EXAM 2

# TMATH 120

Winter 2015

Key

NAME:

1. [8] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let $f$ be a function, and $x$, $y$, and $z$ be real numbers.

T F $x^{-2} = \sqrt{x}$ $\quad x^{-2} = \frac{1}{x^2} \quad \sqrt{x} = x^{\frac{1}{2}}$

T F $x^2$ is a polynomial

T F A savings account starting with \$300 that has an annual interest rate of 5% compounded monthly has $300(1+\frac{.05}{3})^{12*3}$ dollars after three years. $\quad 300(1+\frac{.05}{12})^{12 \cdot 3}$

T F $\log(x+2) = \log(x) + \log(2)$ $\quad \log(x)+\log(2) = \log(2x)$

T F $\dfrac{\log(x)}{\log(y)} = \dfrac{x}{y}$ $\quad \frac{\log x}{\log y} = \log_y(x)$

T F $\log_6(36) = 2$ $\quad \log_6 36 = \log_6 6^2 = 2$

T F $6x - 5$ divides $18x^2 + 15x - 25$

$$\begin{array}{r} 3x+5 \\ 6x-5 \,\big)\, 18x^2+15x-25 \\ -(18x^2-15x) \\ 30x-25 \\ -(30x-25) \\ 0 \end{array}$$

T F $2 \cdot 8^x = 16^x$ $\quad 2 \cdot 8^x = 2(8^x)$ b/c PEMDAS

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [3] Solve for $y$: $10^{xy} = 8$

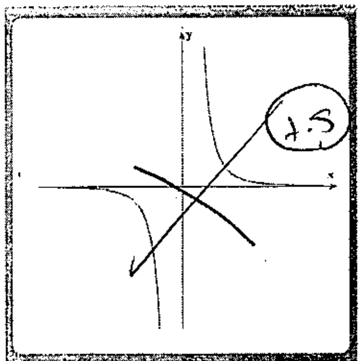
start (+.5)
into log (+.5)
log prop (+1)
algebra (+1)

$\log 10^{xy} = \log 8$

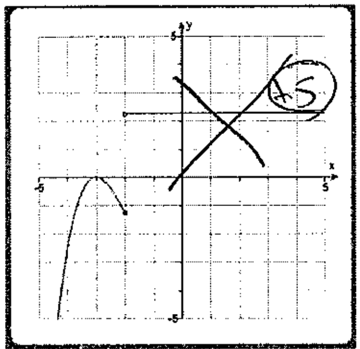
$x \cdot y \log 10 = \log 8$

$xy = \log 8$

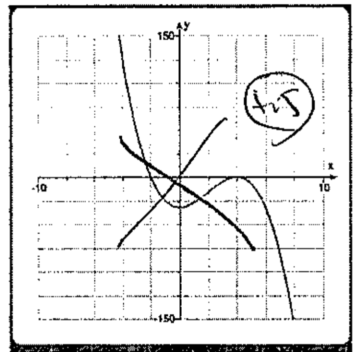
$y = \frac{\log 8}{x}$



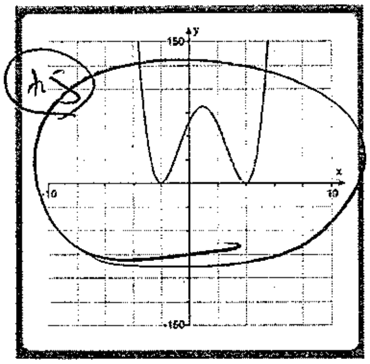

break in graph




break in graph




end behavior
=> odd degree poly




3. [2] (PolyWks #9) Identify all of the above graphs that could be the graph of an even degree polynomial?

