# Contents

things I'd like to add:

1. pictures of the 4 classic bases and nonstandard bases
2. directions for the frog base
3. reading on how to write directions well
4. philosophy/history reading
5. Lockhart's Mathematician's Lament (.pdf only shows first page)

note also that the worksheets need to be edited

# 1. Course Information

## 1.1 Syllabus

MW 1:30-3:35 ADMC BB 106 — Lecturer: Ruth Vanderpool
SLN 18612 C — Phone: 253-692-4310
Office Hours: MW 11:30-12:30 — Office: GWP 430
e-mail: rvanderp@u.washington.edu
Webpage: `http://faculty.washington.edu/rvanderp/index.html`

Core: The Core program consists of a coordinated series of courses that represent the various disciplines in the university. This course, along with the others in your cohort, fulfills one of the university's general education requirements in each of the areas of knowledge plus composition. The courses are designed to both support and challenge you to develop the critical thinking, writing, research, and analytical skills you'll need at UWT while introducing you to relevant topics in the social sciences, humanities, and sciences.

Course Description & Student Learning Objectives: This course will investigate geometry and art through paper folding and origami techniques. We will use this experience to try to answer "What tools or rules should we use to discover geometry?"

Content *and* general skills will be developed.
Students will be able to:

- define origami & provide some history of its development.
- solve open-ended geometric problems involving lines, triangles, and circles.
- express geometric ideas to others through precise writing or speech.
- use geometry to aid in their own origami project.

More generally, by the end of this course students will have the skills to:

- express ideas clearly in writing and speaking in order to synthesize and evaluate information.
- identify, analyze, and summarize/represent the key elements of a text.
- self-assess personal strengths and how they help overcome weaknesses.
- approach a complex issue by breaking it down into manageable pieces.
- make connections among assignments and readings to develop a sense of the "big picture".
- collect, evaluate, and analyze information to solve problems or answer questions.

## Required Items:

- Lang, Robert. *Origami Design Secrets: Mathematical Methods for an Ancient Art* ISBN: 1-568-81194-2
- Wheater, Carolyn. *Practice Makes Perfect Geometry* ISBN: 978-0-07-163814-2
- Okasha, Samir. *Philosophy of Science; A Very Short Introduction* ISBN: 978-0-19-280283-5

## Important Dates:

| | | | |
|---|---|---|---|
| 4/25 | Midterm | 4/1 | Last day to alter your schedule with no fees |
| 6/6 | Final @ 1:30pm | 5/15 | Last day to change your grading option |

## Journals:

You will keep a journal for this class. Journal assignments and questions will be assigned during each class and you are expected to complete the journal work by the next session. You are welcome to put additional thoughts and work in the journal, but keep the assignments in order and at the start of each new journal assignment write the date it was assigned. Bring the journals to class everyday so that they can help inform our discussions.

I will regularly collect the journals every Monday at the start of class and return them before class ends. When collected, the *entire* journal must be turned in and not just the newest entries. *The journal must be kept separate from any course notes* in either its own binder or bound book. The material will never formally graded, but the entries are marked for completion.

## Homework:

Homework will be assigned everyday and collected regularly. Ten minutes will be set aside at the start of every class to address homework questions. Assignments are due by 1:30pm (the start of class) the day they are due. Once I have started marking an assignment, I no longer accept late work.

The homework will be largely comprised of worksheets write-ups and origami work. Given the Student Learning Objectives described above, your work will be largely graded by the *process*, *explanation*, in addition to the *correctness*. Because of this, you will have to spend more time on your homework than you would on a more traditional, computation-based math course.

## Quizzes:

A quiz is given every week at the instructor's discretion. Generally you will be given 30 minutes following the homework question period at the *start* of class every Monday. The quizzes will focus on the material covered in the previous week. No notes or books may be used, but calculators and patty paper are allowed. No make up quizzes, unless previously arranged, will be given, but I drop the lowest scoring quiz to give you some flexibility.

## Papers:

There will be two 2-page papers due this term:

1. A research paper due 5/11 during the seventh week of classes
2. a persuasive paper due 6/1 during the tenth week of classes

More details, including the grading rubrics, will be provided in week five and eight respectively.

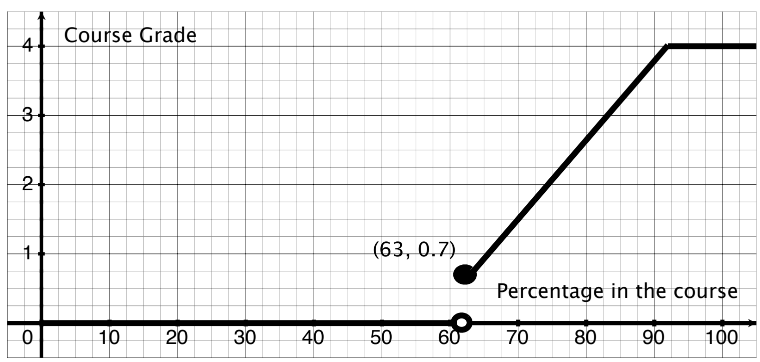
## Origami Projects:

There will be two origami projects due this term:

1. A group project in which you and two others find, present, and help the class fold an origami design. The detailed instructions will be handed out in week two but you may want to start searching for a pattern that you find enjoyable and can teach others.
2. An individual project in which you create your own origami piece (or significantly modify a preexisting design) and provide the directions. A display model will be due before the student showcase and the detailed instructions will be due the day of the final. Work directly related to this project will start in week seven and more details will be provided in week six.

## Grades: The following weights will be used to calculate your grade.

| | |
|---|---|
| Journals | 15% |
| Homework | 15% |
| Quizzes | 15% |
| Origami Projects | 10% |
| Papers | 10% |
| Midterm | 15% |
| Final | 20% |




The above grade assignment is based off the University of Washington, Tacoma's grading scale posted at `http://www.tacoma.washington.edu/enrollmentservices/grading.cfm`.

## Outside Resources:

Come visit me if you have questions! If you are unable to attend my posted office hours but would like to meet, please let me know. I am willing to try and work with your schedule. Also remember that you are not alone in this class and your peers are a valuable (and often underutilized) resource.

The Teaching & Learning Center (TLC) is offering a number of additional instructional services that can help with writing *and* quantitative material. Complete information and hours are posted at: `http://www.tacoma.washington.edu/tlc/`. Also, don't forget about all the resources you learned about in the Fall during Core Lab!

