

NAME: Key

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let  $f$  be a function, and  $x$ ,  $y$ , and  $z$  be real numbers with  $z \neq 0$ .

$$\text{T } (\text{F}) \quad \frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$$

$$\frac{b}{ba} + \frac{a}{ba} = \frac{b+a}{ab}$$

$$(\text{T}) \text{ F } \text{ The degree of } 7x^5 - 4.56x^4 - 7x^5 + 8 \text{ is } 4$$

$$\text{T } (\text{F}) \quad 2 \cdot 3^x = 6^x$$

$$(\text{T}) \text{ F } \log_3 7 \text{ is approximately } 1.771 \quad \log_3(7) = \frac{\log 7}{\log 3}$$

$$(\text{T}) \text{ F } 2 \text{ is a root of } f(x) = x^4 - 3x^2 - x - 2$$

$$(2)^4 - 3(2)^2 - 2 - 2 = 16 - 12 - 4 = 16 - 16 = 0$$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [3] Explain what a logarithmic function is as you would to a 5th grader.

Start  $\frac{1}{5}$   
the  $\frac{1}{1}$   
dec/complete  $\frac{1}{1}$   
sense  $\frac{1}{5}$

When you have a variable in the sky or exponent, such as  $5 = 2^x$  we can use the log to get at the  $x$ . The logarithm can undo the exponentiation so

$$\log(5) = \log(2^x)$$

$\Rightarrow \log(5) = x$  since the log & the expy base 2 undo each other.

By using the exponential function we can point by point build a function  $y = \log_2 x$ .

Note not any number can be a base? Any positive  $^1$  number could work but if the base is 1 then it doesn't either.

3. Let  $f$  be the polynomial function whose graph is below.

- (a) [1] Estimate  $f(0)$ .

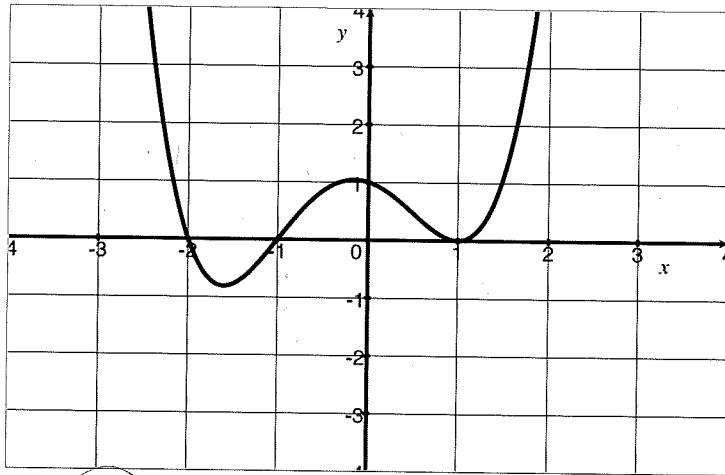
1

- (b) [2] Estimate the range of  $f$ .

$$y \geq -0.7$$

- values  $+5$   
Estimate  $+1$  matches  $+5$

- (c) [3] (polynomialWks #4)



Find the polynomial of least degree with the graph of  $f$ .

- (1)  $\left\{ \begin{array}{l} -2 \text{ is a root} \Rightarrow x+2 \text{ is a factor} \\ -1 \text{ is a root} \Rightarrow x+1 \text{ is a factor} \\ 1 \text{ is a root} \Rightarrow x-1 \text{ is a factor} \end{array} \right.$

- (2)  $\left\{ \begin{array}{l} \text{crosses at } x=-2 \text{ so } (x+2) \text{ is a factor} \\ \text{crosses at } x=-1 \text{ so } (x+1) \text{ is a factor} \\ \text{does not cross at } x=1 \text{ so } (x-1)^2 \text{ is a factor} \end{array} \right.$

4. Simplify the following:

(a) [2] (WebHW10 #6)  $2\sqrt{b}(3a^2b)^2$

$$2\sqrt{b}(3a^2b)(3a^2b)$$

$$2\sqrt{b}9(a^2)^2b^2$$

$$2\sqrt{b}9a^4b^2$$

$$18a^4\sqrt{b}^2b^2$$

(b) [2] (PracticeExam #7)  $\log\left(\frac{10^2 \cdot 10^4}{10}\right)$

$$\log\left(\frac{10^2 \cdot 10^4}{10}\right)$$

$$= \log\left(\frac{10^6}{10}\right)$$

$$= \log(10^5) = 5$$

(1)  $y = ?(x+2)(x+1)(x-1)^2$   
 passes thru  $(0, 1)$   
 $= ?(0+2)(0+1)(0-1)^2$   
 $= ? \cdot 2 \cdot 1 \cdot 1$   
 $\therefore ? = ?$   
 $\therefore y = ?(x+2)(x+1)(x-1)^2$

$$(x^2)^2 = x^2 \cdot x^2 = (xx)(xx) = x^4$$

$$\text{dist exp } +5$$

$$\text{exp to exp } +5$$

$$\text{sqrt } 6^{1/2} +5$$

$$\text{combine } 6b^2 +5$$

$$\rightarrow 18a^4 b^{1/2} b^2$$

$$18a^4 b^{5/2}$$

$$x^2 x^4 = (xx)(xxxx) = x^6$$

$$\frac{x^6}{x} = \cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x} = x^5$$

simplify inside w/ exp  $+1$   
 exp & log's canceling  $+1$

5. [6] Find all real or complex  $x$  values in each of the following:

