tcore 122: Practice Final

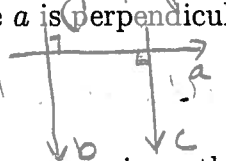
You may find the following table helpful if you did not bring a calculator for the final.

x	22.5°	30°	45°	60°	67.5°	135°
$\cos(x)$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$-\frac{\sqrt{2}}{2}$
$\sin(x)$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2}}{2}$
$\tan(x)$	$\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$	-1

1. TRUE/FALSE: If the statement is true, circle T. If the statement is false, circle F *and* explain briefly why the statement is false.

   T (F) Paper was invented in Japan.
   Paper was invented in China.
   Optional details: Ts'ai Lun presented paper to Emperior Han Hsu Ti in 105

   T (F) 'Origami' translates to cutting & pasting paper.
   'Ori' came from a word meaning 'to fold' and
   'Kami' came from a word meaning 'paper'

   T (F) If line a is perpendicular to line b, and line a is perpendicular to line c, then line b is always perpendicular to line c.
   Consider the following:
   

   note: b is // to c

   (T) F The shortest point between two points on a sphere is on the arc of a great circle.
   If you connect string between the 2 points on the sphere & then make the string as tight as possible - you'll start to see the great arc

   (T) F A triangle on a sphere may have three right angles.
   If we connect the 'northpole' to two points (A+B) on the 'equator'
   

2. What does the word "Origami" translate to in English?

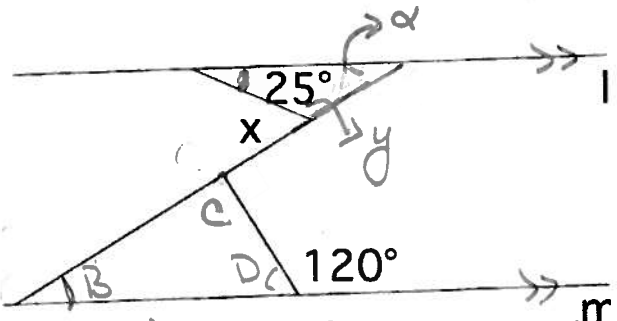
   Origami translates to "to fold paper" where 'ori' comes from a word meaning 'to fold' and 'kami' means paper.

3. Define science *clearly.* (In a way most modern scientists would agree with! Consider referring to David Deutsch's TED talk.)

   There is no perfect answer for this question but the answer should contain a subset of the following characteristics:

   1) Science seeks to explain behaviors using falsifiable and hard to vary hypothesis
      ↳ hypothesis predicts behavior that can be tested
      ↳ all details are necessary / critical to the hypoth
   2) Science results & conclusions should be repeatable (both as experiments and by any scientists i.e. not dogma)

4. The two lines l and m are parallel. Find the measure of angle x. Make sure that your *reasoning* is easy to follow. Note, this diagram is not drawn to scale.



1) Since $l \parallel m$ we know alternating interior angles have the same measure, Thus α and B are the same measure.

Plan: Find the measure of α & then use the fact that the sum of angles in a $\triangle$ (on a flat surface) adds up to 180° to find the measure of y. Then since $x + y = 180°$ I can solve for x.

2) Notice $D + 120°$ is a straight line
$\Rightarrow D = (180 - 120)° = 60°$

3) Notice $C + 90°$ is a straight line
$\Rightarrow C = (180 - 90)° = 90°$

4) Since B, C and D are in a $\triangle$ we know $B = 180 - 90 - 60 = 30°$
Thus, by (1) $\alpha = 30°$. Following the Plan we see $y = 180 - 30 - 25 = 125°$

Since $x + y = 180$ we know
$x = 180 - 125 = \boxed{55°}$

Refer to the diagram on the right when answering the remaining questions on this page. The crease pattern was the result of folding the 'optimum Yoshizawa Split'. Assume the length of the original square is one.