## Notes:

- I do *not* check my email after 4pm. Any questions sent to my email after 4pm may not receive a response until the next morning. The University's e-mail policy is posted at: `http://www.tacoma.washington.edu/policies_procedures/E-mail_Policy.pdf`
- Electronic devices should not be used during class. Activities that are non-relevant to the course, such as checking/sending email, playing games, and surfing the web, are considered disruptive activities when class is in session.
- Don't cheat and don't plagiarize. To plagiarize is to appropriate and to pass off, as one's own ideas, writing or works of another. Ignorance of proper documentation procedures is the usual cause of plagiarism. This ignorance does not excuse the act. Students are responsible for learning how and when to document and attribute resources used in preparing a written or oral presentation. For more information, please refer to the Academic Honesty: Cheating and Plagiarism document prepared by the Committee on Academic Conduct in the College of Arts and Sciences, UW Seattle: `http://depts.washington.edu/grading/issue1/honesty.htm`
- The University of Washington Tacoma is committed to making physical facilities and instructional programs accessible to students with disabilities. Disability Support Services (DSS) functions as the focal point for coordination of services for students with disabilities. In compliance with Title II of the Americans with Disabilities Act, any enrolled student at UW Tacoma who has an appropriately documented physical, emotional, or mental disability that "substantially limits one or more major life activities [including walking, seeing, hearing, speaking, breathing, learning, and working]," is eligible for services from DSS. If you are wondering if you may be eligible for accommodations on our campus, please contact the DSS reception desk at 692-4522. A complete description of services provided is posted at: `http://www.tacoma.washington.edu/studentaffairs/SHW/dss_about.cfm`
- Complete safety information and emergency procedures is available at `http://www.tacoma.washington.edu/security`.
  The highlights are as follows:
  - In case of fire, take your valuables, leave the building, and report to the parking lot next to the Library and across the street from the Mattress factory. Plan to return to class once the alarm has stopped.
  - In case of an earthquake, DROP, COVER, and HOLD. Once the shaking stops, take your valuables, leave the building, and report to the parking lot next to the Library and across the street from the Mattress factory. Do not plan to return for the rest of the day.

  In both of the above cases, do not return until you have received an all clear from somebody "official," the web, or email.
- The Counseling Center offers short-term, problem-focused counseling to UW Tacoma students who may feel overwhelmed by the responsibilities of college, work, family, and relationships. Counselors are available to help students cope with stresses and personal issues that may interfere with their ability to perform in school. The service is provided confidentially and without additional charge to currently enrolled undergraduate and graduate students. To schedule an appointment, please call 692-4522 or stop by the Student Counseling Center (SCC), located in MAT 253.
- While I have attempted to make this syllabus as complete as possible, adjustments will be made throughout the course. Announcements will be made during class and it is the responsibility of the student to keep updated if class is missed.

## 1.2 Schedule

| | | |
|---|---|---|
| Week 1 | 28:Origami: Butterfly<br>Class Orientation & TED talk<br>Math: Lines & Angles<br>Wks: Patty 1 | 30: Discussion: What is science?<br>Technical Writing Example<br>Math: Logic<br>Wks: Patty 2<br>Due: Patty Wks 1 |
| Week 2 | 4: Quiz<br>Technical Writing Example<br>Math: $\parallel$ & $\perp$ Lines<br>Wks: Patty 3<br>Due: Journals & Patty Wks 2 | 6: Origami Presentation Specifications<br>Discussion: Math anxiety<br>Math: $\triangle$ Properties<br>Wks: Patty 4<br>Due: Patty Wks 3 |
| Week 3 | 11: Quiz<br>Two Origami Presentations<br>History: Ancient Mathematicians<br>Due: Journals & Presentations | 13: Three Origami Presentations<br>Peer Eval: Directions<br><br>Due: Elephant Head Directions &Patty Wks 4 |
| Week 4 | 18: Quiz<br>Last Origami Presentation<br>Origami: Bases<br>Wks: Patty 5<br>Due: Journals | 20:<br>Origami: identify bases<br>History: Origami<br>Between the Folds Movie Wks: Patty 6<br>Due: Blintzed Bird Base Directions, Ch4A Origami |
| Week 5 | 25: Midterm<br><br>handout: Paper 1 Specifications | 27: Discuss Lockhart reading<br>Math: Pythagorean Theorem.<br>Wks: Patty 6<br>Due: Patty Wks 5 |
| Week 6 | 2: Quiz<br>Math: trigonometry<br>Origami: Yoshizawa split<br>Due: Journals, Ch4B Origami, & 2 sources | 4: History:Math up to $18^{\text{th}}$ Century<br>Math: Yoshizawa split<br>Paper 1 Drafts work<br>Due: Patty Wks 6, Paper 1 Draft |
| Week 7 | 9: Quiz<br>Wks: Grade Computation<br>Discussion: Course<br>Due: Journals & Ch5 Origami | 11: Origami Project Specifications<br>History: Mathematics up to Gödel<br>Math: Spherical Geometry<br>Due: Paper 1 |
| Week 8 | 16: Quiz<br>Wks: Patty 7<br>Origami: Pattern Grafting<br>Due: Journals | 18: Origami: "Show & Tell"<br>History: Gödel & NPR Story<br>Origami: Pattern Grafting |
| Week 9 | 23: Quiz<br>Origami Directions work<br>Due: Journals , Project Directions, & Wks 7 | 25: NESSI<br>Paper 2 Draft work<br>Due: Paper 2 Draft |
| Week 10 | 30: No Class<br>Memorial Day | 1: Course Evaluations<br>Origami Project Work Day<br>Due: Paper 2 & Display Model |

# 2. Review

## 2.1 Algebra Review

I generally assume proficiency in the skills listed below.

**Basic Simplification**
Apply order of operations to numerical and algebraic expressions; recall the rules for exponents.

Ex 1. Perform the operations: $\frac{3}{5} + 2$

Solution:
$\frac{3}{5} + \frac{2}{1} = \frac{3}{5} + \frac{2}{1} \cdot \frac{5}{5} = \frac{3}{5} + \frac{10}{5} = \frac{13}{5}$
(Another answer is 2.4.)

Ex 2. Simplify: $\sqrt{5} \cdot \sqrt{5} + 2$

Solution:
$\sqrt{5} \cdot \sqrt{5} + 2 = 5 + 2 = 7$

Ex 3. Simplify: $\sqrt{\frac{4}{25}}$

Solution:
$\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$

Ex 4. Simplify: $-3^2 + \left(\frac{3}{2}\right)^2 + (-2)^3$.

Solution:
$-3^2 + \left(\frac{3}{2}\right)^2 + (-2)^3 = -9 + \frac{9}{4} + -8 = -17 + \frac{9}{4} = \frac{-68}{4} + \frac{9}{4} = \frac{-59}{4}$
(Another answer is -14.75. $-14\frac{3}{4}$ is also technically correct, but you should avoid mixed fractions because of potential confusion: Taken out of context, $-14\frac{3}{4}$ could mean *either* $-(14+\frac{3}{4})$ *or* $(-14)\cdot\frac{3}{4}$.)

**Algebraic Manipulation**
Add, subtract, multiply, and divide algebraic expressions; combine algebraic terms that are alike; apply the distributive property to algebraic expressions (the term "FOIL" *only* refers to distribution between two binomials; ask me to show you why FOIL works if you don't remember).

