Exam 2 **TMath 120** Winter 2019

#6b typo

NAME: Key

1. [7] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

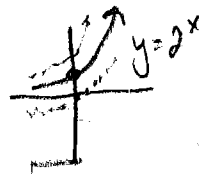
(T) F $\frac{1}{\frac{a}{2}} = \frac{2}{a}$ $1 \div \frac{a}{2} = 1 \cdot \frac{2}{a} = \frac{2}{a}$

T (F) $(x^3)^2 = x^5$ $(x^3)^2 = x^3 x^3 = (xxx)(xxx) = x^6$

T (F) $\frac{\log x}{\log y} = \frac{x}{y}$ ex $\frac{\log 100}{\log 10} = \frac{2}{1}$ but $\frac{100}{10} = 10$

(T) F $\log(\log(10)) = 0$ $\log(10) = 1 \Rightarrow \log(\log(10)) = \log(1) = 0$

(T) F $360° = 2\pi$ radians

T (F) The range of $y = 2^x + 1$ is $0 < y$  $2^x + 1$ vert shift up 1 $\Rightarrow 1 < y$

T (F) $x^{-2} = x^{\frac{1}{2}}$ $x^{-2} = \frac{1}{x^2}$ vs $x^{\frac{1}{2}} = \sqrt{x}$

Show all your work. Reasonable supporting work must be shown to earn credit.

2. [3] (WebHW9 #2) How much money should a couple invest when a child is born in order for them to gift that child with \$5,000 on their 18th birthday? Assume that the interest is 5% compounded twice a year.

$A = P(1 + \frac{r}{n})^{nt}$ (+1)

$P = ?$ (A.S.) $r = .05$ $t = 18$ $n = 2$ $A = 5{,}000$

$5{,}000 = P(1 + \frac{.05}{2})^{2 \cdot 18}$ (+1)

$5{,}000 = P \cdot 2.4325$ alg/eval (+.5)

$\Rightarrow P = \$2{,}055.50$

3. [3] (Quiz3 #1) Explain why "Simplify: $\frac{-1}{9}^2 x^2 y^3 (3x^3)^2$" is a bad quiz question and what you would replace it with.

(+1) $\frac{-1}{9}^2$ is hard to understand what exactly is being squared.

(+1) The negative in the numerator suggests it is part of $\frac{1}{9}$ and perhaps should be included in the square.

(+1) Consider instead $-\frac{1}{9}^2 x^2 y^3 (3x^3)^2$ or $(\frac{-1}{9})^2 x^2 y^3 (3x^3)^2$

4. For each equation below, find the real or complex x:

(a) [3] (exponentActivity pg4) $3(x+1)^{-2} - 7 = 0$

$\frac{3}{(x+1)^2} - 7 = 0$

$\frac{3}{(x+1)^2} = 7$

$3 = 7(x+1)^2$

$\frac{3}{7} = (x+1)^2$

$\pm\sqrt{\frac{3}{7}} = x+1$

$-1 \pm \sqrt{\frac{3}{7}} = x$

(+1) recognize

solve quad (+1)

alg/calc all (+1)

(b) [4] (WebHW8 #27) $\log_{10}(x+4) + \log_{10}(x-5) = 1$

$\log_{10}[(x+4)(x-5)] = 1$

$(x+4)(x-5) = 10$

$x^2 - x - 20 = 10$

$x^2 - x - 30 = 0$

$(x-6)(x+5) = 0$

$x = 6$ or $x = -5$

log prop (+1)

use exp/def log (+1)

solve quad (+1)

check/throw out (+1)

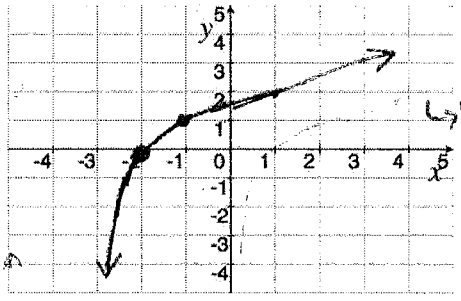
Check:

~~$x = -5$~~

$x = 6$ ✓

5. Provide a graph AND an algebraic rule/expression for each of the functions described:

(a) [3] (WebHW7 #20) A logarithmic function whose domain is $(-3, \infty)$



domain for $y = \log_2(x)$ is $(0, \infty)$

=> shift left 3 units

So $y = \log_2(x+3)$

function (+1)

by (+1)

domain (+1)

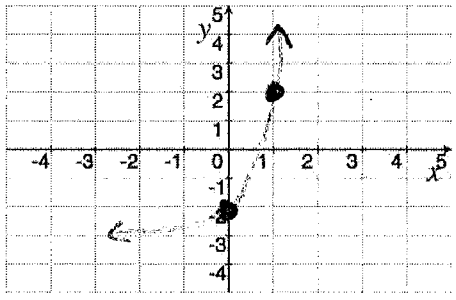
graph (+1)

shape (+1)

domain (+1)

notation/function (+1)

