Exam 1 **TMath 120** 2018 Winter

NAME: Key

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

T (F) $\frac{3x+y}{3z} = \frac{x+y}{z}$ if true for x, y, z then let $x=1, y=1, z=1$ note $\frac{3(1)+(1)}{3(1)} = \frac{4}{3} \neq \frac{2}{1} = \frac{1+1}{1}$

T (F) $f(x+1) = f(x)+1$ let $f(x)=x^2$, $f(x+1)=(x+1)^2$ and $f(x)+1 = x^2+1$

T (F) $\sqrt{-4} = -\sqrt{4}$ note shift ↓ not shift $\sqrt{-4} = \sqrt{-1 \cdot 4} = \sqrt{-1}\sqrt{4} = i\sqrt{4}$

T (F) $\sqrt{x^2+y^2} = x+y$ $\sqrt{(x+y)^2}$ might work but $(x+y)^2 = x^2+2xy+y^2$

(T) F $\frac{1}{1+i} = \frac{1}{2} - \frac{1}{2}i$ $\frac{1}{1+i}\frac{1-i}{1-i} = \frac{1-i}{1-i^2} = \frac{1-i}{1-(-1)} = \frac{1-i}{2}$

(T) F $f(f^{-1}(2)) = 2.$ f and f^{-1} are inverses so f^{-1} "undoes" the effect of f

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [3] (§2.1 #46) Find any real or imaginary x such that $3(x-1)^2 + 4 = 4$.

start (+.5) algebra/graph (+1.5) order of op (+.5) got it (+1.5)

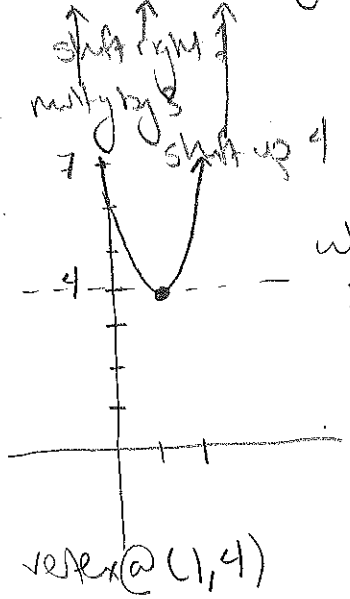
$3(x-1)^2+4 = 4$
$-4 \quad -4$
$3(x-1)^2 = 0$
$\frac{3}{3} \quad \frac{0}{3}$
$(x-1)^2 = 0$
$x-1 = \pm\sqrt{0}$
$x = 1 \pm 0$
$x = 1$

or

$3(x-1)^2+4 = 4$
$3(x^2-2x+1)+4 = 4$
$3x^2-6x+3+4 = 4$
$-4 \quad -4$
$3x^2-6x+3 = 0$
quadratic formula
$\frac{-6 \pm \sqrt{36-4(3)(3)}}{2(3)}$
$\frac{+6 \pm 0}{6} = 1$

or

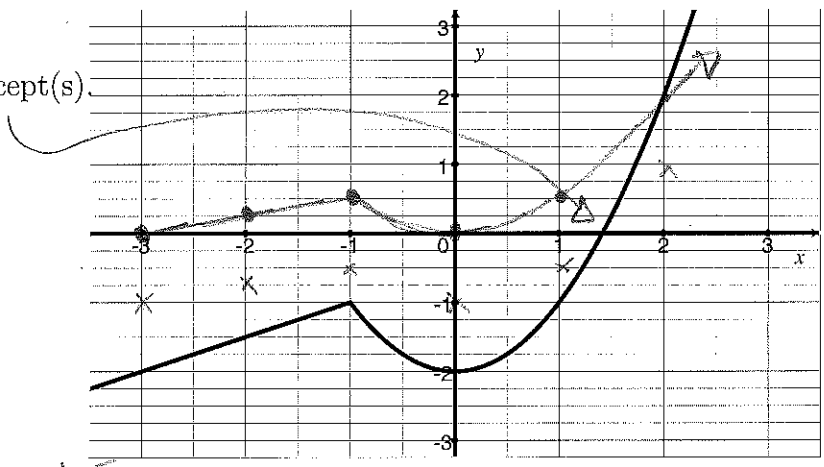
$3(x-1)^2+4 = y$
shift right 1
multiply by 3
shift up 4



when $y=4$, $x=1$

vertex @ (1,4)

3. Let f be the function comprised of a line and a parabola whose graph is below:



(a) [1] (WebHW1 #21)
Estimate the x intercept(s).

≈ 1.4

(b) [1] (§1.3 #56)
Estimate $f(-3)$

-2

(c) [2] (Practice Exam)
$(f \circ f)(0)$ (+.5)

$f(f(0)) = f(-2) = -1.5$

$0 \xrightarrow{f} -2 \xrightarrow{f} -1.5$ (+.5) comp (+1)

(d) [4] (Quiz2 #3) Find the piece-wise defined algebraic rule of f in the form below.