5. Name the intersection of the line $\overline{AB}$ and $\overline{EG}$

C

6. Name two angles with a vertex at D.

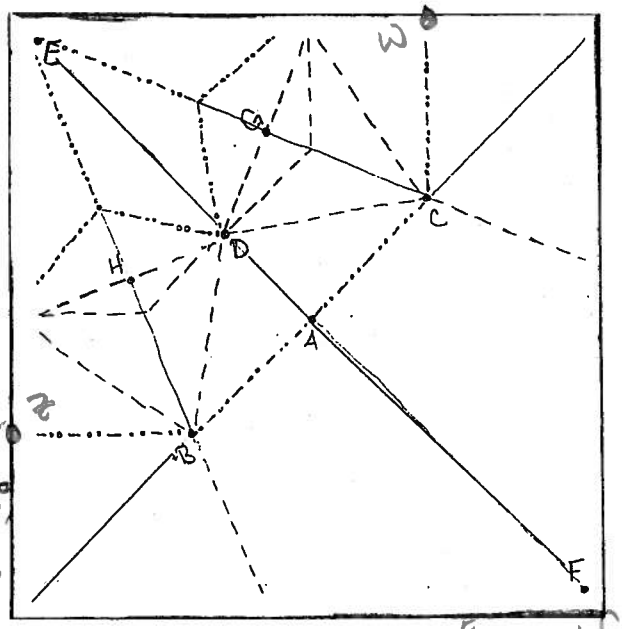
$\angle BDA$, $\angle ADC$, $\angle BDC$ $\angle EDG$ etc

7. Name a pair of lines that are perpendicular (You do *not* need to justify your answer.)

$\overline{EF} \perp \overline{AC}$ $\overline{EF} \perp \overline{BC}$

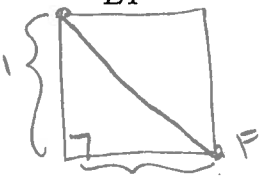
8. Determine the measure of $\angle BEA$ (You do *not* need to justify your answer.)

The creases made at E where reflected angle bisectors.
Since $\angle ZEW = 90°$ we know $\angle ZEA = 45°$
The next fold again bisects the angle
so $\angle BEA = 22.5°$



9. Find the length of:

$\overline{EF}$



$1^2 + 1^2 = (\overline{EF})^2$
$\Rightarrow 2 = \overline{EF}^2$ so $\boxed{\sqrt{2}}$

$\overline{EA}$

since we are working with a square $\overline{BC}$ bisects $\overline{EF}$
so $\boxed{\frac{\sqrt{2}/2}{2}}$

$\overline{EB}$.

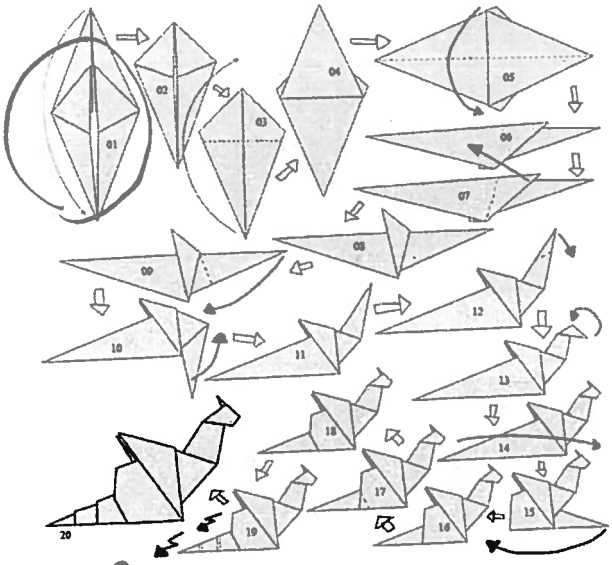
22.5° from #3

Have adj. and hyp
SohCahToa
$\cos 22.5° = \frac{(\sqrt{2}/2)}{\overline{EB}}$

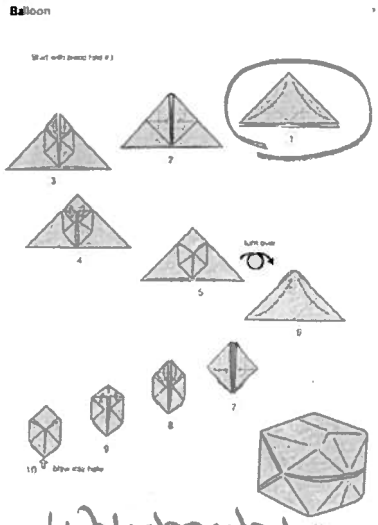
$\Rightarrow \overline{EB} = \frac{(\sqrt{2}/2)}{\cos 22.5°} = \frac{(\sqrt{2}/2)}{\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)} = \boxed{\frac{\sqrt{2}}{\sqrt{2+\sqrt{2}}}}$

