Exam 1 **TMath 120** Autumn 2018

NAME: Key

1. [3] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T (F) $\frac{1}{b+1}+\frac{1}{b}=\frac{3}{b+1}$ b $\frac{1}{b+1}+\frac{1}{b}\cdot\frac{b}{b}=\frac{b+b+1}{b(b+1)}=\frac{2b+1}{b(b+1)}$

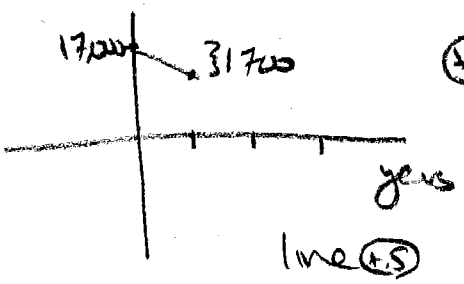
T (F) $(1-2i)(4-i)=6-9i$ $=4-i-8i+2i^2=4-9i-2=2-9i$

(T) F $\frac{-1}{3-i}=\frac{-3}{10}-\frac{1}{10}i$ $\frac{-1}{3-i}\cdot\frac{3+i}{3+i}=\frac{-3-i}{9+3i-3i-i^2}=\frac{-3-i}{9+1}$

Show all your work. Reasonable supporting work must be shown to earn credit.

2. A tractor is purchased for \$17,000 and depreciates linearly at a rate of \$1,700 a year.

   (a) [3] (Quiz1 #1) Write a function that returns the value of the tractor at a certain time. Be sure to define any variables that you create!



(+1) [t = time since bought tractor (in years)
v = value of tractor

$v=\frac{-1700}{1}t+17{,}000$ (+1.5) (+1.5)

line (+.5) start (+.5)

   (b) [2] (WebHW2 #22) When will the tractor have no value?

when will $v=0$ (+1)

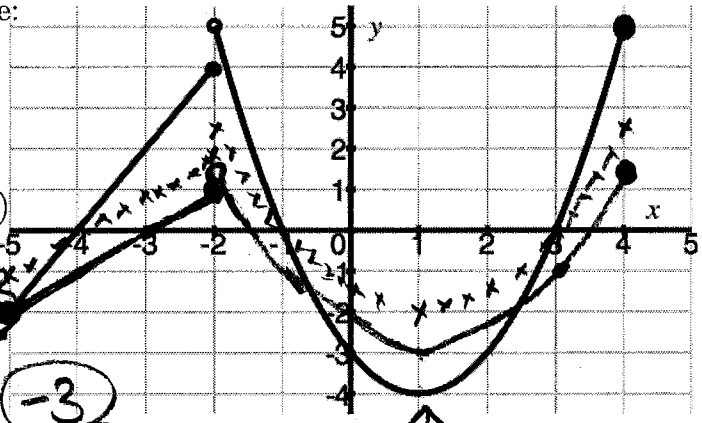
$0=-1700t+17{,}000$

$\Rightarrow \frac{1700t}{1700}=\frac{17{,}000}{1700}$

$t=10$ so in 10 years (+1)

or look for x-intercept (+1)

3. Let f be the piece-wise defined function comprised a line and a parabola shown below.



(a) Estimate the following *if* possible:

i. [1] (Quiz1 #3) $f(-1)$

0

ii. [1] (WebHW3 #1) $(f+f)(2)$

$f(2)+f(2) = -3+-3 = -6$ (+.5) (+.5)

iii. [1] (Quiz2 #2c) $(f \circ f)(-1)$

$f(f(-1)) = f(0) = -3$ (+.5) (+.5)

iv. [1] (§1.1 #104) the minimum of f.

or @ $(1,-4)$

v. [1] (Transformations Activity #5) all possible x such that $f(x) = -4$.

@ $x = 1$

(b) [4] (PracticeExam #4) Find the formula for f in the indicated form:

$$f(x) = \begin{cases} 2x+3 & \text{if } -5 \le x \le -2 \\ (x-1)^2-4 & \text{if } -2 < x \le 4 \end{cases}$$

line (+.5)

$m = \frac{\text{rise}}{\text{run}} = \frac{2}{1}$ (+.5)

thru $(-4,0)$ so (+1)

$0 = 2(-4)+b \Rightarrow b = 8$

parabola (+.5)

vertex @ $(1,-4) \Rightarrow a(x-1)^2-4 = y$ (+1) (+.5)

thru $(2,-3) \Rightarrow a(2-1)^2-4 = -3$

$\Rightarrow a = -3+4 = 1$

(c) [3] (WebHW3 #16) Graph $\frac{1}{2}f(x) - 1$.

