Median 71%

Key

Exam 1

TMath 120

Winter 2023

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

(T) F $\frac{1}{\frac{a}{2}} = \frac{2}{a}$ $\frac{2 \cdot 1}{2 \cdot \frac{a}{2}} = \frac{2}{a}$

T (F) If $r(x) = \frac{x-2}{5x}$, then the inverse to r is $r^{-1} = \frac{5x}{x-2}$.

need composition to be ideally

$r(1) = \frac{1-2}{5 \cdot 1} = \frac{-1}{5}$

$r^{-1}\left(\frac{-1}{5}\right) = \frac{5(-\frac{1}{5})}{-\frac{1}{5}-2} = \frac{-1}{-\frac{11}{5}}$

$= \frac{5}{11} \neq 1$

(T) F $x^2 + 4x + 3$ divides $2x^4 + 6x^3 + 2x + 6$.

$2x^2 - 2x + 2$

$x^2+4x+3 \overline{)2x^4+6x^3+2x+6}$

$-(2x^4+8x^3+6x^2)$

$-2x^3-6x^2+2x+6$

$-(-2x^3-8x^2-6x)$

$2x^2+8x+6$

$-(2x^2+8x+6)$

0

(T) F $(2+i)-(3+i) = -1$.

$2+i-3-i = -1$

T (F) $(2+i)(3+i) = 6+i$.

$6+2i+3i+i^2$

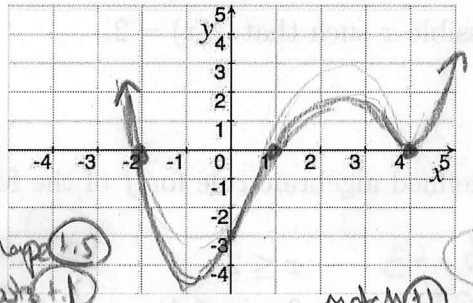
$6+5i-1 = 5+5i$

Show all your work. Reasonable supporting work must be shown to earn credit.

2. Provide a graph AND an algebraic rule/expression for each of the functions described:

(a) [4] (PolyActivity#4) A degree 4 polynomial whose only roots are -2, 1 and 4.

poly +.5, deg +.5, factors +.5



shape 1.5, roots +1, match +1

These are Many Answers?

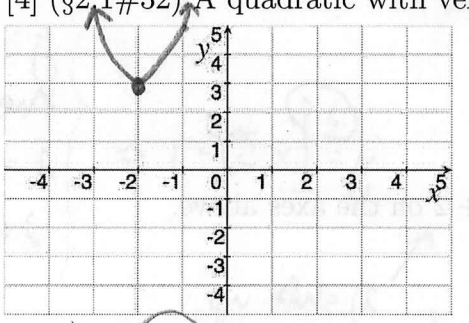
-2 is a root $\Rightarrow$ $(x-(-2))$ is a factor

1 is a root $\Rightarrow$ $(x-1)$ is a factor

4 is a root $\Rightarrow$ $(x-4)$ is a factor

$\frac{1}{10}(x+2)(x-1)(x-4)^2$ need 4th deg poly so inc. the power

(b) [4] (§2.1#32) A quadratic with vertex $(-2, 3)$ and vertically stretched by 2.



shape +.5, vertex +1.5, stretch +.5, match +1

$y = a(x-h)^2 + k$

$y = a(x-(-2))^2 + 3$

$y = 2(x+2)^2 + 3$

quad 1.5, vertex pts +1.5, vert stretch +1.5

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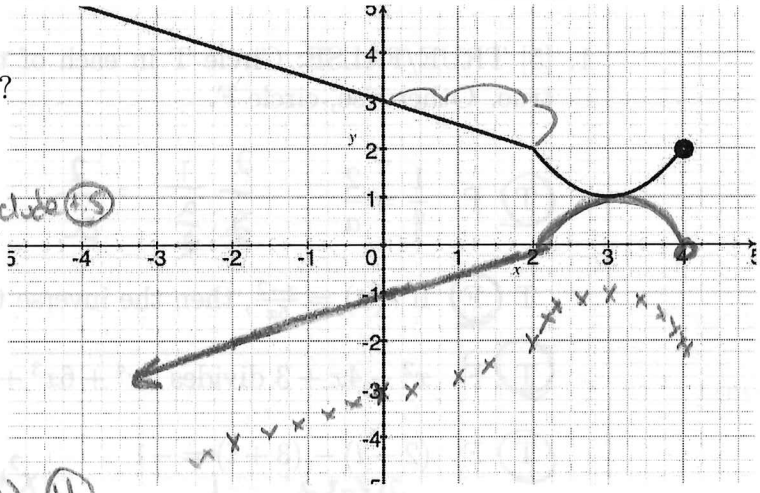
3. Let f be the piece-wise defined graph comprised a line and one parabola below.



