Do # 4b
3b

EXAM 2 # TMath 124 Autumn 2018

Key

Median score: 71%

1. [4] TRUE/FALSE: Let f and g be functions. Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

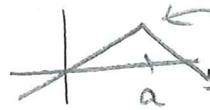
T F $\sqrt{x} = x^{\frac{1}{2}}$

T F If $\lim_{h\to 0} g(h) = 0$ then $\lim_{h\to 0} \frac{f(h)}{g(h)}$ does not exist.

ex Let $f(h)=h(h+1)$ and $g(h)=h$

Then $\lim_{h\to 0} \frac{f(h)}{g(h)} = \lim_{h\to 0} \frac{\cancel{h}(h+1)}{\cancel{h}} = 1$

T F If f is continuous at a, then f is differentiable at a.



Corner is not differentiable

T F $\frac{d}{dz}x^2 = 2x$ $\frac{d}{dx}(x^2) = 2x$

! with respect to z?

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit.

2. (§3.5 #74) Consider the function $f(x) = x^x$.

(a) [1] Explain *why* we cannot use the power rule to find $\frac{dy}{dx}$.

(+1) x is an exponent (in the sky)
the power rule can only be used with real # exponents

(b) [3] Find $\frac{dy}{dx}$ when $y = x^x$

$\ln(y) = \ln(x^x)$

$\Rightarrow \ln y = x\ln(x)$

log both sides (+.5)

log prop (+.5)

$\frac{d}{dx}$

product (+.5)

$\frac{1}{y}\cdot\frac{dy}{dx} = x\frac{d}{dx}(\ln x) + \frac{d}{dx}(x)\ln(x)$

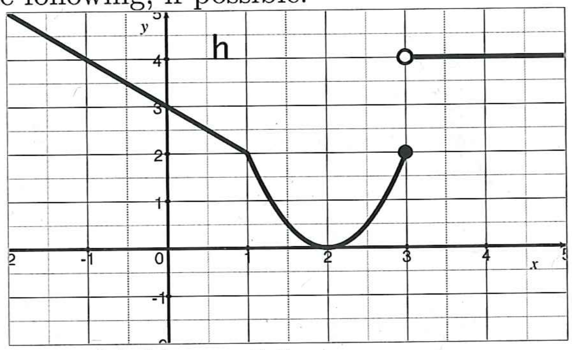
(+1) (+.5)

$\frac{1}{y}\cdot\frac{dy}{dx} = x\cdot\frac{1}{x} + 1\cdot\ln(x)$

$\Rightarrow dy/dx = y[1+\ln(x)]$

3. Let h be the piece-wise defined function comprised of two line segments and a parabola shifted horizontally shown below and to the right. Let f be a continuous function with the characteristics described below. Find the following, if possible.

$f(-1) = -3$
$f(4) = 5$
$f'(-1) = -2$
$f'(4) = 6$



(a) [1] $\frac{d}{dx}(h(x))\,|_{x=0}$

ie $h'(0)$ = slope of line tangent to h @ x=0

-1

[3] (Quiz3 #1)
$(f \circ h)'(-1)$ = $\frac{d}{dx}(f(h(x)))\Big|_{x=-1}$ comp (+.5)

Chain Rule (+1)

$f'(h(-1)) \cdot h'(-1)$

$f'(4) \cdot (-1)$ = $(6 \cdot (-1))$ = -6
(+.5) (+.5) (+.5)

[3] (ProductActivity #1)
$(fh)'(-1)$ product (+.1)

$f(-1)h'(-1) + f'(-1)h(-1)$

$(-3)(-1) + (-2)(4)$
(+.5) (+.5) (+.5) (+.5)