1) vertical stretch by 1/2 / mult y by 1/2 (+.5)

2) move down by 1 (+.5)

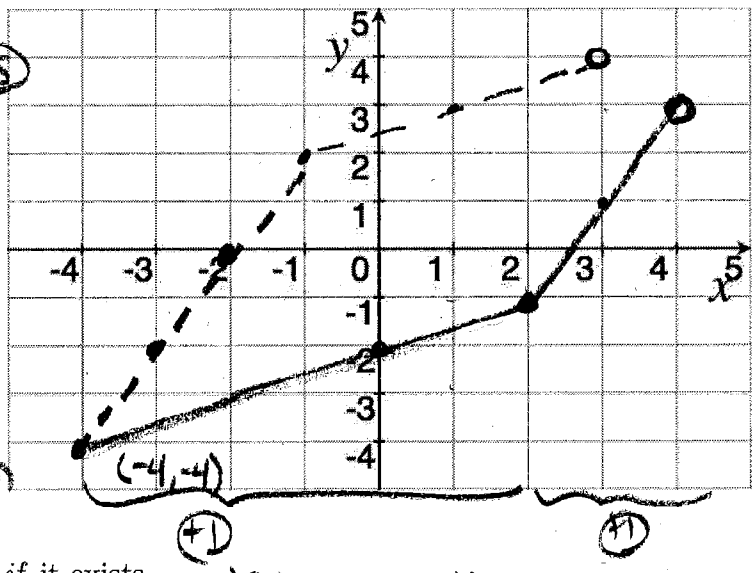
order (+.5)

4. Let h be the *function* defined by: $h(x) = \begin{cases} \frac{1}{2}x - 2 & -4 \le x \le 2 \\ 2x - 5 & 2 < x < 4 \end{cases}$

(a) [1] (§1.3 #32)
Find $h(1)$ $1 \in [-4, 2]$ +.5
$\frac{1}{2}(1) - 2 = -\frac{3}{2}$ +.5

(b) [1] (WebHW1 #13)
What is the domain of h?
$-4 \le x < 4$ +.5
x's +.5

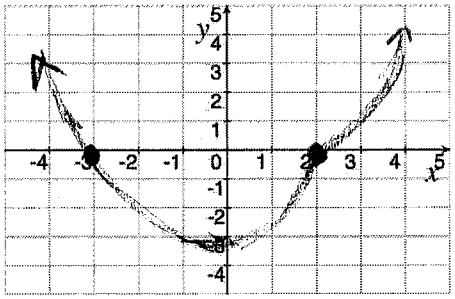
(c) [3] (WebHW2 #11)
Graph h on the axes.
end points/domain
(+1)



(d) [2] (§1.7 #44) Graph h^{-1} *if* it exists.
swap x's & y's (+1)
note: passes the horiz. line test

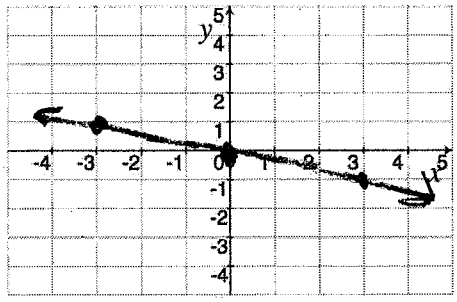
5. Provide a graph AND an algebraic rule/expression for each of the functions described below:

(a) [3] (Polynomial Activity#4) A 4 degree polynomial whose only roots are 2 & -3.



There are LOTS of right answers?
$\frac{1}{8}(x+3)(x-2)^3 = y$
rule
degree (+.5)
factors (+1)
graph
end behavior (+.5)
roots (+.5)
multip (+.5)

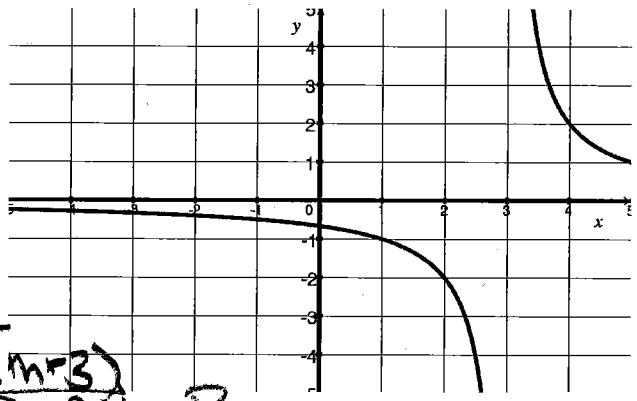
(b) [3] (Line Activity#13) A line perpendicular to $y = 3x - 2$.



$\perp$ to $y = 3x - 2$
$\Rightarrow$ slope is $-1/3$

so
$y = -\frac{1}{3}x$
There are LOTS of right answers?
rule
slope (+1)
line (+.5)
graph
line (+.5)
match (+.5)
slope (+.5)

6. Let $\alpha(x) = \dfrac{2}{x-3}$. The graph of α is shown below.



(a) [2] (§1.3 #32) Find $\alpha(x+h) - \alpha(x)$ and simplify.