(b) [3] (§3.1 #56) A function of the form $b^x + c$ that passes through $(0, -2)$ and $(1, 2)$



passes thru $(0, -2)$

=> $b^0 + c = -2$

=> $1 + c = -2$

=> $c = -3$

So $b^x - 3$

passes thru $(1, 2)$

=> $b^1 - 3 = 2$

=> $b = 5$

So $5^x - 3$

function (+1)

plug in pt (+1)

solve for c (+1)

found c (+1)

graph (+1)

plotted points (+1)

match (+1)

6. Assume that we know $\log_3(x) = 8$ and $\log_3(y) = 49$.

   (a) [1] (WebHW7 #13) Rewrite both the above logarithmic equations as exponential equations.

   $3^8 = x$ $\quad 3^{49} = y$

   (b) [2] (§3.3 #16) Find $\log_3\left(\frac{y}{x^2}\right)$

   log prop (+1) $\log_3\left(\frac{y}{x^2}\right) = \log_3(y) - \log_3(x^2)$ (-1) exp prop

   plug in value (1) $= \log_3(y) - 2\log_3(x)$

   $= 49 - 2 \cdot 8 = 33$ (+1)

   OR $\log_3\left(\frac{3^{49}}{(3^8)^2}\right) = \log_3\left(\frac{3^{49}}{3^{16}}\right)$

   $= \log_3(3^{33}) = 33$ (+1) simplify

7. [4] (WebHW8 #30) Let f be an exponential of the form $f(x) = \frac{12}{2 + be^{kx}}$. It is known that $f(0) = 3$ and $f(1) = \frac{1}{2}$. Find $f(3)$.

   plug in $f(0)$ (+1) Since $f(0) = 3$

   $3 = \frac{12}{2+be^{k0}}$ (+1.5)

   $\Rightarrow 2 + be^{k0} = 4$

   $\Rightarrow 2 + b = 4$ (+1.5)

   $\Rightarrow b = 2$

   So $f(x) = \frac{12}{2+2e^{kx}}$

   Since thru $(1, \frac{1}{2})$

   $\frac{1}{2} = \frac{12}{2+2e^{k1}}$ (+1.5)

   $\Rightarrow 2 + 2e^k = 24$ (+1.5)

   $\Rightarrow e^k = 11 \Rightarrow k = \ln 11$ (+1.5)

   So $f(x) = \frac{12}{2+2e^{x(\ln 11)}}$

   plug in 3 (+1)

   $\Rightarrow f(3) = \frac{12}{2+2e^{3\ln 11}} = \frac{12}{2+2\cdot 11^3}$

   $\approx \frac{12}{2662} \approx .0045$

8. [4] (ExpLogApplications #8) *Richter Scale:* Let I be the intensity of an earthquake X and S be the intensity of a 'standard' earthquake. Then the measurement of an earthquake X as measured on the Richter Scale is:

$$\log\left(\frac{I}{S}\right)$$

In March of 2011 Japan's earthquake was 9.0 on the Richter Scale. February 2001 Tacoma had an earthquake measuring 6.8 on the Richter Scale. How many more times more intense was Japan's earthquake to the one in Tacoma?

variables (1.5) I_J the intensity of Japan's earthquake

I_T the intensity of Tacoma's earthquake

(1.5) Want $? I_T = I_J$

or $? = \frac{I_J}{I_T}$

Japan: $9.0 = \log\left(\frac{I_J}{S}\right)$ (1.5)

$\Rightarrow 10^9 = I_J/S$ (1.5)

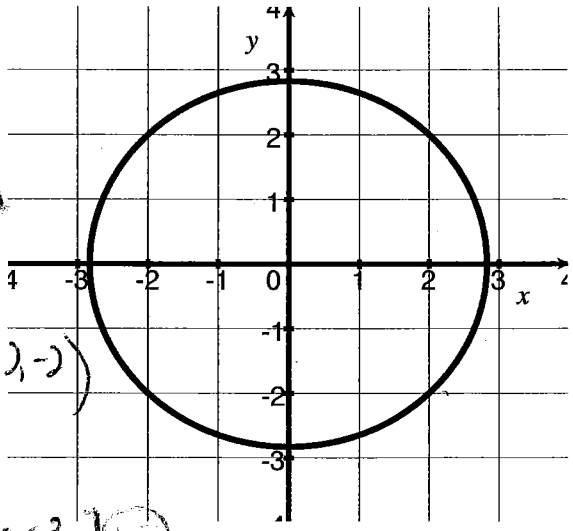
$\Rightarrow I_J = 10^9 S$

Tacoma: $6.8 = \log\left(\frac{I_T}{S}\right)$ (1.5)

$\Rightarrow I_T = 10^{6.8} S$ (1.5)

So $? = \frac{I_J}{I_T} = \frac{10^9}{10^{6.8}} = 10^{2.2} \approx$ (+1)

9. Consider the graph on the right.



(a) [2] (CircleAngles #1) Is the graph a function? Why or why not?

no. (+1) A given input like $x=2$ has 2 outputs/y-values associated with it. i.e. fails vertical line test. (+1)

(b) [1] Find *all* points on the graph when $x = 2$.