(a) [2] (Quiz2#1) Find the range of f?

y values (+.5) $[1, \infty)$ endpoints (+1) include (+.5)

(b) Estimate the following *if* possible:

i. [1] (Quiz1 #1) $f(-2)$

4

ii. [2] (WebHW3 #2) $(f+f)(-2)$

$= f(-2) + f(-2)$ (+1)

$= 4 + 4 = 8$ (+.5) (+.5)

iii. [2] (WrittenHW§1.6 #68) $(f(f(0)))$ $= f(3) = 1$ (+.5)

(+.5) Composition (+.5)

iv. [1] (WrittenHW§1.1 #104) The minimum of f.

@ $x = 3$ and $y = 1$ either answer is ok

v. [2] (PracticeExam1 #6) All possible x such that $f(x) = 2$.

$x = 2$ and 4

(+1) (+1)

(c) [4] (Quiz2#1) Find the piece-wise defined algebraic rule for f of the form:

$$f(x) = \begin{cases} -\frac{1}{2}x + 3 & x \le 2 \\ (x-3)^2 + 1 & 2 < x \le 4 \end{cases}$$

line $y = mx + b$ (+.5)

m = slope $= \frac{\text{rise}}{\text{run}} = \frac{-1}{2}$ (+.5)

y intercept of 3 (+.5)

$\Rightarrow y = -\frac{1}{2}x + 3$ plug in (+.5)

(+.5) parabola: $y = a(x-h)^2 + k$

vertex @ (3, 1)

$y = a(x-3)^2 + 1$ (+1)

thru (2, 2)

$2 = a(2-3)^2 + 1$

$2 = a + 1 \Rightarrow a = 1$

(+.5) $y = (x-3)^2 + 1 \Leftarrow$

(d) [3] (WebHW1.5 #9) Graph $-f(x) + 2$ on the axes above.

shape (+.5)

1) flip over the x axis / mult the y by -1 (+1)

2) shift up 2 units (+1)

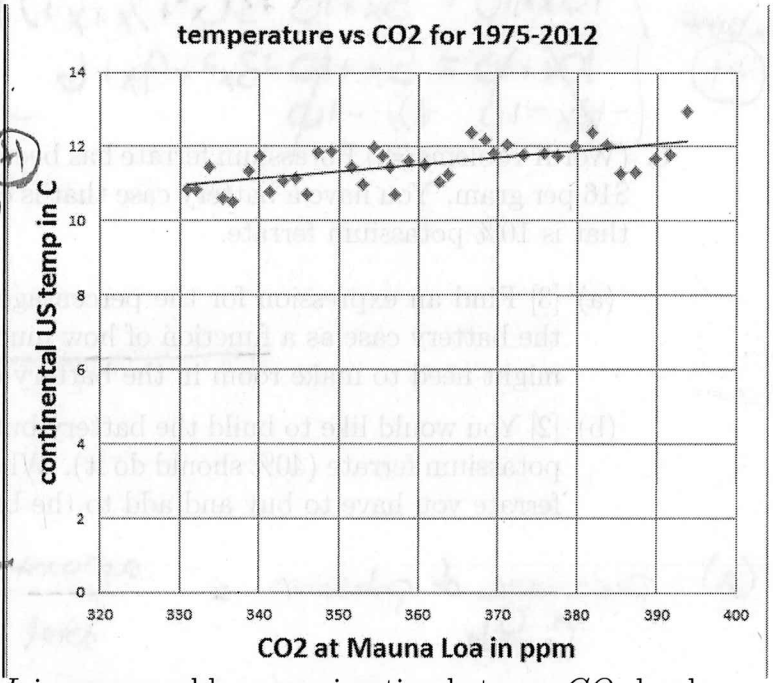
order (+.5)

xxxxxx

+.5 partial if sq +.5 partial if sq

4. (Dr. Archey's Blog) Consider the following data collected from `http://www.esrl.noaa.gov/gmd/dv/data/index.php?site=mlo&parameter_name=Carbon%2BDioxide&frequency=Monthly%2BAveragesandhttp://www.ncdc.noaa.gov/cag/`.

