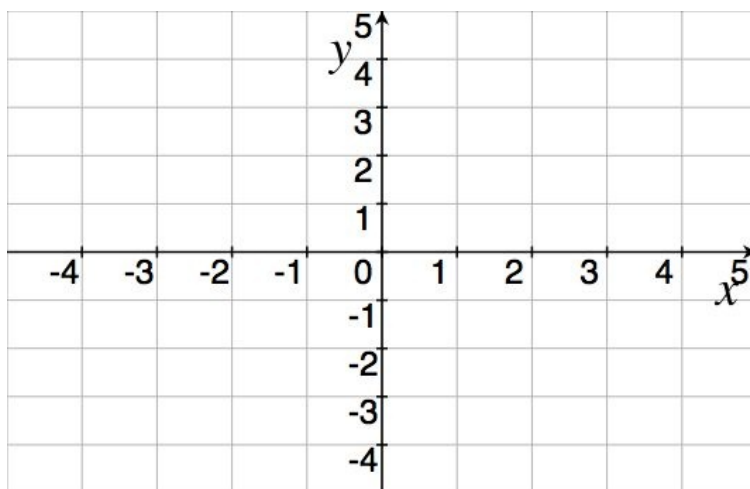   (e) $f'(0) = -{}^{\backprime}1$

   [3] Find a formula for the above graph.

3. Consider the piecewise-defined function f defined below

$$f(x) = \begin{cases} \frac{x^2+5x+6}{x+2} & \text{if } x \leq 1 \\ -\log_3(x) & \text{if } 1 < x \end{cases}$$

(a) [3] (Quiz2 #1) Draw the graph on the axis provided.

(b) Use the graph above to estimate the following (if they exist!):

[1] (WebHW2 #2)
$\lim\limits_{x \to -2} f(x)$

[2] (§2.3 #2e)
$\lim\limits_{x \to 3} x^2 f(x)$

[1] (WebHW2 #2)
$\lim\limits_{x \to 1^-} f(x)$
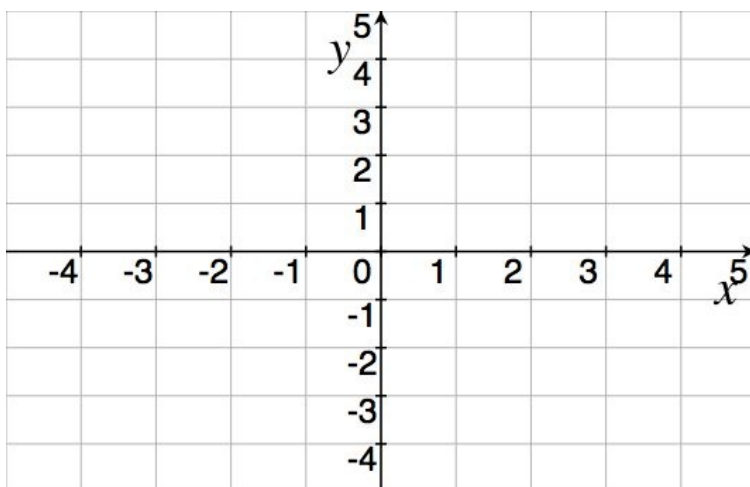
[1] (limit wks #2)
$\lim\limits_{x \to 1} f(x)$

[1] (WebHW5 #9)
$f'(0)$

[1] (WebHW5 #9)
$\frac{d}{dx} f(1)$

(c) [3] (§2.8 #8) Make a rough sketch of the graph of $f'(x)$:

2

4. Find the limit or explain why it does not exist.

[2] (Quiz1 #2)

$$\lim_{x \to 2} \frac{x \ln(x) - 2 \ln(x)}{x^2 - 4}$$

[3] (inf limits wks)

$$\lim_{x \to \infty} \frac{x - 2}{x^2 + 1}$$

[3] (§2.3 #39)

$$\lim_{x \to 0} x^6 \cos\left(\frac{3}{x}\right)$$

[2] (WebHW4 #4)

$$\lim_{x \to 9} \frac{16 + \sqrt{x}}{\sqrt{16 + x}}$$

5. [3] (Winter '12 Exam1) Find an equation of the tangent line to the graph of $y = g(x)$ at $x = 6$ if $g(6) = -2$ and $g'(6) = 3$.

6. Let $f(x) = \frac{1}{x} + e^2$.

   (a) [2] (§3.1 #17) Find $f'(x)$

   (b) [4] (§2.8 #34) Find the derivative of f using the definition of derivative. That is, use $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ and verify your answer to part (a).

7. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (Story Wks #4) A tank contains C liters of pure water. Brine that contains S grams of salt per liter of water is pumped into the tank at a rate of R liters per minute.

   i. [3] Find a formula for the concentration of brine as a function of time t.

   ii. [2] Find out what happens to the concentration as $t \to \infty$.

   iii. [1] Interpret your answer to part ii in terms your 12 year old niece would understand.

(b) A rock thrown upwards on planet Mars with velocity $15\frac{\text{m}}{\text{s}}$ has a height $h(t) = 15t - 1.86t^2$ meters t seconds later.

   i. [2] (Story wks #6) Find the velocity of the rock after 2 seconds.

   ii. [1] (Story wks #5) Recall gravity is the constant acceleration experienced by an object from the planet. Find the gravity on Mars.

   iii. [3] (Story wks 6b) Use calculus to find *when* does the rock reach its maximum height?