$\frac{2}{x+h-3} - \frac{2}{x-3}$]+1

$\frac{2(x-3) - 2(x+h-3)}{[x-3](x+h-3)} = \frac{-2h}{(x-3)(x+h-3)}$ +1 factoring?

(b) [3] (§1.7 #78) Find the algebraic rule/expression for $\alpha^{-1}(x)$.

$x = \frac{2}{y-3}$ +1

$x(y-3) = 2$

$xy - 3x = 2$

$xy = 2 + 3x$

$y = \frac{2+3x}{x}$

y on one side +1.5

algebra +1

signs +1.5

7. [3] (Autumn120Exam1 #7) Find the real or complex solutions to $\frac{2}{5}(x-1)^2 + \frac{7}{2} = 0$.

$\frac{2}{5}(x-1)^2 + \frac{7}{2} = 0$ OR FOIL => quadratic formula

both sides +1

$\frac{2}{5}(x-1)^2 = -\frac{7}{2}$

algebra +1

$(x-1)^2 = -35/4$

complex +1

$x - 1 = \pm i\sqrt{\frac{35}{4}}$

$x = 1 \pm i\frac{\sqrt{35}}{2}$

8. [4] (WebHW6 #17) The area of a rectangle is $3x^4 - 6x^3 + 14x^2 - 4x + 8$ square centimeters. The length is $x^2 - 2x + 4$ cm. Find its width (as a function of x!).

Area +1

Area = length · width +1.5

long ÷ +1.5

$\Rightarrow 3x^4 - 6x^3 + 14x^2 - 4x + 8 = (x^2 - 2x + 4) \cdot \text{width}$

$\Rightarrow \text{width} = \dfrac{3x^4 - 6x^3 + 14x^2 - 4x + 8}{x^2 - 2x + 4}$

$= 3x^2 + 2$

$$\begin{array}{r} 3x^2+2 \\ x^2-2x+4 \overline{)\,3x^4-6x^3+14x^2-4x+8} \\ -(3x^4-6x^3+12x^2) \\ \hline 2x^2-4x+8 \\ -(2x^2-4x+8) \\ \hline 0 \end{array}$$

9. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

   (a) (WebHW4 #14) A rancher with 150 meters of fence would like to enclose a rectangular region next to a river which can serve as a natural boundary meaning no fencing is needed.

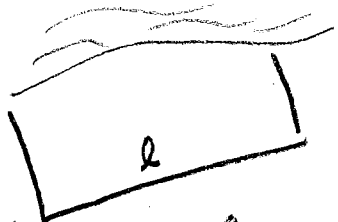
      i. [3] Write a rule/expression that returns the area enclosed by the fence as a function of the width of the rectangular region.

      ii. [2] Find the maximum area that can be enclosed.

   (b) (WordProblems #3)) You have 8oz of coffee which has about .003oz of caffeine sitting in a 24oz cup. You would like a higher dose of caffeine and plan to add espresso which has .0028oz per 2oz shot.

      i. [3] Write a rule/expression that returns the percentage (as a decimal) of caffeine in the cup as a function of espresso that you add.

      ii. [2] Give the domain of this function and justify your answer.

(a) i) 

start (+.5) variables (+.5)

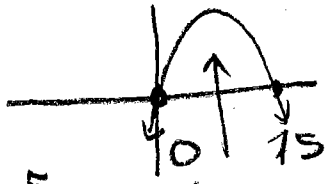
Area = $w \cdot l$ (+1) need in terms of just w.

(+1) [note $2w + l$ = total fencing
$2w + l = 150$
$\Rightarrow l = 150 - 2w$
sub into above Area ∴]

Area = $w(150 - 2w)$

ii) Area = $150w - 2w^2 = -2w^2 + 150w$

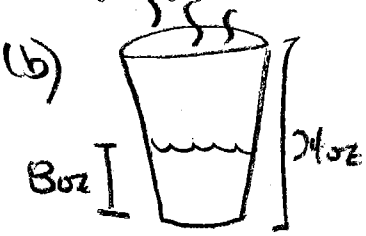
(+1) Max @ vertex



(+1) so @ $w = 37.5$

Area is ≈ 4921

$\frac{1}{2}$ way between

(b) 

start (+.5) variables (+.5)

x = oz of esp. added

i) caffeine % = $\frac{\text{total caffeine}}{\text{liquid in cup}}$ (+1)

$= \frac{\text{original caffeine} + \text{espresso caffeine}}{\text{original coffee} + \text{espresso added}}$

$= \frac{.003 + \frac{.0028 \text{ caffeine}}{2 \text{ esp}} \cdot x \text{ esp}}{8 + x}$ (+.5) (+.5)

$= \frac{.003 + .0014x}{8 + x}$

ii) $0 \le x \le 16\text{oz}$ (+1)

(+.5) $0 \le x$ b/c we are adding liquid

(+.5) $x \le 16\text{oz}$ b/c the liquid needs to fit inside the cup

~~8~~ 20
~~10~~ 25
~~15~~ 5
~~10~~ ---
5 50