Exam 2 **TMath 120** 2018 Winter

NAME: Key

1. [7] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let x & y be positive real numbers.

   T (F) $\frac{3x+y}{3z} = \frac{x+y}{z}$ $\frac{3x+y}{3z} \neq \frac{3(x+y)}{3z} = \frac{x+y}{z}$

   T (F) $2^x + 5x = 8$ is a polynomial x is an exponent in 2^x

   T (F) $4^{\frac{1}{2}} = 4^{-2}$ $4^{\frac{1}{2}} = \sqrt{4} = 2$ x $\frac{1}{16} = \frac{1}{4^2} = 4^{-2}$

   (T) F $\log_5(\log_5(5)) = 0$ $\log_5(\log_5(5)) = \log_5(1) = 0$

   (T) F $(x^2)^3 = x^6$ $x^2 x^2 x^2 = (x\,x)(x\,x)(x\,x) = x^6$

   T (F) $x^0 = 0$ $x^0 = 1$

   T (F) $\log_2(x+y) = \log_2(x) + \log_2(y)$ $\log_2(x) + \log_2(y) = \log_2(xy)$

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [3] Explain what an exponential function is as you would to a 5th grader.

true (+1)
complete (+1)
correct level (+.5)
start (+.5)

3. [4] Write a polynomial p that satisfies the following criteria:

- as x goes to ∞, then y goes to $-\infty$
- -2, 1, and 3 are the only roots.

Note: there is more than one right answer!

$p(x) = (x+2)(x-1)(x-3)$

wrong end behavior => mult by negative

$p(x) = -2(x+2)(x-1)(x-3)$

or $= -2(x^2+x-2)(x-3)$

$= -2(x^3-2x^2-5x+6)$

4. [3] (LogPractice #1) If you invest \$5,500 with an annual rate of 7% compounded quarterly, how much money do you have after 10 years?

amount of \$ after t years $= P\left(1+\frac{r}{n}\right)^{nt}$

$= 5{,}500\left(1+\frac{.07}{4}\right)^{4 \cdot 10}$

$= 11{,}008.79$

sole formula for 1.5 pts

5. [5] The area of a rectangle is $5x^4 - 15x^3 + 22x^2 - 6x + 8$ cm^2. Its length is also a function of x and is $x^2 - 3x + 4$ cm. Find the width as a function of x.

start

area = length · width

$5x^4 - 15x^3 + 22x^2 - 6x + 8 = (x^2 - 3x + 4) \cdot \text{width}$

$\Rightarrow \text{width} = \dfrac{5x^4 - 15x^3 + 22x^2 - 6x + 8}{x^2 - 3x + 4}$

long division? algorithm

setup

$$\begin{array}{r} 5x^2 + 2 \text{ R } 0 \\ x^2-3x+4 \overline{)\,5x^4 - 15x^3 + 22x^2 - 6x + 8} \\ -(5x^4 - 15x^3 + 20x^2) \\ \hline 2x^2 - 6x + 8 \\ -(2x^2 - 6x + 8) \\ \hline 0 \end{array}$$

$\Rightarrow \text{width} = 5x^2 + 2$

6. Simplify:

(a) [2] (WebHW7 #7)

$$\frac{5(x^{-3}y)^{-2}}{(5x)^2} = \frac{5(x^{-3})^{-2}y^{-2}}{5^2x^2} = \frac{5x^6y^{-2}}{5^2x^2}$$

$$= \frac{x^4}{5y^2}$$

(+1) (+1.5) (+.5)

(b) [2] (PracticeExam #4)

$$4 + \log_3\left(\frac{1}{9}\right)$$

$4 + \log_3\left(\frac{1}{3^2}\right)$

$4 + \log_3(3^{-2})$

$4 - 2 = 2$

or $4 + \log_3(1/9)$

$\log_3(3^4) + \log_3(1/9)$

$\log_3\left(\frac{3^4}{9}\right) = \log_3(3^2) = 2$ (+.5)

use log/exp undoing (+.5)

log/exp property (+1)

7. Find all x that satisfy:

(a) [4] (WebHW8 #22) $\log_5(2x+5) - \log_5(x+5) = 1$

log prop (+1)

use exp to undo (+1)

alg/order of op (+1)

$\log_5\left(\frac{2x+5}{x+5}\right) = 1$

$\frac{2x+5}{x+5} = 5^1$

$2x+5 = 5(x+5)$

$2x+5 = 5x+25$

$-20 = 3x$

$-20/3 = x$

check (+.5)

(+1.5) No solution?