Ex 5 Solve for $x$: $\frac{3}{x} = \frac{1}{6}$.

Solution:
Multiply both sides by $x$: $x \cdot \frac{3}{x} = \frac{1}{6} \cdot x \Rightarrow 3 = \frac{x}{6}$.
Multiply both sides by 6: $6 \cdot 3 = \frac{x}{6} \cdot 6 \Rightarrow 18 = x$.
Both of these steps done at the same time is often called "cross multiplication".

Ex 6 Simplify by combining like terms: $(9x^2 + 4xy - 7y^2) - (5xy - 6x^2 - 1)$.

Solution:

$$(9x^2 + 4xy - 7y^2) - (5xy - 6x^2 - 1) = 9x^2 + 4xy - 7y^2 - 5xy + 6x^2 + 1 = 15x^2 - xy - 7y^2 + 1$$

Ex 7 Expand and simplify completely: $3(4v - 2a)^2$.

Solution:

$$\begin{aligned} 3(4v - 2a)^2 = 3(4v - 2a)\cdot(4v - 2a) &= 3[(4v)(4v) - (4v)(2a) - (2a)(4v) + (2a)(2a)] \\ &= 3[16v^2 - 16av + 4a^2] = 48v^2 - 48av + 12a^2 \end{aligned}$$

**Rational Expressions**

Simplify fractions by finding factors in common; add/subtract fractions by first rewriting with the least common denominator; multiply/divide rational expressions; simplify complex fractions.

Ex 9 Reduce to lowest terms: $\frac{10x - 20}{2x^2 - 8}$.

Solution:

$$\frac{10x - 20}{2x^2 - 8} = \frac{10(x-2)}{2(x^2 - 4)} = \frac{10(x-2)}{2(x+2)(x-2)} = \frac{2(x-2)}{2(x-2)} \cdot \frac{5}{x+2} = \frac{5}{x+2}$$

Ex 8 Perform the indicated operations and simplify: $\frac{2}{3x+2} - \frac{1}{3x+1}$.

Solution:

$$\begin{aligned} \frac{2}{3x+2} - \frac{1}{3x+1} &= \frac{3x+1}{3x+1} \cdot \frac{2}{3x+2} - \frac{1}{3x+1} \cdot \frac{3x+2}{3x+2} \\ &= \frac{6x+2}{(3x+1)(3x+2)} - \frac{3x+2}{(3x+1)(3x+2)} = \frac{3x}{(3x+1)(3x+2)} \end{aligned}$$

(There is no further simplification here, $3x$ has *no* factors in common with $3x + 1$ or $3x + 2$.)

Ex 9 Perform the indicated operations and simplify: $\frac{2z+6}{12z} \div \frac{z^2 - 9}{9z^3 + 18z^2}$.

Solution:

$$\begin{aligned} \frac{2z+6}{12z} \div \frac{z^2-9}{9z^3+18z^2} = \frac{2z+6}{12z} \cdot \frac{9z^3+18z^2}{z^2-9} &= \frac{2(z+3)}{12z} \frac{9z^2(z+2)}{(z+3)(z-3)} \\ &= \frac{2\cdot 3\cdot z(z+3)}{2\cdot 3\cdot z(z+3)} \cdot \frac{3z(z+2)}{2(z-3)} = \frac{3z(z+2)}{2(z-3)} \end{aligned}$$

**Solving Equations**

Solve linear equations; or solve for two unknowns when given two equations.

Ex 10 Solve for $m$: $3(m + 4) + 2m = 4 - 3m$.

Solution

$$3(m+4) + 2m = 4 - 3m \Rightarrow 3m + 12 + 2m = 4 - 3m \Rightarrow 8m = -8 \Rightarrow m = -1$$

Ex 11 Solve for $q$ when $q + 7 = 9p$ and $\frac{2}{p} = 2q$.

# 3. Readings

## 3.1 Quotes

Geometry, throughout the $17^{th}$ and $18^{th}$ centuries, remained, in the war against empiricism, an impregnable fortress of the idealists. Those who held ... that certain knowledge, independent of experience, was possible about the real world, had only to point to Geometry: none but a madman, they said would throw doubt on its validity, and none but a fool would deny its objective reference.

-Bertrand Russell

I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world.

-G.H. Hardy

From this proposition it will follow, when arithmetical addition has been defined, that

$$1 + 1 = 2.$$

-Principia Mathematica
A.N. Whitehead & B. Russell

Euclidean Axioms:

1. We can draw a straight line from any point to any other point.
2. We can extend a finite straight line continuously into a straight line.
3. We can completely describe a circle by specifying the center and distance [radius].
4. All right angles are equal to one another.
5. Through a point not on a given straight line, at most one line can be drawn that never meets the given line.

-Elements
Euclid

[Euclidean geometry] was rigorous and definite. Sure theorems about lines and triangles, circles and squares, following with unimpeachable logic from clearly stated assumptions ... [it] remained unchanged for more than a hundred years.

-Pi in the Sky
John Barrow

## 3.2 A Mathematician's Lament

### A Mathematician's Lament

by Paul Lockhart

A musician wakes from a terrible nightmare. In his dream he finds himself in a society where music education has been made mandatory. "We are helping our students become more competitive in an increasingly sound-filled world." Educators, school systems, and the state are put in charge of this vital project. Studies are commissioned, committees are formed, and decisions are made— all without the advice or participation of a single working musician or composer.

Since musicians are known to set down their ideas in the form of sheet music, these curious black dots and lines must constitute the "language of music." It is imperative that students become fluent in this language if they are to attain any degree of musical competence; indeed, it would be ludicrous to expect a child to sing a song or play an instrument without having a thorough grounding in music notation and theory. Playing and listening to music, let alone composing an original piece, are considered very advanced topics and are generally put off until college, and more often graduate school.

As for the primary and secondary schools, their mission is to train students to use this language— to jiggle symbols around according to a fixed set of rules: "Music class is where we take out our staff paper, our teacher puts some notes on the board, and we copy them or transpose them into a different key. We have to make sure to get the clefs and key signatures right, and our teacher is very picky about making sure we fill in our quarter-notes completely. One time we had a chromatic scale problem and I did it right, but the teacher gave me no credit because I had the stems pointing the wrong way."

In their wisdom, educators soon realize that even very young children can be given this kind of musical instruction. In fact it is considered quite shameful if one's third-grader hasn't completely memorized his circle of fifths. "I'll have to get my son a music tutor. He simply won't apply himself to his music homework. He says it's boring. He just sits there staring out the window, humming tunes to himself and making up silly songs."