← from #9

10. Identify the base used in each of the origami designs shown below.

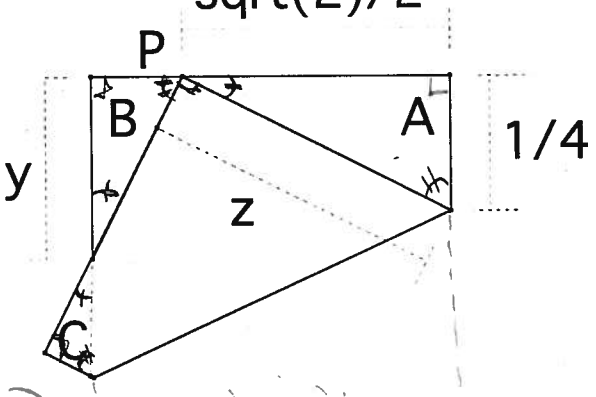


fish base



Waterbomb base

11. Suppose you folded a patty paper so that the measurements shown in the diagram to the right were satisfied (where the length of the original patty paper has length one).



(If you are more comfortable with decimals you may use the approximation: $\frac{\sqrt{2}}{2} \approx .707$.)

(a) Find the length of z.

By construction we know $1/4 + z = 1$ so $z = 3/4$

or

Since Δ is a right triangle

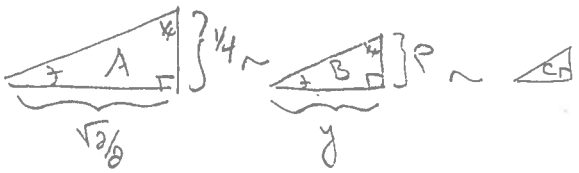
$(1/4)^2 + (\sqrt{2}/2)^2 = z^2$

$\Rightarrow 1/16 + 2/4 = 1/16 + 8/16 = 9/16 = z^2$

$\Rightarrow z = \sqrt{9/16} = 3/4$

(b) Find the length of y.

Note $\Delta A \sim \Delta B \sim \Delta C$



Since we have similar triangles we know $\frac{y}{\sqrt{2}/2} = \frac{P}{1/4}$

or $1/4\, y = (\sqrt{2}/2)P$ or $y = 4(\sqrt{2}/2)P = (2\sqrt{2})P$.

Since the length of P and $\sqrt{2}/2$ make the top of the square

$P + \sqrt{2}/2 = 1 \Rightarrow P = 1 - \sqrt{2}/2$

Thus $y = 2\sqrt{2}(P) = 2\sqrt{2}(1 - \sqrt{2}/2)$ or $2\sqrt{2}\left(\frac{2-\sqrt{2}}{2}\right)$ or $\sqrt{2}(2-\sqrt{2})$

my of these answers are OK?

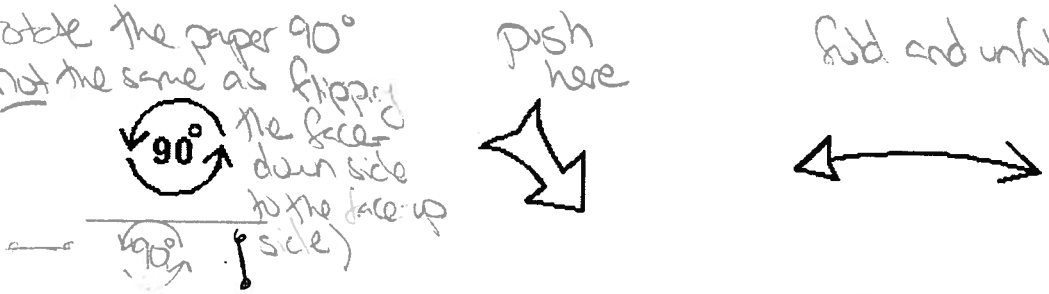
12. Write the converse to the following conditional statement. Determine the truth value of both conditional statements.