(a) (exponent wks #3)  $(x-3)^{-1} + 1 = (x-1)^{-1}$

$$\frac{1}{x-3} + \frac{1}{x-2} = \frac{1}{x-1}$$

$$\frac{1}{x-3} + \frac{1}{1-x-3} = \frac{1}{x-1}$$

$$\frac{x-2}{x-3} = \frac{1}{x-1}$$

negative exp +5  
did neg exp correctly +5  
simplifies +1  
quadratic +1  
alg/answer +5  
 $x = \frac{4 \pm \sqrt{16-4(5)}}{2} = \frac{4 \pm 2i}{2}$

$= 2 \pm i$

order of operations +1

use log +5

use 'log' correctly +5

finish up +5

(b) (WebHW13 #3)  $4 \cdot 2^{x-1} + 4 = 15$

$$4 \cdot 2^{x-1} + 4 = 15$$

$$\frac{4}{4} \cdot 2^{x-1} = \frac{11}{4}$$

$$2^{x-1} = \frac{11}{4}$$

$$\log_2(\frac{11}{4}) = x-1$$

$$x = \log_2(\frac{11}{4}) + 1$$

$$\text{or } (x-1)\ln 2 = \ln(\frac{11}{4})$$

$$x = \frac{\ln(\frac{11}{4}) + 1}{\ln 2} \approx 2.46$$

6. Let  $h$  be the function defined by:  $h(x) = \begin{cases} \log_3(x+1) & x \leq 2 \\ -x+3 & 2 < x \end{cases}$

Graph consists of solid curves.

(a) [1] Find  $h(0)$  if possible.

$0 \leq 2$  so 1st line +5

$$\log_3(0+1) = \log_3(1) = 0$$

(b) [1] Find  $h(3)$  if possible.

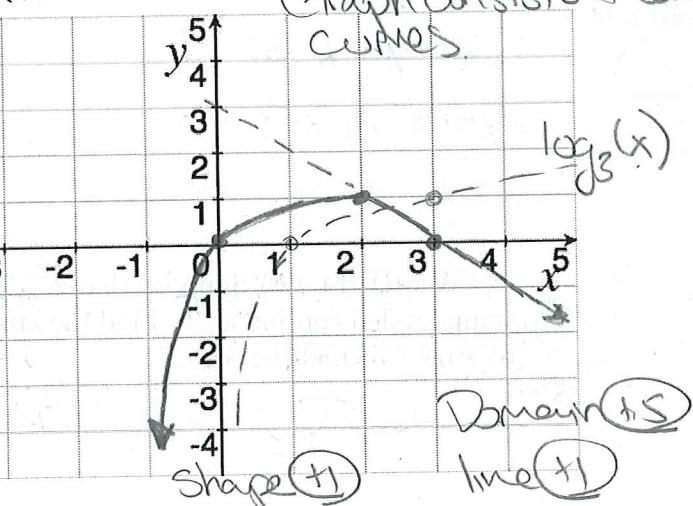
$2 < 3$  so 2nd line +5

$$-3+3=0$$

(c) [1] Find  $h(-1)$  if possible.

$-1 \leq 2$  so 1st line +5

$$\log_3(-1+1) = \log_3(0)$$



ERROR  $0$  is not in the domain of  $\log_3$  function?

(d) [4] (PracticeExam2 #9) Graph  $h$  on the axes above.

$\log_3(x+1)$  looks like  $\log_3(x)$  shifted LEFT one unit.

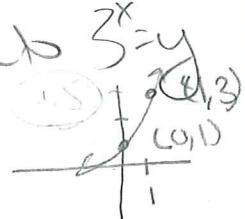
plot points from a,b,c +1

$$-x+3$$

line w/ y-intercept 3

slope of -1

Graph of  $\log_3(x)=y$  is inverted  $3^x=y$



7. [2] (§3.3 #14) Let  $3 = \log_2(x)$  and  $8 = \log_2(y)$ . Find  $\log_2\left(\frac{y}{x}\right)$ .

$$\log_2\left(\frac{y}{x}\right) = \log_2(y) - \log_2(x) \text{ or } 3 = \log_2(x) \text{ & } 8 = \log_2(y)$$

$$= 8 - 3 \quad \left\{ \begin{array}{l} \Rightarrow 2^3 = x \\ \text{So} \end{array} \right. \Rightarrow 2^8 = y$$

$$= 5 \quad \text{property } 2.5 \quad \text{used log property } +.5$$

$$\log_2\left(\frac{2^8}{2^3}\right) = \log_2(2^5) = 5 \quad \text{plugged in } \log_2(x) = 3 \text{ in etc } +.5$$

$$\text{got it } +.5$$

8. The area of a rectangle is  $5x^4 - 15x^3 + 22x^2 - 6x + 8$ . Its length can be computed by  $x^2 - 3x + 4$ .

(a) [2] If the length of the rectangle is 4, what is  $x$ ?