In the higher grades the pressure is really on. After all, the students must be prepared for the standardized tests and college admissions exams. Students must take courses in Scales and Modes, Meter, Harmony, and Counterpoint. "It's a lot for them to learn, but later in college when they finally get to hear all this stuff, they'll really appreciate all the work they did in high school." Of course, not many students actually go on to concentrate in music, so only a few will ever get to hear the sounds that the black dots represent. Nevertheless, it is important that every member of society be able to recognize a modulation or a fugal passage, regardless of the fact that they will never hear one. "To tell you the truth, most students just aren't very good at music. They are bored in class, their skills are terrible, and their homework is barely legible. Most of them couldn't care less about how important music is in today's world; they just want to take the minimum number of music courses and be done with it. I guess there are just music people and non-music people. I had this one kid, though, man was she sensational! Her sheets were impeccable— every note in the right place, perfect calligraphy, sharps, flats, just beautiful. She's going to make one hell of a musician someday."

# 4. Patty Paper Worksheets

Welcome to geometric investigations with patty paper!

Patty paper is a thin wax paper commonly used between uncooked hamburger patties. They are conveniently cut into 5.5" or 6" squares which is useful to have during origami practice or for geometric investigations.

## Patty Paper Rules

Just as video games, car drivers, and people in general have rules that must be respected, our geometric investigations are going to have a set of rules. A few years ago most geometry classes would only let students use a compass and straightedge (no rulers or protractors!). The calculus classes on campus will let students use rulers, protractors, and calculators but no graphing calculators. Every class and in particular, every math class, is usually explicit about what tools the students are allowed to use in order to solve problems, that is, they lay out the 'rules'.

In this class we will have the patty paper game rules:
You are allowed to use:

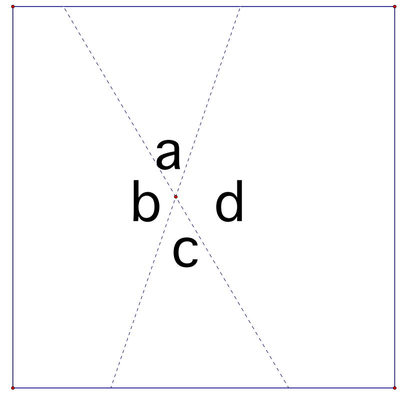
1. lots of patty paper (notice that they are semi-translusent and have straight edges),
2. a pencil, and
3. a calculator (of any kind).

Explicitly that means you are *not* allowed to use: an already made ruler or an already made protractor.

Despite these restrictions you will be able to complete many tasks. For example, while distances can't be measured, we can make a record of it by making marks on the paper. With this record of distance we can compare distances to each other to determine which is longer or if they are the same length. As the investigations go on you will discover your own methods and can perhaps share your techniques with those around you.

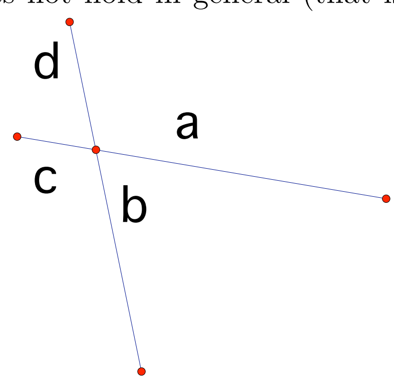
## 4.1 Patty Paper Worksheet 1

### Vertical Angles




inspired by Michael Serra's *Patty Paper Geometry*.

1. Fold a line on a patty paper. Unfold.
   Fold a second line intersecting the first line. Unfold.
2. Notice there are four angles surrounding the intersection of the two lines. Label them $a$, $b$, $c$, and $d$ so that you can refer to each by name.
3. Are any angles the same? Are all the angles different?
4. Explain *carefully* what steps you took so that you could compare one angle to another. Consider drawing pictures if that helps clarify your steps.
5. Repeat steps 1 and 2 and see if the observations you make in Step 3 still apply to a different pair of lines. If your conclusion in Step 3, does not hold in general (that is for any pair of intersecting lines that make angles $a$, $b$, $c$, and $d$), correct Step 3 so that it is true in general.

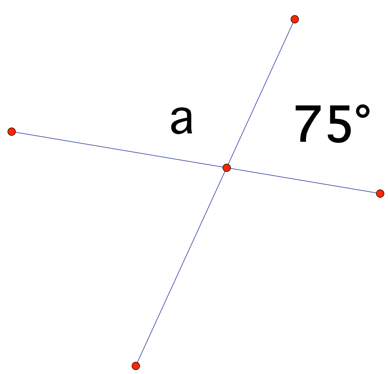


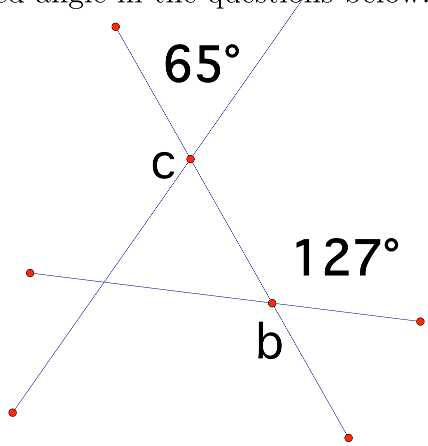

6. The pairs of opposite angles formed by two intersecting lines are called *vertical angles*. For example, in the diagram to the right, $\angle a$ and $\angle c$ are a pair of vertical angles. Restate your conclusions in 3 using this new terminology.

7. Two angles that have a common vertex, share a side, and do not overlap are *adjacent angles*. For example, $\angle a$ and $\angle b$ in the diagram on the bottom of the front page, are adjacent angles. Can you say anything about sum of the pairs of adjacent angles above?

8. Two angles whose measures add up to 180° are called *supplementary* angles. Are all adjacent angles also supplementary? Either justify yourself of provide a counter example.

Use your conclusions on the front page of this worksheet and any other geometric knowledge to calculate the measure of each lettered angle in the questions below.




9. Origami directions usually start with a square piece of paper, but 8.5”×11” paper is more commonly available today. How can you determine what amount to cut off of a rectangular 8.5”×11” (remember the rules! no rulers!) to make a square? Give the steps below (you may want another sheet of paper if you need more room) and then *justify* that your steps guarantee a square.

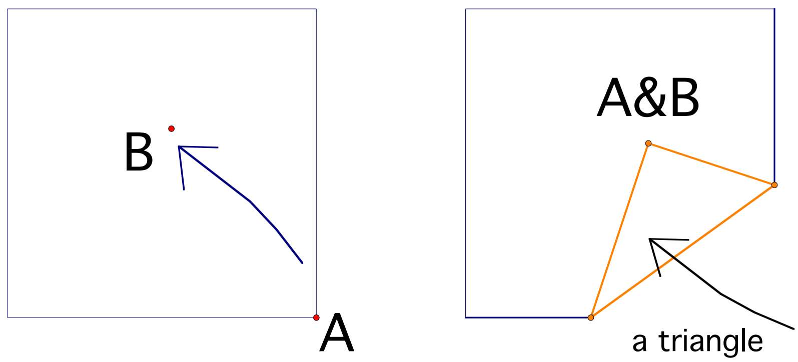
## 4.2 Patty Paper Worksheet 2

# Folding TUPs

inspired by Kazuo Haga's "Folding Paper and Enjoy Math: Origamics" in *Origami: Third International Meeting of Origami Science, Mathematics, and Education.*

1. Take a piece of patty paper and label the lower right-hand corner $A$. Pick a random point on the paper and label that point $B$.