"If an object is hot, then the object is on fire. False

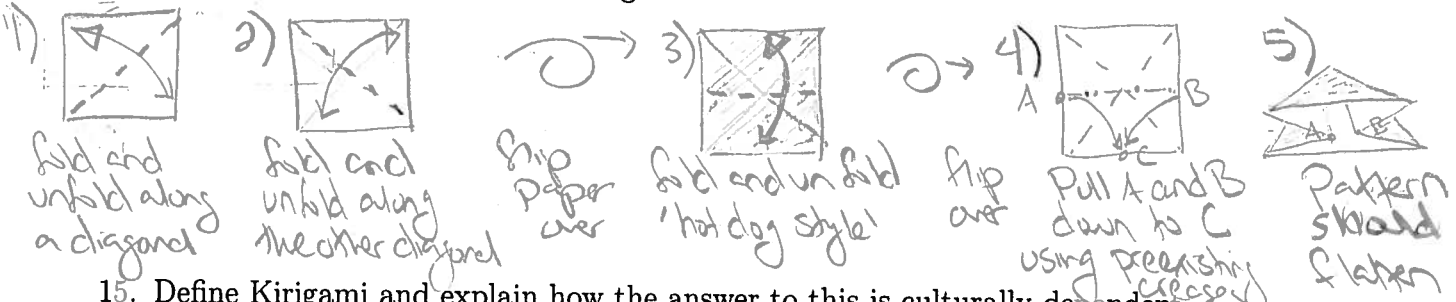
If the object is on fire, then the object is hot. True

Which highlights the very important fact that the converse of a conditional statement is not the same as the original? Be watchful of this during political debates & in the media!

13. Identify what the origami symbols mean below.



14. Write down instructions for folding the bomb base.



15. Define Kirigami and explain how the answer to this is culturally dependent.

In Japan: Kirigami is art and designs made from cutting paper and often lays flat.

In the U.S. Kirigami was introduced by a man combining Japanese Kirigami and origami so Americans think of Kirigami as origami 'with cuts'.

16. Describe two of the people interviewed for the "Between the Folds" movie and identify an opinion that they share or that they disagree over.

There are so many correct answers here! Note: you do not need to name two people, but could describe them instead.

17. Identify the thesis from Lockhart's *A Mathematician's Lament* and explain the mathematical example he refers to in the first few pages when making this point.

Mathematics is an art in just the same way that painting or music is. The mathematical example he refers to in the first few pages is the discovery of the area formula for a triangle. Given any triangle 

Lockhart drew a box around it & then with the addition of one line we could see the area formula?

18. Describe four characteristics and guidelines for technical writing as described from Johnson-Sheehan's Technical Communication Today sections assigned.

Focuses on: action with words & images
achieving a specific purpose
anticipates the needs of readers
Tends to give step-by-step pattern
where the steps are sequenced (order matters)

Guidelines: use command voice
say only one action per step
be concise (but still precise)
number the steps
refer to graphics

19. Consider Robert Lang's TED talk. How does Lang defined origami? Briefly explain the mathematical approach Lang takes to creating origami patterns.

"One sheet of paper, no cuts."
Lang chooses an object, draws a stick figure of the object (where limbs are reduced to lines), then reserves/arranges circles on paper (since each circle will give rise to a limb in his stick figure)

20. Write down an argument supporting the position "math is science".

1) Math makes use of the scientific method (requires experimentation & repeatability)
2) Math uses logic as a tool when confronted with problems
3) Both are trying to solve problems in a precise (hard to vary) way

Note: there are many more arguments to make?

21. Write down an argument against the position "math is science".

1) Math is not 'grounded' in the real world. For example, there are no actual 2 dimensional lines in reality
2) Math theories are actually provable (as opposed to relying only on repeated experiments)

Note: there are many more arguments to make?