$\log_5(-20/3+5) = \log_5 -5/3$ Error

(b) [3] (LogPropertySheet#4) $\frac{11}{1+e^{2x}} = 3$

alg/order of op (+1)

use log to undo (+1)

alg/order of op after log (+.5)

$1+e^{2x}\cdot\frac{11}{1+e^{2x}} = 3(1+e^{2x})$

$\frac{11}{3} = \frac{3(1+e^{2x})}{3}$

$11/3 = 1 + e^{2x}$

$-1 \quad -1$

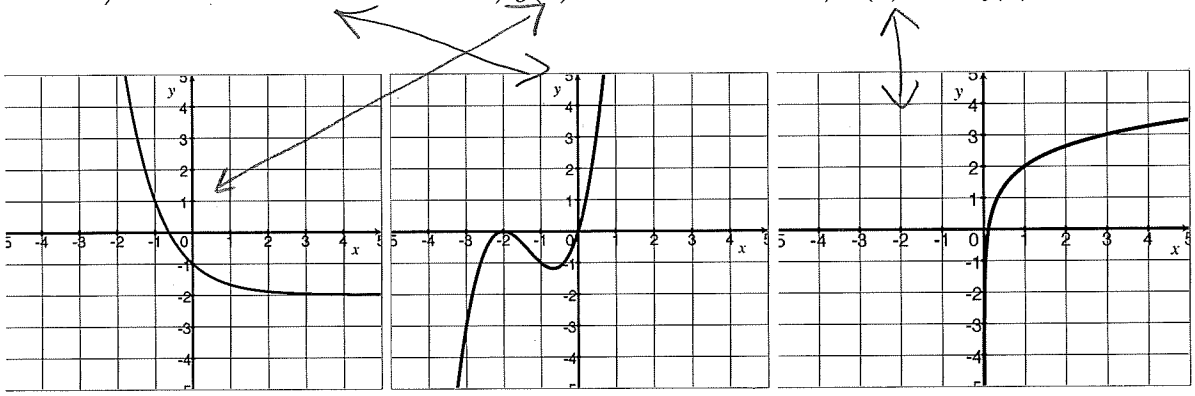
$8/3 = e^{2x}$

$\ln(8/3) = 2x \Rightarrow x = \frac{1}{2}\ln(8/3) \approx .4904$

(+1.5) Check ✓

8. [3] (Quiz3 #1 &4) Three different function forms are given below. Match each with the graph it could make if the parameters (a, b, c, etc) were chosen correctly.

i) $ax^3 + bx^2 + cx + d$ ii) $g(x) = b^x + c$ iii) $h(x) = \log_b(x) + c$



9. [3] For one of the above, find the parameters (a, b, c, etc) that describes the graph that is given. No, doing more will not earn you extra credit.

correct form (+.5)

(+1) roots $-2, 0 \Rightarrow (x+2)x$
touches @ $-2 \Rightarrow (x+2)^2 x$
passes thru $(-1,-1)$
(+1.5) $-1 = a(-1+2)^2(-1)$
$-1 = -a \Rightarrow a = 1$
$y = (x+2)^2 x = x^3 + 4x^2 + 4x$

ii) normally passes thru $(0,1)$
$\Rightarrow$ vert shift down 2
$g(x) = b^x - 2$
passes thru $(-1,1)$
$1 = b^{-1} - 2 \Rightarrow 3 = 1/b$
$g(x) = (\frac{1}{3})^x - 2$

iii) normally thru $(1,0) \Rightarrow$ up 2
$h(x) = \log_b(x) + 2$
passes thru $(3,3)$
$3 = \log_b(3) + 2$
$1 = \log_b(3) \Rightarrow b = 3$
$h(x) = \log_3(x) + 2$

10. [6] (§3.4 #98) UW Tacoma has about 5000 students. At noon two students noticed food in the SIAS office and heard it was free for the taking. A rumor began to spread and by 1pm, 200 students had heard it. It is reasonable to assume that the number of people who have heard the rumor is modeled by $\frac{5000}{1 + ae^{kt}}$ where t is the number of hours since noon and $f(t)$ is the number of students who have heard the rumor. Find when $\frac{3}{4}$ of the student body at UW Tacoma has heard the rumor.