4. Find all $x$ that satisfy the following:

(a) [3] (WebHW8 #13) $4 \cdot 3^{2x-3} + 4 = 15$

$4 \cdot 3^{2x-3} + 4 = 15$
$\quad -4 \quad -4$

$\dfrac{4 \cdot 3^{2x-3}}{4} = \dfrac{11}{4}$

$3^{2x-3} = 11/4$

$\ln 3^{2x-3} = \ln 11/4$

$\dfrac{(2x-3)\ln 3}{\ln 3} = \dfrac{\ln 11/4}{\ln 3}$

$2x - 3 = \dfrac{\ln 11/4}{\ln 3}$
$\quad +3 \qquad +3$

$2x = \dfrac{\ln 11/4}{\ln 3} + 3$

$x = \left(\dfrac{\ln 11/4}{\ln 3} + 3\right) / 2$

use log (+.5)
properties of log (+1)
order of op (+1)
arithmetic (+.5)

(b) [4] (PracticeExam #8) $\log(x - 16) = 2 - \log(x - 1)$

$\log(x-16) = 2 - \log(x-1)$

$\log(x-16) + \log(x-1) = 2$

$10^{\log(x-16)(x-1)} = 10^2$

$(x-16)(x-1) = 10^2$

$x^2 - 17x + 16 = 100$

$x^2 - 17x - 84 = 0$

$(x-21)(x+4) = 0$

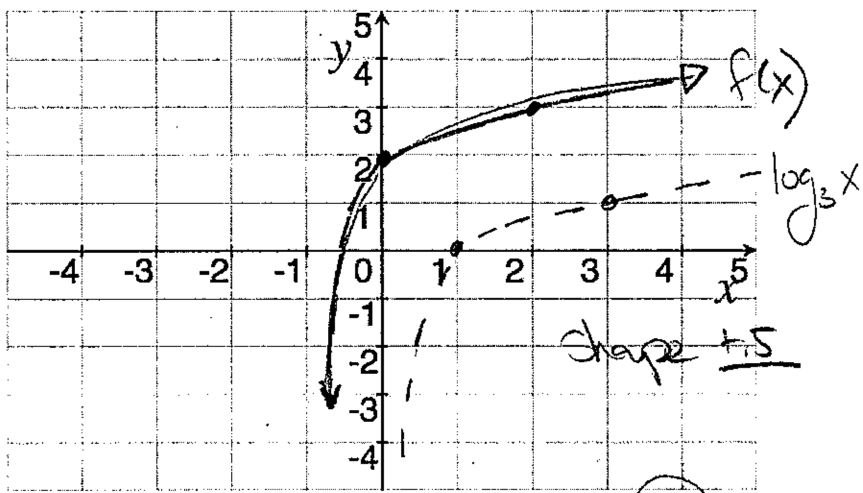
$\Rightarrow (x = 21)$ or $\cancel{-4}$

log properties (+1)
use exp (+.5)
used right (+.5)
~~add whole exp~~ (+.5)
check (+.5)
quadratic (+1)

5. Let $f(x) = \log_3(x+1) + 2$.

(a) [3] (PracticeExam #9) Carefully graph $f$.

up 2 left 1 +.5




(b) [2] What is the domain of $f$? +1

$(-1, \infty)$ or $-1 < x$

(c) [3] (PracticeExam #9) Find the inverse of $f$.

$x = \log_3(y+1) + 2$

$x - 2 = \log_3(y+1)$

$3^{x-2} = y+1$

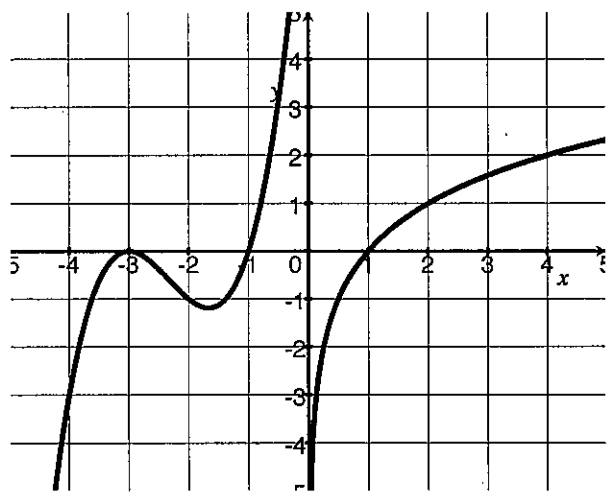
$3^{x-2} - 1 = y$

algebra +1
exponent +.5
exp prop +.5

6. (Quiz3 #3) The piece-wise defined function $h$ is graphed to the right and is of the form:

$$h(x) = \begin{cases} p(x) & \text{if } x < 0 \\ \log_b(x) & \text{if } 0 \leq x \end{cases}$$

where $p(x)$ is a polynomial that when completely factored is of the form $(x-c)^m$ and is of degree three.