Typos?

$$f(x) = \begin{cases} (\frac{1}{2})x - \frac{1}{2} & \text{if } x < -1 \\ x^2 - 2 & \text{if } -1 \le x \end{cases}$$

line: $y = mx + b$ (+.5)

slope $= \frac{\text{rise}}{\text{run}} = \frac{1}{2}$ (+.5)

thru $(-1,-1)$

$\Rightarrow -1 = (\frac{1}{2})(-1) + b$

$\Rightarrow -1 = -\frac{1}{2} + b$

$+\frac{1}{2} \quad +\frac{1}{2}$

$\Rightarrow b = -\frac{1}{2}$ (+1)

So $y = \frac{1}{2}x - \frac{1}{2}$

quadratic: $y = a(x-h)^2 + k$ (+.5)

vertex $= (h,k)$ $(0,-2)$ (+1)

$\Rightarrow y = a(x-0)^2 - 2$

or

vert shift down 2
no horiz shift
no vert stretch

thru $(1,-1)$

$\Rightarrow -1 = a(1-0)^2 - 2$

$\Rightarrow -1 = a \cdot 1 - 2$

$+2 \quad +2$

$\Rightarrow a = 1$

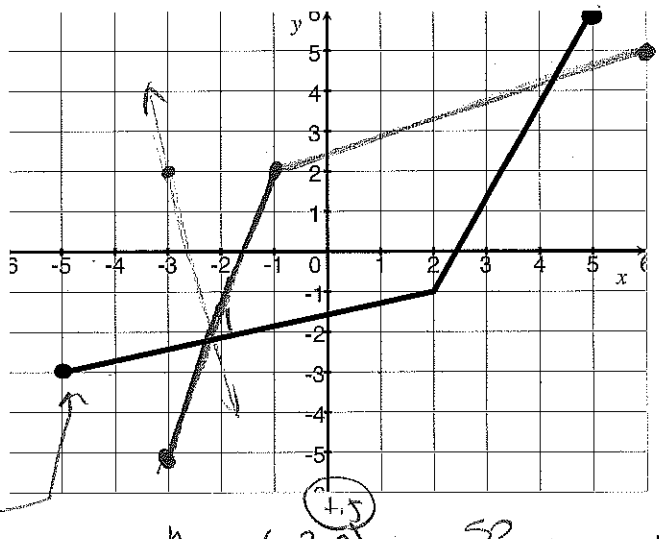
So $y = (x-0)^2 - 2$ (+.5)

(e) [3] (WebHW3 #19) Graph the function $g(x) = \frac{1}{2}f(x) + 1$ on the axes above.

(+.5) multy by $\frac{1}{2}$ order (+.5)

(+.5) shift up 1 get it (+1.5)

4. Let z be the function whose graph is shown to the right.



(a) [1] (TransformationSheet #5) Find the range of z. (+.5)

y-values (+.5) $[-3, 6]$

(b) [2] (PracticeExam #8) Find the equation of the line that passes though $(-3, 2)$ and makes a right angle when intersecting z.

looking for $y = mx + b$ (+.5)

m = slope of $\perp$ to

= opp. reciprocal of $2/7$ (.5)

$= -7/2$ (+.5)

thru $(-3,2)$

$2 = -7/2(-3) + b$

$\Rightarrow 2 - 21/2 = -17/2 = b$

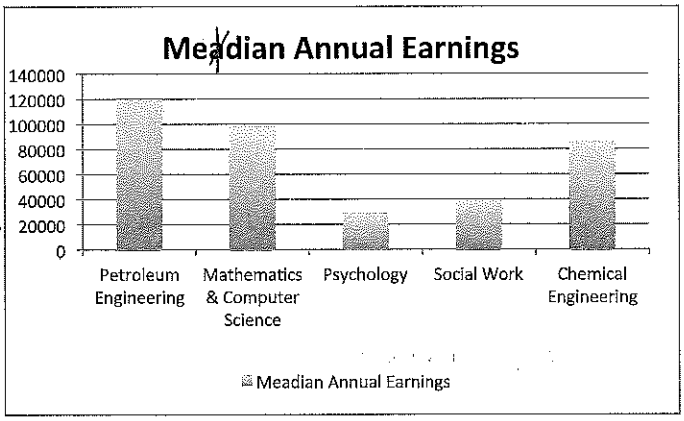
so $y = -7/2\,x - 17/2$

or $y - 2 = -7/2(x+3)$

(c) [2] (§1.7 #40) Draw the graph of z^{-1} on the above axes *if* it exists.

Sample x's and y's (+.5)

5. Let the domain of f be undergraduate majors and $f(x)$ be the median annual earnings of people with the the undergraduate major x.



(a) [2] (PracticeExam #9) Is f a function? Why are why not?

yes (1)

(1) Only one median annual earning will be returned for each major. (1)

(b) [1] (PracticeExam #9) Find an x such that $f(x) \leq 40,000$. Note: the median household income in the US is \$59,039.

