Key

Exam 2 **TMath 120** Winter 2023

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T F $\frac{1}{a^2} + \frac{1}{a} = \frac{2}{a^2}$ $\frac{1}{a^2} + \frac{1}{a}\frac{a}{a} = \frac{1+a}{a^2}$

T F $\log(a+b) = \log(a) + \log(b)$ for positive a and b. $\log(a) + \log(b) = \log(a \cdot b)$

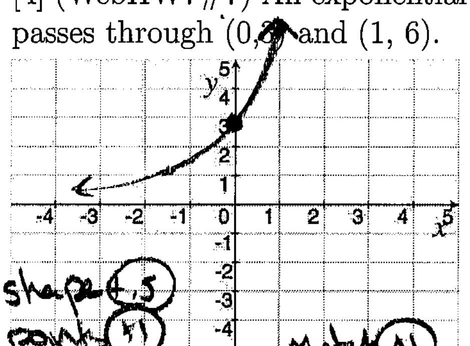
T F The equation $x^2 + (y-2)^2 = 9$ defines a circle centered at $(0,-2)$ centered @ $(0,2)$

T F The distance between $(-2,5)$ and $(2,2)$ is 5 units. 3 4 $dist^2 = 3^2 + 4^2 = 9 + 16 = 25$

Show all your work. Reasonable supporting work must be shown to earn credit.

2. Provide a graph AND an algebraic rule/expression for each of the functions described:

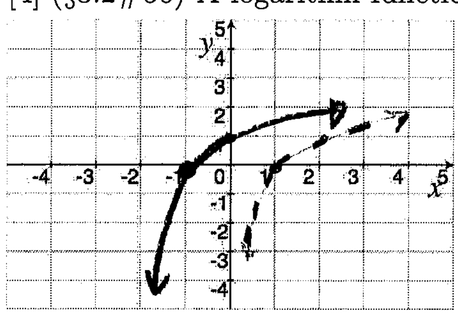
(a) [4] (WebHW7#7) An exponential function that has been vertically stretched and passes through (0, 3) and (1, 6).



shape (+.5)
points (+1)
match (+1)

$y = A \cdot b^x$ (+1)
thru $(0,3) \Rightarrow 3 = A \cdot b^0 \Rightarrow A = 3$
thru $(1,6) \Rightarrow \frac{6}{3} = \frac{3 \cdot b^1}{3} \Rightarrow 2 = b^1$ (+.5)
so $y = 3 \cdot 2^x$

(b) [4] (§3.2#56) A logarithm function with a domain of $(-2, \infty)$.



(+.5) shifted left 2 units
There are many answers!
$y = \log_2(x+2)$ (+.5) use logs (+.5)

shape (+1)
domain (+.5)
match (+1)

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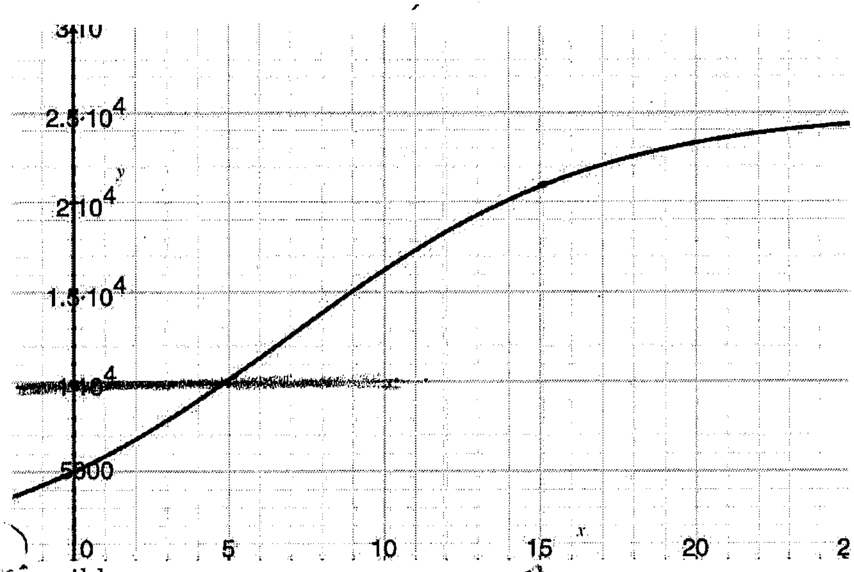
3. The number of people in a community who became infected during an epidemic x weeks after its outbreak is modeled by the function
$$f(x) = \frac{25,000}{1 + ae^{kx}}$$
for some parameters a and k. This is graphed to the right.