(a) [1] Find $h(-2)$.

$-1$ +1

(b) [2] Does $h$ have an inverse? Why or why not?

no, h fails the horizontal line test. +1

(c) [5] Find the (algebraic rule for the) function $h$.

$-3$ is a root $\Rightarrow$ $(x+3)$ is a factor
$-1$ is a root $\Rightarrow$ $(x+1)$ is a factor +1

So $p(x) = \alpha(x+3)^m(x+1)^n$

graph does not cross at $x=-3$
$\Rightarrow$ degree of $(x+3)$ is even +1

So $\alpha(x+3)^2(x+1)$

passes thru $(-2,-1)$ so
$\alpha(-2+3)^2(-2+1) = -1 \Rightarrow \alpha = 1$ +.5

So $p(x) = (x+3)^2(x+1)$ Poly +.5

Note $(2,1)$ is on the graph of
$\log_b(x) = y$ so
$\log_b 2 = 1 \Leftrightarrow b^1 = 2$ +1 +2
So $b = 2$.

$$h(x) = \begin{cases} (x+3)^2(x+1) & \text{if } x<0 \\ \log_2(x) & \text{if } 0<x \end{cases}$$

25
25
125
500
625

7. [4] (expwks #2) Assume that $s$ and $t$ are positive, simplify: $\left(\frac{3s}{5s^{-2}\sqrt{t}}\right)^4$

(+.5) $\frac{3^4 s^4}{5^4 (s^{-2})^4 (t^{1/2})^4}$ (+.5) (+.5) $= \frac{9 \cdot 9 \cdot s^4}{25 \cdot 25\, s^{-8}\, t^{4/2}}$ (+1) $= \frac{81 s^4}{625\, s^{-8}\, t^2}$

$= \frac{81 s^{4-(-8)}}{625 t^2}$ (+1) $= \frac{81 s^{12}}{625 t^2}$ (+.5) #'s in final

$(x^2)^3 = x^2 x^2 x^2 = xxxxxx = x^6$ $\qquad \frac{x^2}{x^3} = \frac{xx}{xxx} = \frac{1}{x} = x^{-1}$

8. [4] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

   (a) (WordWks #10) How many times stranger is stomach acid than vinegar? Assume that vinegar has a pH level of 3.2 and stomach acid has a pH level of 1.6. Now that the pH level of an object is computed by $-\log[H^+]$ where $[H^+]$ is the concentration of hydrogen ions (in Moles) per liter.

   (b) (§3.5 #54a) Compare the intensity of the Tacoma February 2001 earthquake that measured 6.8 on the Richter scale, to the intensity of the March 2011 earthquake in Japan that measured 9.0 on the Richter scale. Recall that the magnetite $M$ of an earthquake is a function of its intensity $I$ and is defined by $M = \log\left(\frac{I}{I_0}\right)$ where $I_0$ is the intensity of the zero-level earthquake.

Start (+.5)

(a) $pH = -\log[H^+]$

let $[H^+]_s$ = concentration of stomach acid (+.5)

$[H^+]_v$ = concentration of vinegar

want $?[H^+]_v = [H^+]_s$ (+.5)

arithmetic (+.5)

note

$3.2 = -\log[H^+]_v$ & $1.6 = -\log[H^+]_s$

$\Rightarrow -3.2 = \log[H^+]_v$ & $-1.6 = \log[H^+]_s$ (+1)