22. How did origami spread across the globe and how was Yoshizawa's involved?

We think origami began shortly after paper was invented (in 105 AD). This may have happened in Japan or China. After a few centuries it traveled to the Arabic world (9th century), and made it to Spain (13th century). Yoshizawa was 'modernized' and highly popularized Origami by 1) creating a language to help communication, 2) creating new patterns that appealed to people 3) traveling & dedicating his life to sharing Origami with others

23. Describe Gödel's role in the history of mayhematics?

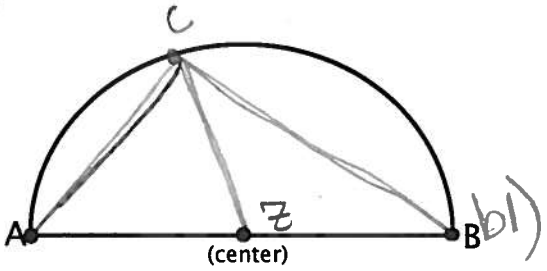
Gödel proved that mathematics as a system can't be complete (know exactly which statements are true and which are false)

and consistent (once known as true, the statement remains true)

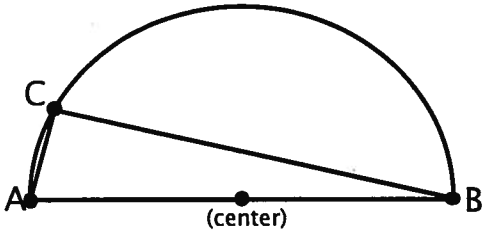
Prior to this mathematicians were trying to 'fill in the gaps' and identify exactly what axioms where necessary for this.

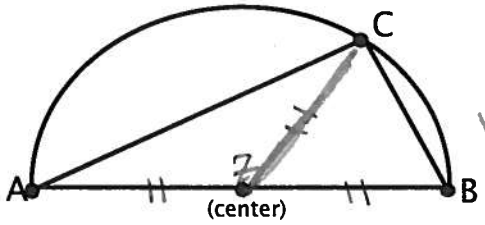
23. Mark the ends of the diameter on a semicircle as A and B as shown on the right.

Consider the triangles show below that have $\overline{AB}$ as one side, and a third vertex C on the semicircle. Notice that both angles $\angle ACB$ measure 90°!



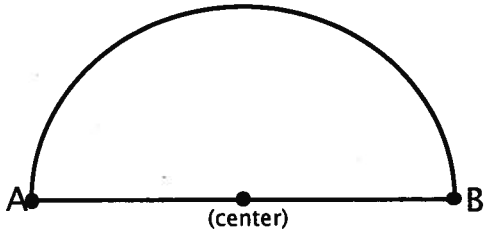
b1)

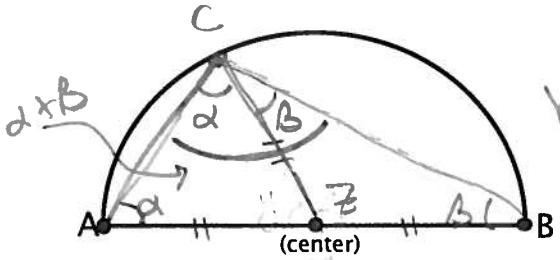




b2)

Claim: In general, if a triangle has all of its vertices on a circle and one of its sides is also a diameter, the resulting triangle is a right triangle.





b3)

(a) *Explain* a process to use patty paper to verify the above claim.

(b) Find a way to *justify* the above claim without using a preexisting 90° corner.

(a) Given a semicircle
1) Identify any point along the arc, call it C. (besides the endpoints of the arc)
2) Use the sides of the patty paper to trace lines between the end points of the arc (A and B) and the point C.
3) Compare the corner of a patty paper (known to be 90°) with ∠ACB by lining up the edges of the patty paper with line AC and line BC.
4) If the patty paper's corner matches we know we have a 90° △.

(b) 1) Given a triangle of the form described above, add a line from the center to C. Let the center of the circle be denoted Z. (see figure above)
2) Use $\overline{ZB} = \overline{ZA} = \overline{ZC}$ since these are all the radius of the same circle (see figure above)
3) So △AZC and △BZC are isosceles △'s, implying each has a pair of equal angles (shown above)
4) Since the sum of angles in a △ is 180°
Then △ABC implies
$\alpha + (\alpha+\beta) + \beta = 180°$
$\Rightarrow 2(\alpha+\beta) = 180° \Rightarrow \alpha+\beta = 90°$