2. Fold the paper so that $A$ lies on top of $B$. This creates a flap of paper, called the Turned-Up Part (or TUP for short).




3. How many edges does your TUP have? Three? Four? Five?

   For instance, in the example depicted above, the TUP has three sides.

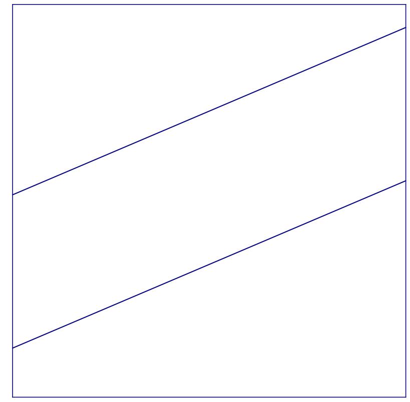
4. Experiment with many TUPs to find an answer to the question, "How can we tell how many edges a TUP will have before we even fold the paper?"

5. What if we allowed the point $B$ to be outside the square? "How can we tell how many edges a TUP will have before we even fold the paper?"

## 4.3 Patty Paper Worksheet 3

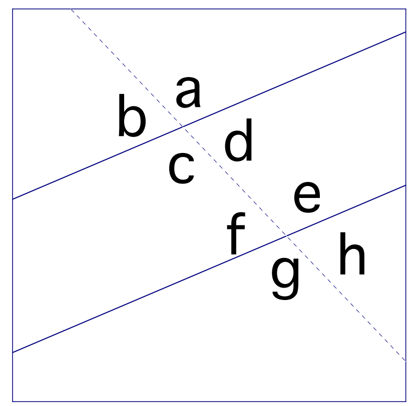
### Angles & Parallel Lines

inspired by Michael Serra's *Patty Paper Geometry*.



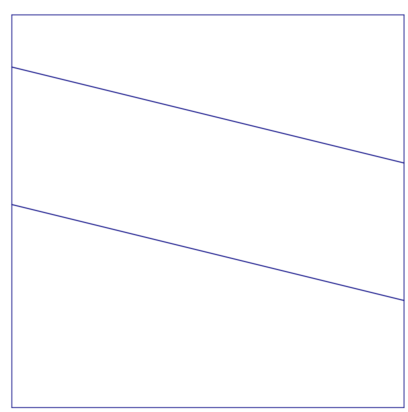
1. Trace the pair of parallel lines shown on the right, onto a patty paper.

2. Fold or draw a transversal line.
   Label the angles as shown in the diagram
   so that you can refer to each by name.




3. Are any angles the same? Are all the angles different?

4. Explain *carefully* what steps you took so that you could compare one angle to another. Consider drawing pictures if that helps clarify your steps.



5. Repeat steps 1 through 3 with a different pair of parallel lines (another set is provided on the right if you are not sure how to manufacture your own) and see if the observations you make in Step 3 still apply. If your conclusion in Step 3, does not hold in general (that is for any set of parallel lines intersected by a transversal), correct Step 3 so that it is true in general.

6. Recall the definition of corresponding angles provided in class. Restate and, if necessary, expand your statement in Step 3 that that you use this term.

7. Assume you are given a set of lines with a transversal. If the corresponding angles have the same measure, explain why the alternating interior angles must have the same measure.

8. Restate and, if necessary, expand your statement in Step 3 that that you use the terms alternating interior angles and alternating exterior angles.

9. Your statement in Step 6 should be written as a conditional statement. Identify the hypothesis and the conclusion.

10. Write the converse of your statement in Step 6.

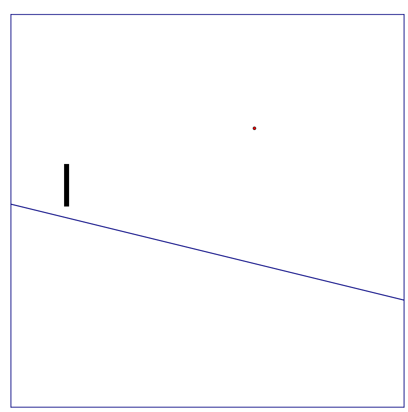
11. Is that statement in Step 10 true? If true, provide some justification, if false, provide a counterexample.

## 4.4 Patty Paper Worksheet 4

### Making Parallel & Perpendicular Lines

inspired by Michael Serra's *Patty Paper Geometry*.

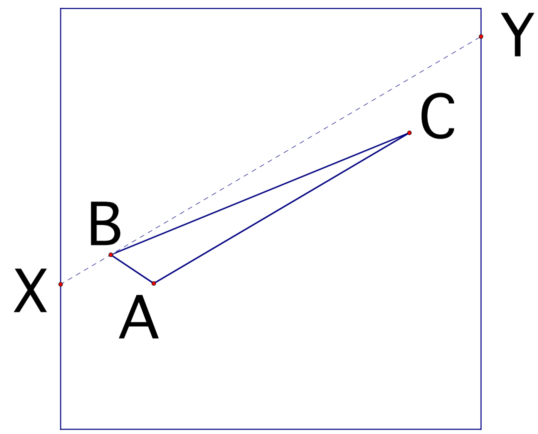
1. Fold a line on a patty paper. Unfold. Mark this line $l$.
2. Discover a method for making a line that is perpendicular to $l$. Remember that this is a patty paper investigation so you can only make use of a pencil and patty paper(s).
3. Make another line on your patty paper and try to use your process in Step 2 to make a perpendicular line. Once you are sure your process in Step 2 works for *any* line on the patty paper, describe your process and *justify* why your method works.
4. Discover a method for making a line that is parallel to $l$. Make sure that your process works for *any* line on your patty paper, and then describe your process. *Justify* why your method works.
5. Use your pencil to make a point not on the line $l$. Discover a method for folding a line through the point so that it is parallel to the line $l$. Compare your method with other groups and, if different, determine which method you like better.
6. Describe your favorite process of making a line parallel to a given line $l$ that also runs through a specified point.



# Sum of Angles in a Triangle

1. Draw a triangle on your patty paper near the center and identify the vertices $A$, $B$, and $C$.

2. Use the method described on the front of this worksheet to make a line parallel to $AC$ that also passes through $B$. Label the points where this new line intersects with the edges of the patty paper as $X$ and $Y$, as shown.




3. There should be at least five angles on your patty paper now. Are any of the angles on the patty paper are the same? If so, mark them.