Psychology and Social Work both work (1)

6. Let $q(x) = \dfrac{9x+2}{x+6} + 3$ and $r(x) = x + 3$

(a) [2] (WebHW2 #16) Find the average rage of change of q from 0 to 2.

slope of line between $(0, q(0))$ and $(2, q(2))$ $= \dfrac{q(2)-q(0)}{2-0}$ (+.5) $= \dfrac{\left(\frac{13}{8}+3\right)-\left(\frac{2}{6}+3\right)}{2}$ (+.5)

$= \left(\frac{10}{4} - \frac{1}{3}\right) \div 2 = \left(\frac{5}{2} - \frac{1}{3}\right)\left(\frac{1}{2}\right)$

$= \left(\frac{15-2}{6}\right) \cdot \frac{1}{2} = \frac{13}{12}$ (+.5) $= 1.08333$

(b) [1] (§1.6 # 20) Find $(q - r)(x)$.

$\left(\frac{9x+2}{x+6} + 3\right) - (x+3)$ (+1)

(c) [3] (WebHW5 #16) Given that q is one-to-one (ie has an inverse), find q^{-1}.

$y = \frac{9x+2}{x+6} + 3$

(+.5) inverse swaps x's & y's

(+.5) $x = \frac{9y+2}{y+6} + 3$

now solve for y algebra (+1) order of op (+1)

$x - 3 = \frac{9y+2}{y+6}$

$(x-3)(y+6) = 9y+2$

$xy + 6x - 3y - 18 = 9y + 2$

$xy - 3y - 9y = -6x + 18 + 2$

$xy - 12y = -6x + 20$

$y(x-12) = -6x + 20$

$y = \frac{-6x+20}{x-12}$

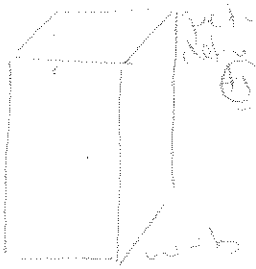
7. [2] (§A.8 #18) Compute the product of $-1 + i$ and $2 - 10i$.

$(-1+i)(2-10i) = -2 + 10i + 2i - 10i^2$ FOIL (+1)

$= -2 + 12i - 10(-1)$ (+.5)

$= 8 + 12i$ simplify (+.5)

8. [4] (PracticeExam #11) A rectangular box with a volume of 60 ft^3 has a square base. Find a function that models its surface area S in terms of the length of one side of its base.



9. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) You have 8oz of coffee which has about .003oz of caffeine sitting in a 24oz cup. You would like a higher dose of caffeine and plan to add $x$oz of espresso which has .0028oz per 2oz shot.

- Write an expression that returns the percentage (as a decimal) of caffeine in the cup as a function of espresso that you add.
- Give the domain of this function and justify your answer.

(b) You have \$5500 in a ROTH retirement fund and would like a return of 4% (to do slightly better than inflation which has been 2.9% lately). There are 5 year CDs (certificate of deposits) being offered with an annual rate of 3.05% and index funds (a collection of stocks from companies included in measures like the S&P 500) that returned 8.2% since the 1990's (Thomson Reuters, 2010 S&P 500 Composite Index total return for the period 12/31/1989 to 12/31/2009). How much money do you relegate to a CD and how much money do you put in an index fund to get an annual return of 4%?

a) let x be the amount of espresso added

percentage of caffeine = $\frac{\text{Caffeine in cup}}{\text{total liquid in cup}}$ (+1.5)

$= \frac{\text{original caffeine} + \text{added caffeine}}{\text{original liquid} + \text{new liquids}}$ (+1.5)

start (+1.5) variables (+1.5)

$= \frac{.003 + \left(\frac{.0028 \text{ caffeine}}{2 \text{ oz esp}}\right) \cdot x \text{ esp}}{8 \text{ oz} + x}$ (+1.5)

$= \frac{.003 + .0014x}{8 + x}$

Domain $8 + x \neq 0 \Rightarrow x \neq -8$ (+1 pt)

but we can't have negative espresso so that doesn't make sense

(+1) $x \geq 0$ because we can only add esp.

(+1) $x \leq 16$oz since that is all of the room in the cup

so $[0, 16]$

b) let x = \$ into CD
y = \$ into index funds

start (+1.5) variables (+1)

(+.5) $x + y = 5500$ (+1)

Interest want = Interest from CD + Interest Index

$.04 \cdot 5{,}500 = .0305x + .082y$ (+.5) (+1.5)

Since $x + y = 5500 \Rightarrow x = 5500 - y$ substitute a (+1.5)

$220 = .0305(5500 - y) + .082y$

$220 = 167.75 - .0305y + .082y$

$-167.75 \quad -167.75$

$52.25 = .0515y$

$\Rightarrow y = 1{,}014.56$ into index funds

So $x = 5{,}500 - 1{,}014.56$ (+1.5)

$= 4{,}485.44$ into CD's.

20
13
11

44