$3 - 8 = -5$

[3] (WebHW7 #7)
$$\frac{d}{dx}\left(\frac{f(x)}{3+h(x)}\right)\Bigg|_{x=-1}$$ quotient (+1)

$$\frac{[3+h(-1)]f'(-1) - f(-1)[3+h(x)]'\big|_{x=-1}}{[3+h(-1)]^2}$$

(+.5) (+.5) (+.5)

$$= \frac{(3+4)(-2) - (-3)(0+h'(-1))}{(3+4)^2}$$ (+.5)

$$= \frac{-14+3(-1)}{7^2} = \frac{-17}{49}$$

(b) [3] (ApproxActivity #3) The linearization of f at $x = -1$

ie, line tangent to f @ x = -1 (+.5)

Looking for $y - y_1 = m(x - x_1)$ (+.5)

m = slope of line tangent (+.5) to f @ x=-1

$= f'(-1)$ (+.5)

$= -2$

passes thru $(-1, f(-1)) = (-1, -3)$ (+.5)

So $y - -3 = -2(x - -1)$] plug in (+.5)

$\gg$ $y + 3 = -2(x+1)$

or $y = -2x - 5$

4. Find each of the following.

[3] (WebHW8 #18)

$(\log_2 \sqrt{x^2-6})'$

(+.5) decompose one chain +.5 der $\log_2(u)$

$\left[\log_2 (x^2-6)^{1/2}\right]'$ log prop (+1)

$= \left[\frac{1}{2}\log_2(x^2-6)\right]'$

$= \frac{1}{2}\left[\log_2(x^2-6)\right]'$

Chain $f(u)=\log_2(u)$, $g(x)=x^2-6$ (+1.5)

$= \frac{1}{2} f'(x^2-6)\cdot 2x$

$= \frac{1}{2}\frac{1}{(x^2-6)(\ln 2)}\cdot 2x$

OR (+1) Chain $f(u)=\log_2(u)$, $g(x)=\sqrt{x^2-6}$

(+1) need Chain for g: $f(u)=\sqrt{u}$, $g(x)=x^2-6$

put back together (+.5)

$\frac{1}{(\ln 2)\sqrt{x^2-6}}\cdot\frac{1}{2}(x^2-6)^{-1/2}\cdot 2x$

[4] (Spring15Exam2 #4)

$$\frac{d}{dy}\left(\frac{\sin(y)+y\cos(y)}{\cos(y)}\right)$$

$\frac{d}{dy}\left(\frac{\sin y}{\cos y}+\frac{y\cos y}{\cos y}\right)$

$=\frac{d}{dy}(\tan y + y)$

$=\sec^2 y + 1$ (+.1) (+1)

Simplify (+1.5)

notation (+.5)

OR Quotient Rule (+1)

$\frac{\cos y[\sin y + y\cos y]' - (\sin y + y\cos y)(\cos y)'}{(\cos y)^2}$

product rule (+1)

$\frac{\cos y(\cos y + y(-\sin y) + (\cos y)1) + (\sin y + y\cos y)\sin y}{\cos^2 y}$ (+.5) (+.5)

wrt y (+.5)

notation (+.5)

5. Each of the following derivatives is *wrong*. Please explain why and provide the correct derivative.

(a) [3] (WebHW8 #17) $\frac{d}{dx}(2^x) = x2^{x-1}$

(+1) x is an exponent so we can't use the power rule here?

$\frac{d}{dx}(2^x) = (\ln 2)\,2^x$ (+1) (+1)

(b) [2] (Quiz3 #2) $\frac{d}{dx}\left(\frac{\pi x}{2}\right) = \frac{(2)(\pi)-(\pi x)(\cancel{1})}{4}$

(+1) $\frac{d}{dx}(2)=0$ so the second term in the numerator is zero?