4. Repeat Steps 1 and 2 with a different triangle to see if the observations you made in Step 3 still apply. If your statements in Step 3 does not hold in general (for any $\triangle ABC$ and line parallel to $\overline{AC}$), correct Step 3.

5. The $\angle XBY$ is a straight line and thus measures $180°$, but it is also equal to the sum three other angles. What three angles make up $\angle XBY$?

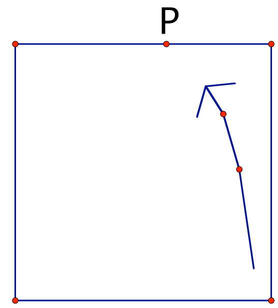
6. Use Step 5 and your observations made in Step 3 to *justify* that the sum of angles in a triangle is $180°$.

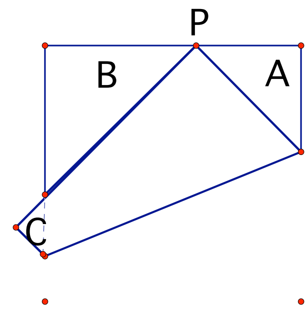
## 4.5 Patty Paper Worksheet 5

# Haga's Theorem

inspired by Kazuo Haga's "Folding Paper and Enjoy Math: Origamics" in *Origami: Third International Meeting of Origami Science, Mathematics, and Education*.

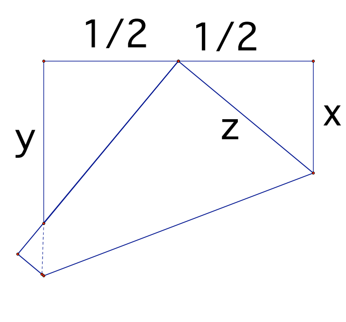
1. Take a patty paper and mark a point $P$ at random along the top edge of the paper.
2. Fold the lower right corner to the point $P$ as depicted.




3. Notice that there are (usually) three triangles that are formed by the fold you made in step two. Label the triangles $A$, $B$, and $C$ as done above.
4. What nice relationship must be true about triangles $A$, $B$, and $C$? Justify your conclusions.

5. Suppose that you took the point $P$ to be the midpoint of the top edge. Use your above observation (or generate more observations) to find out what lengths $x$, $y$, and $z$ must be in the figure to the right.



## 4.6 Patty Paper Worksheet 6

The Largest Equilateral Triangle

inspired by Thomas Hull's "What's the biggest Equilateral Triangle in a Square" in *Project Origami, activities for exploring mathematics.*

1. First fold you square to produce a $30° - 60° - 90°$ triangle inside it. Explain your method and *justify* that you made a $30° - 60° - 90°$ triangle . Hint: these special triangles have the property that the hypothenuse is twice as long as one of its sides.

2. Find a way to fold an equilateral triangle inside a patty paper. Explain your method and *justify* that you made an equilateral triangle.

3. If the side length of your original square is 1, conjecture if it is possible for you to make an equilateral triangle who's side length is greater than 1?

4. If an equilateral triangle is maximal (the equilateral triangle is the largest equilateral triangle that will fit on a patty paper), can we assume that one of its vertices will coincide with a vertex of the patty paper? Why? Could we say anything about the other vertices of the equilateral triangle?

5. Draw a picture of the situation in question 4 where the patty paper shares the lower left vertex with the maximal equilateral triangle.

6. Let $\theta$ denote the angle between the bottom of the square and the bottom of the triangle. What angle should $\theta$ be to maximize the size of the equilateral triangle in the patty paper? *Justify* your conclusion. Hint: consider symmetry!

## 4.7 Patty Paper Worksheet 7

Spherical Geometry

1. Explain in your own words what a geodesic is.

2. Complete each of the following two sentences:

   Geodesics in the plane look like:

   Geodesics on the sphere look like:

3. Answer each of the following:

   How many geodesic paths are between two points in a plane?

   How many geodesic paths are between the 'north pole' & 'south pole' on a sphere?

4. Given a geodesic on the plane, you can make a geodesic perpendicular to the first with techniques from Patty Paper Worksheet 4. Given a geodesic on the sphere, find a technique that you can use on the sphere to make a geodesics perpendicular to the first.

5. Given a geodesic on the plane, you can make a geodesic parallel to the first with techniques from Patty Paper Worksheet 4. Given a geodesic on the sphere, is it possible to find a geodesic on the sphere that is parallel to the first?

6. Answer each of the following:

   How many times do two perpendicular geodesics intersect in the plane?

   How many times to two perpendicular geodesics intersect on the sphere?

   If two geodesics are perpendicular on a plane, how many 90° angles are made?

   If two geodesics are perpendicular on a sphere, how many 90° angles are made?

7. Define *triangle*.

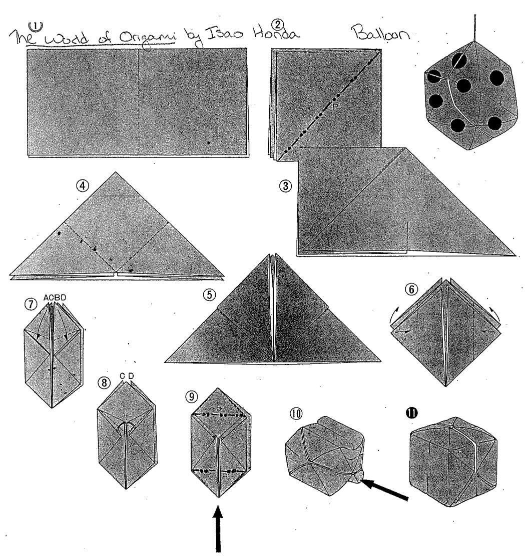
8. Can you make a right triangle on the sphere? If so, explain how, and if not, explain why you can't.

9. Can you make a right triangle on the sphere with more than one 90° angle? If so, draw a picture of it.

10. Can you make a rectangle on the sphere? If so, explain how, and if not, explain why you can't

# 5. Origami Patterns

## 5.1 Bases




**72. Balloon**

The balloon base is an origami that Japanese children learn in kindergarten. With it we can make a number of interesting folds.

Use a six-inch square of paper.

1. Prepare the paper by book folding it in half. Valley fold on the dotted line.
2. Open the square into a triangle by peak and valley folding as indicated.
3. Repeat step 2 on the underside.
4. Valley fold the top folds on the dotted lines, bringing the bottom edges up to the center line.
5. Turn the model over and repeat.
6. Valley fold the right and left points into the center line. Turn the model over and repeat.
7. Valley fold points A and B down into the pockets shown.
8. Repeat with points C and D.
9. Peak and valley fold as shown to flatten out the top and bottom of the cube shape.
10. Blow air into the indicated hole.
11. The finished balloon.