$\Rightarrow 10^{-3.2} = [H^+]_v$ & $10^{-1.6} = [H^+]_s$

So $\frac{[H^+]_s}{[H^+]_v} = \frac{10^{-1.6}}{10^{-3.2}} = 10^{1.6}$ or $\approx 40$ times (+1)

start (+.5) (b) $M = \log\left(\frac{I}{I_0}\right)$

(+.5) let $I_T$ = intensity of tacoma 2001 earthquake

$I_J$ = intensity of Japan's 2011 earthquake

(+.5) want $? I_T = I_J$

arithmetic (+.5)

note

$6.8 = \log\left(\frac{I_T}{I_0}\right)$ & $9.0 = \log\left(\frac{I_J}{I_0}\right)$

(+1) $\Rightarrow 10^{6.8} = \frac{I_T}{I_0}$ & $10^{9.0} = \frac{I_J}{I_0}$

$\Rightarrow I_0 10^{6.8} = I_T$ & $I_0 10^{9.0} = I_J$

So $\frac{I_J}{I_T} = \frac{\cancel{I_0} \cdot 10^{9.0}}{\cancel{I_0} \cdot 10^{6.8}} = 10^{2.2}$ or 158 times (+1)

9. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (WebHW9 #5) The population of a town was 6,000 in 1990 and grew to 8,000 in 2000. Assume that the population will continue to grow exponentially.

i. [4] What will the population be in 2015?

ii. [2] When will the population hit 12,000?

(b) (WordWks #12) In a large late, one-fifth of the water is replaced by clean water each year. A chemical spill deposits 60,000 cubic meters of soluble toxic waste into the late.

i. [4] How much of this toxin will be left in the late after four years?

ii. [2] When will the toxic chemical be reduced to 6000 cubic meters?

(a)

| (t) years since 1990 | population |
|---|---|
| 0 | 6000 |
| 10 | 8,000 |

stzA(+.5) (+.5) (+.5)

Use $Pe^{rt}$

(+.5) { we need to find r before letting t=25

(+1) $\frac{8000}{6000} = \frac{6000 e^{r(10)}}{6000}$

$\frac{4}{3} = e^{10r}$

$\ln 4/3 = 10r$

$\left(\frac{1}{10}\ln 4/3\right) = r$ So $6000 e^{\left(\frac{1}{10}\ln \frac{4}{3}\right)t}$ (+.5)

(+.5) { So, $6000 e^{\left(\frac{1}{10}\ln 4/3\right)\cdot 25} \approx 12,316.$

ii) Find t so that

$6000 e^{\left(\frac{1}{10}\ln \frac{4}{3}\right)t} = \underbrace{12,000}_{+.5}$

(+.5) alg to solve

$\Rightarrow e^{\left(\frac{1}{10}\ln 4/3\right)t} = 2$

(+1) exp+log prop. $\Rightarrow \left(\frac{1}{10}\ln \frac{4}{3}\right)t = \ln 2$

$\Rightarrow t = \frac{10\ln 2}{\ln 4/3} \approx 24$ years

2014

(b)

| years since spill (t) | amount of chemicals |
|---|---|
| 0 | 60,000 |
| 1 | $\frac{4}{5}(60,000) = 52,000$ |

stzA(+.5) (+.5)

Use $Pe^{rt}$ (+.5)

(+.5) we need to find r befor letting t=4

(+1) $52,000 = 60,000\, e^{r(1)}$

$\frac{4}{5} = e^{r}$

$\Rightarrow \ln \frac{4}{5} = r$ So $\underbrace{60000 e^{\left(\ln \frac{4}{5}\right)t}}_{+.5}$

(+.5) { So $60,000 e^{(\ln 4/5)(4)} \approx 24,576$

ii) find t so that

$60,000 e^{(\ln 4/5)t} = \underbrace{6000}_{+.5}$

(+.5) alg

(+1) exp+log prop

$\Rightarrow e^{\left(\ln \frac{4}{5}\right)t} = \frac{1}{10}$

$\Rightarrow (\ln 4/5)t = \ln 1/10$

$\Rightarrow t = \frac{\ln 1/10}{\ln(4/5)} \approx 10$ years