start (+1.5) Need to find t so that $(3/4)5000 = \frac{5000}{1+ae^{kt}}$ (+1)

(+.5) First need to find a and k

$(0,2)$ (+.5) $\Rightarrow 2 = \frac{5000}{1+ae^{k\cdot 0}} \Rightarrow 2 = \frac{5000}{1+a} \Rightarrow 1+a = 2500 \Rightarrow a = 2499$ (+.5)

$(1,200)$ (+.5) $\Rightarrow 200 = \frac{5000}{1+2499e^{k(1)}} \Rightarrow 200(1+2499e^k) = 5000 \Rightarrow 1+2499e^k = 25$ (+1)

$\Rightarrow 2499e^k = 24 \Rightarrow e^k = 24/2499 \Rightarrow k = \ln(24/2499) \approx -4.65$

use values (+.5) So find t given $(3/4)(5000) = \frac{5000}{1+2499e^{-4.65t}}$

(+1) $3750(1+2499e^{-4.65t}) = 5000$
$\Rightarrow 1+2499e^{-4.65t} = 4/3$
$\Rightarrow 2499e^{-4.65t} = 1/3$
$\Rightarrow e^{-4.65t} = \frac{1}{3\cdot 2499}$

$\frac{-4.65t}{-4.65} = \frac{\ln(\frac{1}{3\cdot 2499})}{-4.65}$

$t = 1.9$ hrs before 2 pm

11. [5] (LogsInPracticeSheet #9&5) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit. No, doing both questions will not earn you extra credit.

(a) *Decibels:* The loudness of a sound is measured in decibels and is related to the intensity I by

$$10 \log\left(\frac{I}{S}\right)$$

where $S = 10^{-12}$ W/m^2.

Normal conversation has a sound level of about 65 decibels. How many more times intense than normal conversation is the sound an iPod operating at the French maximum of 100 decibels?

(b) *Newton's Law of Cooling:* If D is the initial temperature difference between an object and its surroundings, and if its surroundings have a temperature T, then the temperature of the object A and time t is modeled by:

$$A = T + De^{-kt}$$

where k is a positive constant that depends on the type of object.

Initially coffee has a temperature of 200°F in a room that is 70°. After ten minutes the temperature is 150°. When will the coffee get down to 100°?

(a) $?: I_{conv} = I_{max} \Rightarrow ? = \frac{I_{max}}{I_{conv}}$ (+.5)

I_{conv}: intensity of normal conversation; I_{max}: intensity of max level

start (+.5) do this (+.5) understand input (+.5)

Given $65 = 10\log\left(\frac{I_{conv}}{S}\right)$

$\Rightarrow 6.5 = \log\left(\frac{I_{conv}}{S}\right)$ (+.5)

$\Rightarrow 10^{6.5} = \frac{I_{conv}}{S} \Rightarrow I_{conv} = S10^{6.5}$

use one or the other (+1)

Given $100 = 10\log\left(\frac{I_{max}}{S}\right)$

$\Rightarrow 10 = \log\left(\frac{I_{max}}{S}\right)$ (+.5)

$\Rightarrow 10^{10} = \frac{I_{max}}{S} \Rightarrow I_{max} = S10^{10}$

So (+1) $? = \frac{I_{max}}{I_{conv}} = \frac{S10^{10}}{S10^{6.5}} = 10^{3.5} \approx 3162$ times

(b) $A = 70 + (200 - 70)e^{-kt}$ (+.5) (+.5)

want to find t so $100 = 70 + 130e^{-kt}$ (+.5)

need to find k first....

$(10, 150) \Rightarrow 150 = 70 + 130e^{-k \cdot 10}$ (+.5)

$\Rightarrow 80 = 130e^{-10k}$

$\Rightarrow 8/13 = e^{-10k}$ (+1)

$\Rightarrow \ln(8/13) = -10k$

$\Rightarrow k = -\frac{1}{10}\ln(8/13) \approx .04855$

So $100 = 70 + 130e^{-.04855t}$ (+.5)

$\Rightarrow 30 = 130e^{-.04855t}$

$\Rightarrow 3/13 = e^{-.04855t}$

$\Rightarrow \ln(3/13) = -.04855t$ (+1)

$\Rightarrow t = \frac{-1}{.04855}\ln(3/13)$

≈ 30.2 about 30 min