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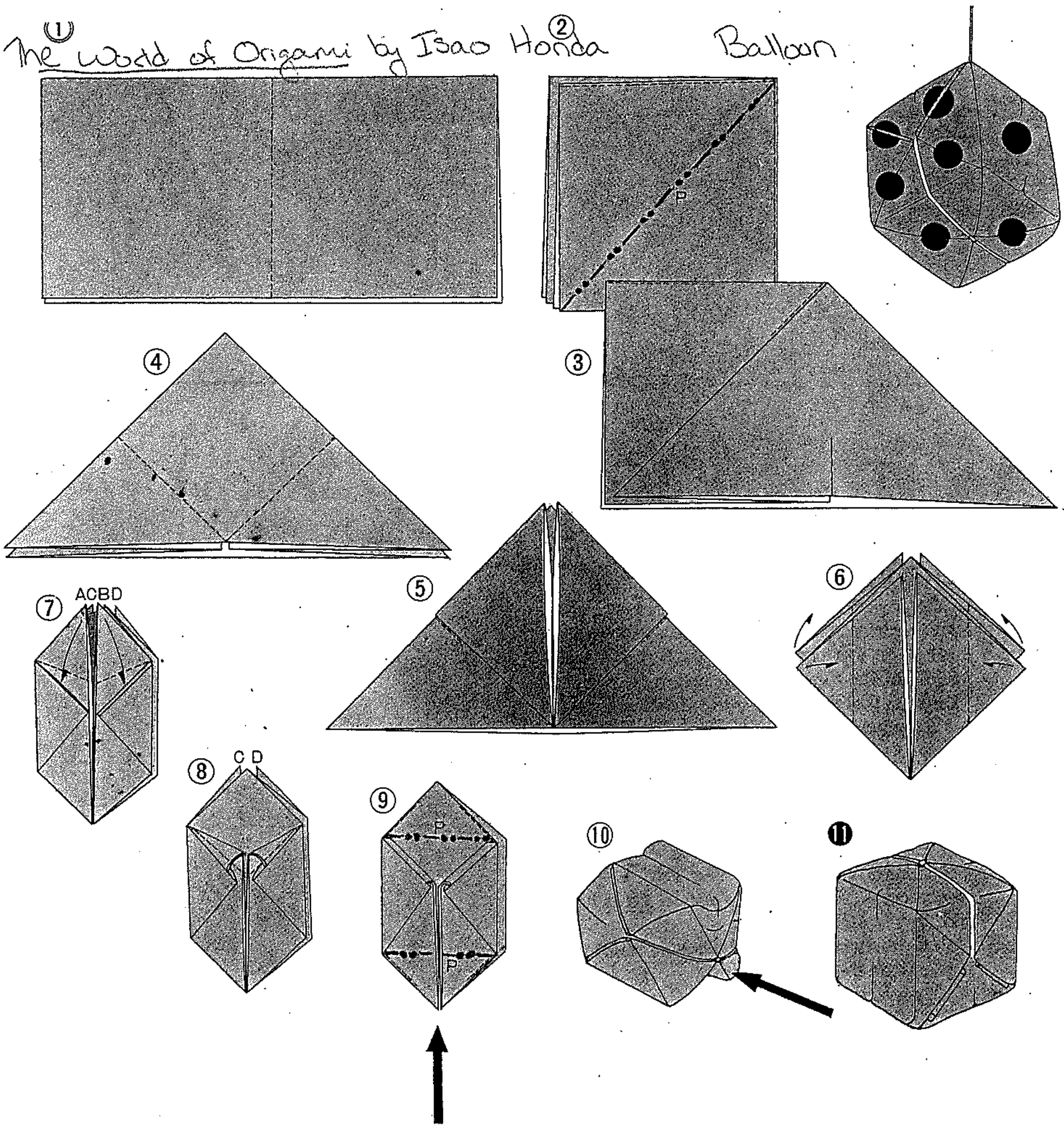
# 5.2 Balloon

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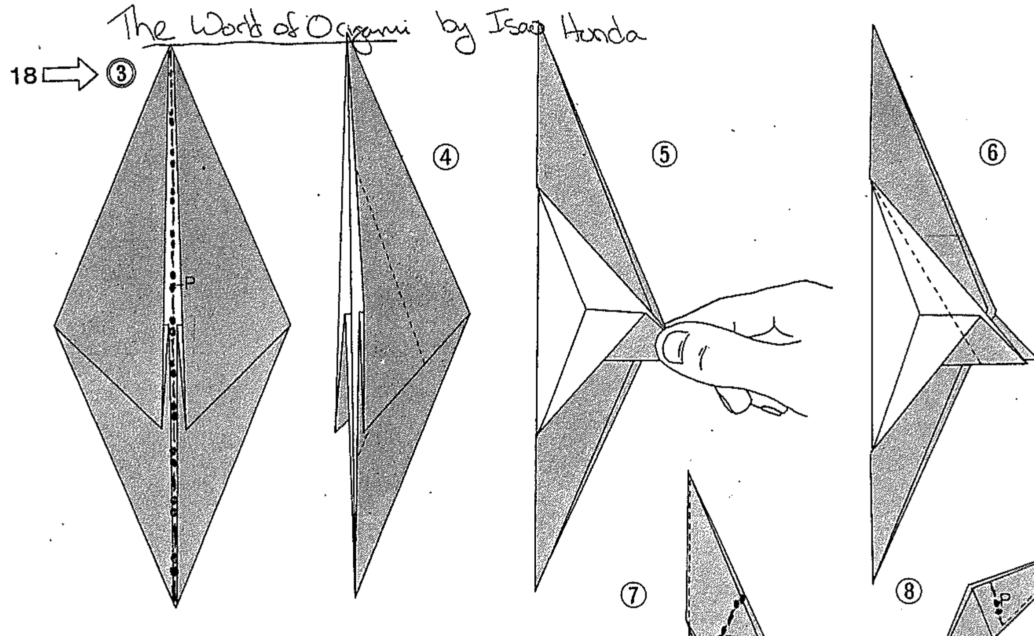
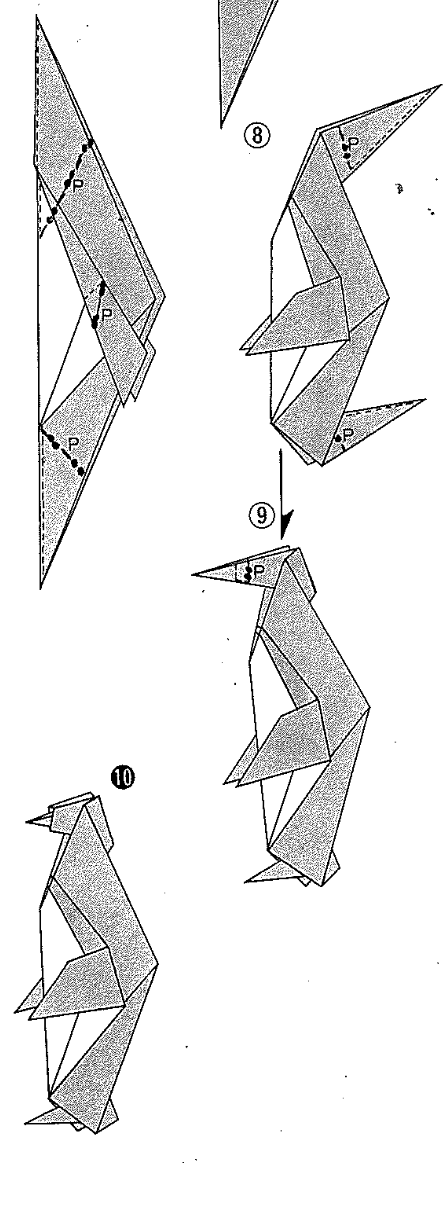
7. Valley fold points A and B down into the pockets shown.
8. Repeat with points C and D.
9. Peak and valley fold as shown to flatten out the top and bottom of the cube shape.
10. Blow air into the indicated hole.
11. The finished balloon.