$\frac{d}{dx}\left(\frac{\pi x}{2}\right) = \frac{d}{dx}\left(\frac{\pi}{2}\cdot x\right) = \frac{\pi}{2}$ (+1) (+1)

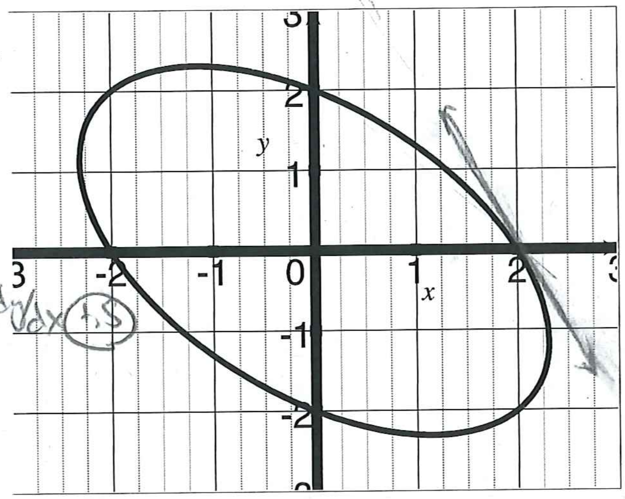
or

$= \frac{2\frac{d}{dx}(\pi x) - (\pi x)\frac{d}{dx}(2)}{2^2}$ quotient (+1)

$= \frac{2\cdot\pi - \cancel{\pi x\cdot 0}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$ got it (+1)

-4

6. The graph of the equation $x^2 + xy + y^2 = 4$ is shown to the right.



d/dx

(a) [4] (WebHW9 #1) Find $\frac{dy}{dx}$ as a function of x and y.

product rule

$2x + x\frac{dy}{dx} + y(1) + 2y\frac{dy}{dx} = 0$ (+.5) (+1.5) (+1) (+.5)

solve for dy/dx (+.5)

$\Rightarrow \frac{dy}{dx}x + 2y\frac{dy}{dx} = -y - 2x$

$\Rightarrow \frac{dy}{dx} = \frac{-y-2x}{x+2y}$

(b) [3] (§3.5 #46) Find the equation of the line tangent to the curve at $(2, 0)$.

Looking for $y - y_1 = m(x - x_1)$ (+.5)

m = slope of line tang to curve at x=2 (+.5)

$= \left.\frac{dy}{dx}\right|_{x=2}$ (+.5)

$= \frac{-0-2(2)}{2+2(0)} = -2$ (+.5)

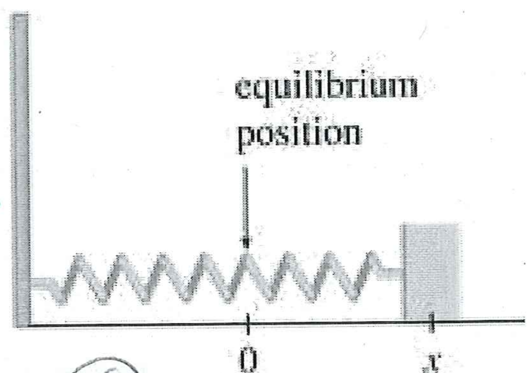
passes thru (2,0) so

$y - 0 = -2(x-2)$ plug in (+1)

$y = -2x + 4$

7. (Spring15Exam2 #6) A mass on a spring vibrates horizontally on a smooth level surface with the equation $x(t) = 10\cos(2t)$ where t is in seconds and x is in centimeters.

Note: x should not be a function of itself..



(a) [2] Find the velocity of the spring at time t.

velocity $= \frac{d}{dt}(\text{position}) = \frac{dx}{dt}$

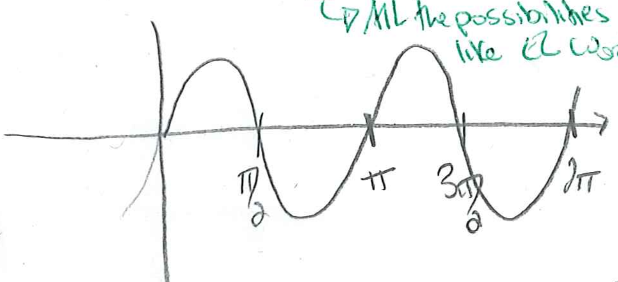
$= \frac{d}{dt}(10\cos(2t))$ (+.5) Chain (+.5)

$= 10(-\sin(2t)) \cdot 2 = -20\sin(2t)$ (+.5) (+.5)

(b) [3] When is the spring at rest?