(a) [2] (Quiz3#1) Find a realistic range for f.

graph reading (.5) y-values (+.5)

[5,000, 25000] (+1)

(b) Estimate the following *if* possible:

i. [2] (§3.1 #84) $f(15)$ $\approx$ 22,000 (+.5) decode $21 \cdot 10^4$ (+.5)

start (+.5)

graph reading (+.5)

ii. [1] (Quiz3#1) The y intercept of f.

5000 (+1)

iii. [2] (LogActivity#4) All possible x such that $f(x) = 10,000$.

(+.5) ie inputs w/ y-value of 10,000 (+.5) decode 10,000 to 1·10

$\approx$ 5 weeks (+1)

iv. [2] (WebHW9#1) The eventual number of infected people as $x \to \infty$ (+.5)

$2.5 \cdot 10^4$ or 25000 (+1) graph reading (+.5)

(c) [6] (WebHW8#32) Find the parameters a and k so we have the complete algebraic rule of f.

thru (+.5) (0,5000) $\Rightarrow$ plug in (+.5) $5000 = \dfrac{25000}{1+ae^{k \cdot 0}} \Rightarrow 5000 = \dfrac{25000}{1+a \cdot 1}$

alg (+1) $\Rightarrow \dfrac{5000(1+a)}{5000} = \dfrac{25000}{5000} \Rightarrow 1+a = 5$

$\Rightarrow a = 4$ (+.5) So $f(x) = \dfrac{25000}{1+4e^{kx}}$

thru (+.5) (5, 10,000) $\Rightarrow$ plug in (+.5) $10,000 = \dfrac{25000}{1+4e^{k \cdot 5}} \Rightarrow \dfrac{10,000(1+4e^{k5})}{10000} = \dfrac{250}{10}$

So

$f(x) = \dfrac{25,000}{1+4e^{-.196x}}$

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alg before (+1) $\Rightarrow 1 + 4e^{5k} = 2.5 \Rightarrow \dfrac{\cancel{4}e^{5k}}{\cancel{4}} = \dfrac{1.5}{4}$

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use log (+1) $\Rightarrow e^{5k} = \dfrac{1.5}{4} \Rightarrow \ln e^{5k} = \ln \dfrac{1.5}{4} \Rightarrow \dfrac{5k}{5} = \dfrac{\ln\left(\frac{1.5}{4}\right)}{5}$ (+.5) calc

4. [3] (WebHW9#2) How much should a guardian invest at the time their son's birth in order to afford (one year of) $14,000 tuition at the University of Washington in 18 years? Assume 5% compounded monthly.

Sold for 1 point

$A = P(1+\frac{r}{n})^{nt}$ (+.5)

find P so that

$14{,}000 = P(1+\frac{.05}{12})^{12 \cdot 18}$ (+.5) (+.5) (+.5)

$\frac{14000}{2.455} = \frac{P(\cancel{2.455})}{\cancel{2.455}}$ ✓ compute (+.5)

$\$5703 \approx P$ (+.5) solve for P

5. [4] (LogInPractice#2) How long will it take to take for $2,500 to amount to $14,000 if invested at an annual rate of 5% compounded continuously?

Sold for 1 point

$A = Pe^{rt}$ (+.5)

$\frac{14000}{2500} = \frac{2500e^{.05t}}{2500}$ (+.5) (+1) alg (+.5)

$5.6 = e^{.05t}$

$\frac{\ln 5.6}{.05} = \frac{\cancel{.05}t}{\cancel{.05}}$ use log (+1)

alg (+.5)

$t \approx 34$ years

6. Assume that $\log_2(a) = 5$ and $\log_2(ab) = 7$.

(a) [2] (§3.2 #28) Find a.

$\log_2(a) = 5 \Rightarrow 2^5 = a \Rightarrow 4 \cdot 4 \cdot 2 = a \Rightarrow 32 = a$ (+.5)

(+1) exp

realize a/5 (+.5)

(b) [2] (§3.3 #16) Find $\log_2(b)$.