# 5.3 Penguin

The World of Origami by Isao Honda




### 19. Penguin (I)

Use a five-inch sheet of black paper with a white underside.

1~3. Begin with the [redacted] base. Peak fold on the dotted line.

4. Valley fold on the dotted line (top two layers only). Be careful to note that there is a dotted line extending beneath the first fold. Turn the model over and repeat.
5. Pull the flipper sections out to reveal the penguin's white belly.
6. Valley fold each flipper on the dotted line.
7. Peak and valley fold the neck section, and reverse fold it outside in. Peak and valley the foot section, and reverse fold it outside in. Peak and valley fold the flippers on the dotted lines.
8. Peak and valley fold the neck on the dotted lines, and reverse fold it outside in to form the head. Peak and valley fold the feet section, and reverse fold it outside in.
9. Peak and valley fold the head, and reverse fold it in to form the beak.

# 6. Other Worksheets

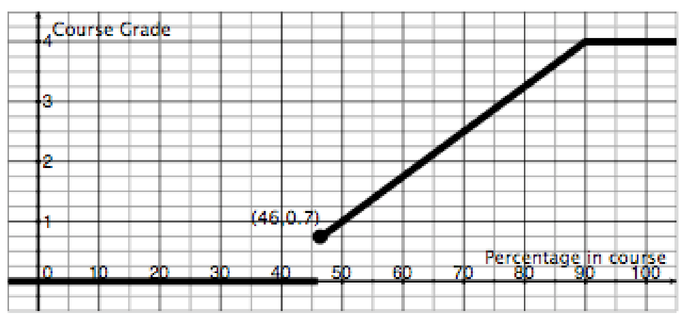
## 6.1 Computing Grades

1. Clark Kent goes to college and on the first day of precalculus class he is handed the following information on the syllabus:

   **Grades:**

   The following weights will be used to calculate your percentage in the course. The function graphed takes your percentage in the course and returns your grade on a 4. scale.

| | |
|---|---|
| Mini-Quizzes (top score) | 5% |
| WeBWork assignments | 10% |
| Handwritten assignments | 15% |
| Quizzes | 15% |
| 2 Midterms | 30% |
| Final | 25% |




   In week 7 Clark begins to get concerned about his grades (being a super hero is still new to him so he has missed a few assignments and a test). He logs onto Catalyst and finds the following information:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Mini-Quiz: | (ea. worth 10pts) | 9.5 | | | | | | | | | |
| WebWork: | (ea. worth 10pts) | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Written HW: | (ea. worth 10pts) | 10 | 8 | 0 | 7 | 9 | 0 | 6 | | | |
| Quizzes: | (percentage) | 100 | 0 | 100 | 80 | | | | | | |
| 2 Exams | (percentage) | 92 | 0 | | | | | | | | |

   (a) Compute Clark's current averages for each of the categories below:

   Mini-Quiz
   WeBWork
   WrittenHW

   Quizzes

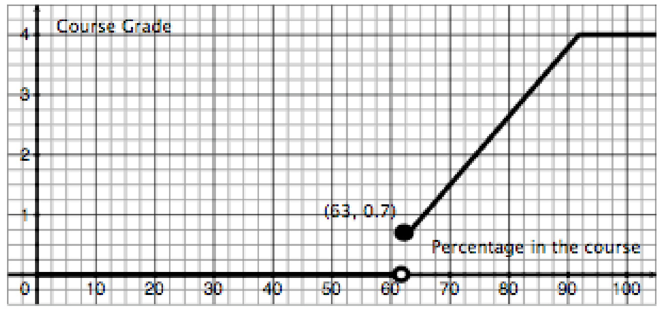
   2 Exams

   (b) Assuming his performance in each category will not change in the remaining weeks, what grade must Clark get on his final to get a 2.5 in the class?

2. Lois Lane goes to college and on the first day of origami math class is handed the following information on the syllabus:

**Grades:** The following weights will be used to calculate your grade.

| | |
|---|---|
| Journals | 15% |
| Homework | 15% |
| Quizzes | 15% |
| Origami Project | 10% |
| Papers | 10% |
| Midterm | 15% |
| Final | 20% |




The above grade assignment is based off the University of Washington, Tacoma's grading scale posted at http://www.tacoma.washington.edu/enrollmentservices/grading.cfm.

In week 7 Lois begins to get concerned about her grades. She logs onto Catalyst and finds the following information:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Journals: | (ea. worth 10pts) | 10 | 10 | 10 | 7 | 10 | 10 | 10 | 10 | 10 | 10 |
| Homework: | (ea. worth 15pts) | 15 | 11 | 10 | 12 | 10 | 15 | 15 | 11 | | |
| Quizzes: | (ea. worth 20pts) | 18 | 13 | 17 | 13 | | | | | | |
| Origami Proj. | (ea. worth 40pts) | 37 | | | | | | | | | |
| Midterm: | (percentage) | 82 | | | | | | | | | |

Recall that the lowest scoring quiz is dropped when computing grades. Assuming her journal, homework, quiz, and origami project averages do not change much in the remaining weeks, what scores could Lois Lane get on her papers and final to make sure she gets a 2.0 in the class?

# 7. Practice Problems

## 7.1 Spherical Geometry Problems

Spherical Geometry Practice Probems

TRUE/FALSE

1. The shortest point between two points on a sphere is on the arc of a great circle.
2. If two great circles intersect to form a right angle, they are perpendicular.
3. On a sphere there is only one distance that can be measured between two points.
4. A great circle is infinite.
5. Every two unique geodesics on a sphere must cross at two points
6. Two perpendicular 'lines' on a sphere create four right angles.
7. A triangle on a sphere may have three right angles.
8. On a sphere if two 'lines' are perpendicular to a given line, the two lines are parallel.
9. Rectangles can be drawn on a sphere.

1. true
2. true
3. false (the great circle connects the points in two ways)
4. false
5. true
6. false (there is another set of 4 at the other point the two great circles intersect)
7. true
8. false
9. false