(b) [3] Find the polynomial function that outputs the width of a rectangle as a function of  $x$ .

(a)  $\text{length} = x^2 - 3x + 4$

$$+ .5 \cancel{x^2} - \cancel{3x} + \cancel{4}$$

$$- x^2 - 3x + 4$$

start +.5

Solve quadratic  
+1

$$0 = x^2 - 3x$$

$$0 = x(x-3)$$

$$\Rightarrow x=0 \text{ or } x=3$$

$$x=0 \text{ or } x=3$$

(b)  $\text{length} \cdot \text{width} = \text{area} \Rightarrow \text{width} = \frac{\text{area}}{\text{length}}$

$$\text{width} = \frac{5x^4 - 15x^3 + 22x^2 - 6x + 8}{x^2 - 3x + 4}$$

$$+ .5 \quad \text{long division}$$

$$x^2 - 3x + 4 \overline{) 5x^4 - 15x^3 + 22x^2 - 6x + 8}$$

$$- (5x^4 - 15x^3 + 20x)$$

$$- (2x^2 - 6x + 8)$$

$$\text{width} = 5x^2 + 2$$

9. [2] (WebHW14 #1) Suppose that \$ 2,500 is invested in an account that pays interest compounded continuously. Find the amount of time that it would take for this account to grow to \$4,500 at 5.25 %.

$$\rightarrow Pe^{rt} = A \quad +.5$$

$$4500 = 2500 e^{.0525t}$$

$$+ .5$$

$$\frac{45}{25} = e^{.0525t}$$

order of op +.5  
used ln's +.5

$$\ln\left(\frac{9}{5}\right) = .0525t$$

4

$$\frac{\ln\left(\frac{9}{5}\right)}{.0525} = t \approx 11.2 \text{ years}$$

10. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.  
No, doing both questions will not earn you extra credit.

- (a) Given a mortgage  $M$  we can compute the regular payments  $P$ . Let  $r$  be the annual interest rate,  $t$  be the number of years, and  $n$  be the number of payments per year. Then we can find:

$$P = \left[ \frac{rM}{1 - (1 + \frac{r}{n})^{-nt}} \right] \div n$$

- i. (§3.3 #95) [2] What is the monthly payment on a mortgage of \$120,000 with a 6% interest rate for 20 years?  
ii. (§3.3 #97) [3] The First National Bank offers Andy an 8.5% interest rate on a 30-year mortgage to be paid back in monthly payments. The most Andy can afford to pay in monthly payments is \$850.00. What mortgage amount can Andy afford?  
(b) (WordProblem2 #3) Entropy  $S$  is a function of the number of possible states  $W$ , that are accessible to a system with a given amount of energy. We can explicitly compute entropy by

$$S = k \ln(W)$$

where  $k$  is Boltzmann's constant and equal to  $1.38064852 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$ .

- i. [2] Find the entropy  $S$  of flipping one coin where the states are counting what side is up.  
ii. [3] If liquid  $A$  has 100,000 times more possible states than liquid  $B$ , which liquid has a higher entropy and what is the difference?

*(a) i)* 
$$\left[ \frac{0.06 \cdot 120,000}{1 - (1 + \frac{0.06}{12})^{-12(20)}} \right] \div 12$$
 *use .06 +5* *use .06 +5* *plug in correctly +5* *algebra for sure +5* *\$850*

*(b) i)* 
$$(1.38064852 \cdot 10^{-23}) \ln(2)$$
 *there are 2 states: heads up or tails up* *so* *2 states +1* *plug in +3* *or*  *$= 9.5697263 \cdot 10^{-24}$*  *notation +5*

*ii)* 
$$850 = \left[ \frac{.085 \cdot M}{1 - (1 + \frac{.085}{12})^{-30}} \right] \div 12$$
 *use .085 +5* *use .085 +5* *plug in correctly +5* *algebra for sure +5*  *$W_A = \text{possible states of A}$*   *$W_B = \text{possible states of B}$*   *$W_A = 100,000 W_B$*  *we want to compare  $S_A$  to  $S_B$  +5* *we want to find difference or  $S_A - S_B$  +5*

$$12000 = \frac{.085M}{1 - (1.007083)^{-360}}$$

$$12000 (1 - (1.007083)^{-360}) = \frac{.085M}{.085^5}$$

$$M = \$110,545.59$$

$$S_A - S_B = k \ln W_A - k \ln W_B$$

$$= k \ln 100,000 W_B - k \ln W_B$$

$$= k (\ln 100,000 + \ln W_B) - k \ln W_B$$

$$= k \ln 100,000 + k \ln W_B - k \ln W_B$$

$$\approx 1.539534 \cdot 10^{24}$$