2 and -2 or $(2,2)$ & $(2,-2)$ (+1)

(c) [2] Find an equation for the graph.

eq of circle $(x-h)^2+(y-k)^2=r^2$ (+0.5)

center @ $(0,0)$ $\Rightarrow$ $(x-0)^2+(y-0)^2=r^2 \Rightarrow x^2+y^2=r^2$ (+0.5)

thru $(2,2)$ $\Rightarrow$ $2^2+2^2=r^2 \Rightarrow 8=r^2$ (+1)

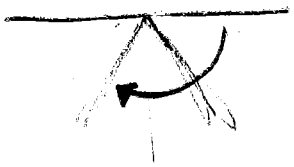
$\therefore$ $x^2+y^2=8$

10. Let $\theta = -\frac{2\pi}{3}$.

(a) [2] (WebHW10 #7) Draw the angle θ.

direction (+1)

$|2\pi/3|$ (+1)



(b) [1] (§4.1 #40) Convert θ into degrees.

$-\frac{2\pi}{3}\text{ rad} \cdot \frac{180^\circ}{\pi\text{ rad}} = -2 \cdot 60^\circ = -120^\circ$ (+1)

(+0.5)

conv. factor of same units

11. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) [5] (Word Problem2 #4) You have four ten-year subsidized loans you took out to pay for college. Below is a table of the loans taken and their respective effective annual interest rates (AIR):

loan ($)	8,000	9,000	10,000	12,000
AIR (%)	3.51	4.22	5.01	6.31

After graduation you are given the option of consolidating (that is take out one loan to pay off *all* the balances on your current loans). Assume all loans are continuously compounded once interest start accruing. You do not have a job lined up yet so you doubt you will be able be to make any payments for the three years, what rate would you need to consolidate your loans at to be in a better position three years from now?

(b) (§3.4 #98) UW Tacoma has about 5000 students. At noon two students noticed food in the SIAS office and heard it was free for the taking. A rumor began to spread and by 1pm, 200 students had heard it. It is reasonable to assume that the number of people who have heard the rumor is modeled by $\frac{5000}{1+ae^{kt}}$ where t is the number of hours since noon and $f(t)$ is the number of students who have heard the rumor. Find when $\frac{3}{4}$ of the student body at UW Tacoma has heard the rumor.

(a) start (+.5) use Pe^{rt} (+.5)

without consolidating:

$8000e^{.0351\cdot 3}$
$9000e^{.0422\cdot 3}$
$10{,}000e^{.0501\cdot 3}$
$+12{,}000e^{.0631\cdot 3}$

$\$45{,}228.75$ (+1)

to be in a better position, find r so

$45{,}228.75 = (8000+9000+10{,}000+12{,}000)e^{r\cdot 3}$ (+1.5)

$45{,}228.75 = 39{,}000e^{3r}$

$\Rightarrow 1.15971 = e^{3r}$ (+1)

$\Rightarrow \ln(1.15971) = 3r$ (+1)

$\Rightarrow \frac{1}{3}\ln(1.15971) = r$

$.04989 = r$

so less than 4.989% (+.5)

use values (+.5)

alg (+1)

(b) start (+.5)

need to find t so that $(3/4)5000 = \frac{5000}{1+ae^{kt}}$ (+.5)

First we need to find a & k

$(0,2) \Rightarrow 2 = \frac{5000}{1+ae^{k\cdot 0}} \Rightarrow 1+a = 2500 \Rightarrow a = 2499$ (+.5) (+.5)

$(1,200) \Rightarrow 200 = \frac{5000}{1+2499e^{k(1)}} \Rightarrow 200(1+2499e^{k}) = 5000$ (+.5)

$\Rightarrow 1+2499e^{k} = 25 \Rightarrow 2499e^{k} = 24$

$\Rightarrow e^{k} = 24/2499 \Rightarrow k = \ln\left(\frac{24}{2499}\right) \approx -4.65$ (+1)

So now return to original

$(3/4)5000 = \frac{5000}{1+2499e^{-4.65t}} \Rightarrow 3750(1+2499e^{-4.65t}) = 5000$

$\Rightarrow 1+2499e^{-4.65t} = 4/3 \Rightarrow 2499e^{-4.65t} = \frac{1}{3}$

$\Rightarrow e^{-4.65t} = \frac{1}{3\cdot 2499} \Rightarrow -4.65t = \ln\left(\frac{1}{3\cdot 2499}\right)$

$\Rightarrow t = 1.9$ hrs before 2pm (+1)

~~13~~ ~~13~~ ~~11~~ ~~8~~ ~~5~~

$$\begin{array}{r} {}^{1}26 \\ 24 \\ \hline 50 \end{array}$$