$7 = \log_2(ab)$

(+1) property $7 = \log_2(a) + \log_2(b)$

$7 = 5 + \log_2(b)$ (+.5)

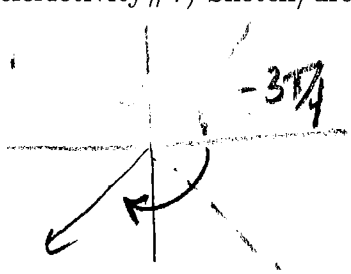
$-5 \quad -5$

$2 = \log_2(b)$ got it (+.5)

7. [2] (§4.1 #40) Convert $\frac{-3\pi}{4}$ radians into degrees.

$\frac{-3\pi}{4}\text{ rad} \cdot \underbrace{\frac{180°}{\pi\text{ rad}}}_{\text{conversion (+1)}} = \frac{-3\pi \cdot 180}{4\pi}° = -3 \cdot 45° = -135°$ (+1)

8. [2] (CircleActivity#7) Sketch/draw the angle $\frac{-3\pi}{4}$.



direction (+1)

got it (+1)

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$$\begin{array}{r} 27 \\ 23 \\ \hline 50 \end{array} \Rightarrow 100\%$$

9. [3] (PracticeExam) Simplify

start (.5) $\left(\frac{3x}{2y^{1/2}}\right)\left(\frac{4}{x}\right)^2$ neg exp (.5) $\left(\frac{3x}{2\sqrt{y}}\right)\left(\frac{x}{4}\right)^{-2}$ $\frac{3x\cdot 16^{8}}{2y^{1/2}x^2} = \frac{24}{y^{1/2}x}$

$\frac{3x}{2y^{1/2}}\ \frac{16}{x^2}$ dist over ÷ (+1) (+1) canceling coef + x

10. (§3.5 #52) Let I be the intensity of an earthquake X and S be the intensity of a 'standard' earthquake of 10^{-4} cm on a seismograph. Then the measurement of an earthquake X as measured on the Richter scale is:

$$\log\left(\frac{I}{S}\right)$$

(a) [2] New standards were put into place for King County in 2018 which were designed so that buildings have a low chance of collapse (10%) if caught in an earthquake with a Richter scale of 8.9 or less. Find the intensity of the earthquake that would increase the chance of this building's collapse.

(b) [3] February 6th 2023 Turkey had massive earthquake that measured 7.5 on the Richter scale. February of 2001 Tacoma had an earthquake measuring 6.8 on the Richter scale. How many more times intense was Turkey's earthquake than the one in Tacoma? notation (+1)

(a) chances increase if Richter scale ≥ 8.9

start (.5) find I so that $\log\left(\frac{I}{S}\right) \geq 8.9$

(+1) $\Rightarrow \log\left(\frac{I}{10^{-4}}\right) \geq 8.9$ lets look at

what I is $\log\left(\frac{I}{10^{-4}}\right) = 8.9$

alg/def of log (.5)

or $10^{8.9} = \frac{I}{10^{-4}}$

$\Rightarrow I = 10^{-4}\cdot 10^{8.9}$

$I = 10^{4.9}$

$\approx 79{,}432$

So intensities higher than 79,432 would increase chance of building collapse

(b) let I_{01} be intensity of 2001 earthquake and I_{23} be intensity of 2023 earthquake

Want $?\, I_{01} = I_{23}$ or $? = I_{23}/I_{01}$ (1.5)

Given

$7.5 = \log\left(\frac{I_{23}}{10^{-4}}\right)$ set up (.5) $6.8 = \log\left(\frac{I_{01}}{10^{-4}}\right)$

$10^{7.5} = \frac{I_{23}}{10^{-4}}$ $10^{6.8} = \frac{I_{01}}{10^{-4}}$

alg (.5)

$I_{23} = 10^{7.5}\cdot 10^{-4}$ $I_{01} = 10^{6.8}\cdot 10^{-4}$

$= 10^{3.5}$ $= 10^{2.8}$

So $\frac{I_{23}}{I_{01}} = \frac{10^{3.5}}{10^{2.8}} = 10^{.7}$ (1.5)

≈ 5 times more intense

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