All the possibilities like $\mathbb{Z}$ Word Prob?

ie when is velocity $= 0$ (+1)



$0 = -20\sin(2t)$

$0 = \sin(2t)$

$0 + k \cdot 2\pi = 2t$ or $\pi + 2\pi k = 2t$

$\Rightarrow 0 + k\pi = t$ or $\frac{\pi}{2} + \pi k = t$

where k is an integer

every integer multiple of $\frac{\pi}{2}$ (+1) (+1)

4

find 1 sol +1.5

8. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (§3.7 #24) A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 2 m higher then the bow of the boat. The rope is being pulled in at a rate of 1.5m/s.

i. [3] Find an equation relating the speed of the boat to other variables.

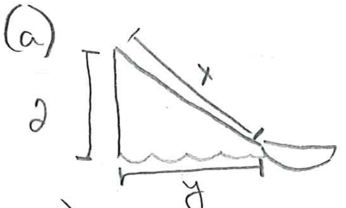
ii. [2] How fast is the boat approaching the dock when it is 3 m from the dock?

(b) (§3.7 Example#4) An airplane is flying on a flight path that will take it directly over a radar tracking station. Thus the airplane is still flying *towards* the radar tracking station. It is discovered that this distance from the plane to the radar tracking station is decreasing at a rate of 400 miles per hour. The altitude of the plane is 6mi

i. [3] Find an equation relating the *speed* of the airplane to other variables.

ii. [2] How fast is the airplane traveling when the airplane is 10 miles from the radar tracking station?

(a)



Start (+.5)

$\frac{dx}{dt} = -1.5\,m/s$

picture (+.5)

variables (+.5)

i) want a function for $\frac{dy}{dt}$ (+.5)

note $2^2 + y^2 = x^2$ (+.5)

$\Rightarrow 0 + 2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

$\Rightarrow \frac{dy}{dt} = \frac{x}{y}\cdot\frac{dx}{dt}$

or $\frac{dy}{dt} = \frac{-1.5x}{y}$ (+.5)

ii) want $\left.\frac{dy}{dt}\right|_{y=3} = \frac{-1.5x}{3}$ (+.5)

we need to know x when y=3 (+.5)

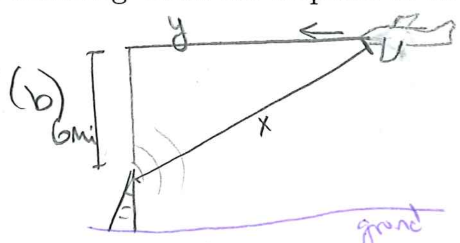
$2^2 + 3^2 = x^2 \Rightarrow 13 = x^2$ (+.5)

$\Rightarrow x = \sqrt{13}$

So $\left.\frac{dy}{dt}\right|_{y=3} = \frac{-1.5\cdot\sqrt{13}}{3}$ (+.5)

$\approx -1.803\,m/s$

(b)



Start (+.5)

$\frac{dx}{dt} = -400\,mi/hr$

picture (+.5)

variables (+.5)

i) want a function for $\frac{dy}{dt}$ (+.5)

note $6^2 + y^2 = x^2$ (+.5)

$\Rightarrow 0 + 2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$

or $\frac{dy}{dt} = \frac{-400x}{y}$ (+.5)

ii) want $\left.\frac{dy}{dt}\right|_{x=10} = \frac{-400(10)}{y}$ (+.5)

we need to know y when x=10 (+.5)

$6^2 + y^2 = 10^2 \Rightarrow y^2 = 64$

$\Rightarrow y = 8$ (+.5)

So $\left.\frac{dy}{dt}\right|_{x=10} = \frac{-400(10)}{8}$ (+.5)

$\approx -500\,mi/hr$