   Each year's average CO_2 levels measures a Mauna Loa, HI and average continental US temperature (in Celsius) were plotted on the axes below. Additionally the linear model that best fits the data was graphed. The equation of the line is $L(x) = .019x + 4.76$.



   (a) [2] (Quiz1#1) Is L a function? Why or why not?

   yes L passes the vertical line test (+1) (+.5)

   Start (1.5)

   (b) [1] (§1.2#106) What are the units for the slope?

   $\frac{\text{rise}}{\text{run}} = \frac{\text{temp in °C}}{CO_2 \text{ levels ppm}}$

   (c) [1] (LineActivity#7) What does the slope mean in real world terms?

   (+.5) a ratio

   The change in temperature as the concentration of CO_2 levels change

   (d) [2] (WebHW2#1) Assuming L is a reasonable approximation between CO_2 levels and average continental temperatures, what would the continental temperature of the US be when/if CO_2 levels reach 400 ppm?

   $L(400) = .019(400) + 4.76$

   $= 12.36°C$ (+1)

   plug in 400 to L (+1)

   (e) [3] Find the y intercept and interpret in real world terms. Do you think your interpretation is reasonable? Why or why not?

   y-intercept is when $x = 0$ (+.5)

   $L(0) = .019(0) + 4.76$

   $= 4.76$ (+1)

   (+1) When CO_2 levels are nonexistent, the ave continental temp in US will by 4.76°C

   (.5) Not reasonable - CO_2 levels will never be 0

   (f) [2] (InverseActivity#1) Describe the inverse function L^{-1} by identifying the inputs and outputs.

   (+.5) CO_2 levels → (L) → temp in US (°C)

   temp in US (°C) → (L^{-1}) → CO_2 levels

   understand L (+.5)

   Start (+.5)

   (+.5) is finding rule

L^{-1} takes ave temp rates in continental US (°C) and returns CO_2 levels at Mauna Loa in ppm

30
20
50 => 100%

5. [4] (PracticeExam #3) Find any real or imaginary solutions to

start +1.5
factors +1

$$(x+2)5(x+1)\left(\frac{2}{x+2}\right)=\left(\frac{1}{x+1}+\frac{3}{5}\right)(x+2)5(x+1)$$

dist/combine +1

$$\begin{cases} 10(x+1) = 5(x+2) + 3(x+2)(x+1) \\ 10x+10 = 5x+10+3(x^2+2x+x+2) \\ 10x+10 = 5x+10+3x^2+9x+6 \\ -10x-10 \quad -10x-10 \end{cases}$$

$$0 = 3x^2+4x+6$$

quadratic formula, factoring

$$x = \frac{-4\pm\sqrt{16-4(3)(6)}}{2(3)} = \frac{-4\pm i\sqrt{56}}{6}$$

Decimals +1

Complex

both answers +1.5

6. (WordProblems#8) Potassium ferrate has been considered for use in batteries but costs \$16 per gram. You have a battery case that is currently *full* with 50 grams of a mixture that is 10% potassium ferrate.

   (a) [3] Find an expression for the percentage (as a decimal) of potassium ferrate in the battery case as a function of how much potassium ferrate is added. Note you might need to make room in the battery case before something can be added.

   (b) [2] You would like to build the battery but you need a higher concentration of the potassium ferrate (40% should do it). What is the minimum amount of potassium ferrate you have to buy and add to the battery case to get the cathode to work?

(a) percentage of potassium ferrate = $\frac{\text{amount of potassium ferrate}}{\text{total amount of mix}}$ +1.5 +1.5

$= \frac{\text{potassium ferrate from original mix + added pot. fe}}{\text{total amount of mix}}$

$= \underbrace{\frac{.10\cdot y + x}{50}}_{+1.5}$

+1 where y = potassium ferrate from original mix; x = the added potassium ferrate

start +1.5 if [illegible]

note $x+y=50 \Rightarrow y=50-x$ +1.5

$= \frac{.10(50-x)+x}{50}$

(b) need % of pot ferrate = .4

$.4 = \frac{.10(50-x)+x}{50} = \frac{5-.1x+x}{50}$ +1

$20 = 5-.9x$ alg +1

$15 = .9x$

$x = 15/.9$

$x = 16.7\text{g}$

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