EXAM 1 **TMATH 120** Winter 2015

NAME: Key

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers.

T (F) $\frac{1}{x+2}+\frac{1}{x}=\frac{1}{x+2}+\frac{1+2}{x+2}=\frac{4}{x+2}$ $\times\ x\ \frac{1}{x+2}+\frac{1}{x}\ \frac{x+2}{x+2}$ $=\frac{x}{x(x+2)}+\frac{x+2}{x(x+2)}$

T (F) $(x+2)^2=x^2+4$ $(x+2)^2=(x+2)(x+2)\Rightarrow x^2+2x+2x+4$ $=\frac{2x+2}{x(x+2)}$

T (F) $f(x+2)=f(x)+2$ let $f(x)=x^2$ then $f(x+2)=(x+2)^2$ but $f(x)+2=x^2+2$

(T) F A eighth degree polynomial will have eight complex roots.

(T) F $\frac{3+5i}{1-2i}=-\frac{7}{5}+\frac{11}{5}i$ ✓ ✓ $\frac{3+5i}{1-2i}\cdot\frac{1+2i}{1+2i}=\frac{3+6i+5i+10i^2}{1+2i-2i-4i^2}=\frac{3+11i-10}{1+4}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. (Practice exam #3) [4] Find any real or imaginary x such that $\frac{1}{x+1}+\frac{1}{2}=\frac{1}{x+3}$

start (+1.5) solve quadratic (+1.5) +.5 both answers +.5 use quad tricks +.5 use tricks correctly

alg (+1) $\frac{1}{x+1}+\frac{1}{2}=\frac{1}{x+3}$

factors (+1) $2\ \frac{1}{x+1}+\frac{1}{2}\ \frac{x+1}{x+1}=\frac{1}{x+3}$

$\frac{2+x+1}{2(x+1)}=\frac{1}{x+3}$

$(x+3)\frac{3+x}{2(x+1)}=\frac{1}{x+3}(x+3)$

$2(x+1)\frac{(x+3)(x+3)}{2(x+1)}=1\ 2(x+1)$

$(x+3)(x+3)=2(x+1)$

$x^2+6x+9=2x+2$

$x^2+4x+7=0$

$x^2+4x=-7$

$+4\qquad +4$

$(x+2)^2=-3$

$x+2=\pm\sqrt{-3}$

$x=-2\pm\sqrt{-3}$

$=-2\pm i\sqrt{3}$

or quadratic formula

$x=\frac{-4\pm\sqrt{4^2-4(1)(7)}}{2(1)}$

$=\frac{-4\pm\sqrt{16-28}}{2}$

$=\frac{-4\pm\sqrt{-12}}{2}$

$=\frac{-4\pm 2\sqrt{-3}}{2}$

$=-2\pm i\sqrt{3}$

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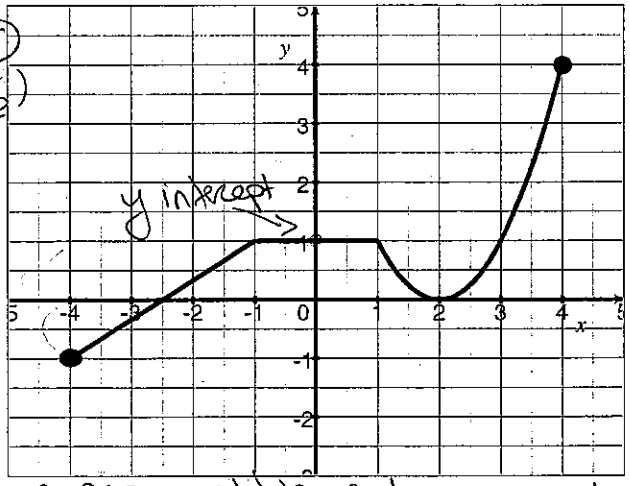
3. Let g be the function comprised of two lines and a parabola that has only been shifted (not vertically stretched):



y-coord (+.5)
2rd pts (+1.5)
#'s (+.5)

(a) (Quiz1 #3d) [2]
Find the range of g.

$[-1, 4]$

(b) (§1.1 #48) [1]
Find the y intercept(s).

1 (+1)

(c) (Inverse Wks #2) [2]
Does g have an inverse?
Why or why not?

no (+1) since $1 \xrightarrow{g} 1$ (+1) so the inverse would have to $1 \to -1$ ie not be a function

(d) (S1.3 #56 &) [6] Estimate the following *if* possible:

$$\frac{2g(2)+1}{g(-1)}$$

$\frac{2 \cdot 3 + 1}{1} = 1$ (+1) (+1) alg (+1)

$(g \circ g)(-4)$

$g(g(-4)) = g(-1) = 1$

$-4 \xrightarrow{g} -1$ (+1) $\xrightarrow{g}$ (+1)

composition (+1)

(e) [4] (WebHW1 #21 & WebHW2 #5 & WebHW3 #12) Find the piece-wise defined rule of g in the indicated form.

$$g(x) = \begin{cases} 2/3\,x + 5/3 & \text{if } -4 \le x < -1 \\ 1 & \text{if } -1 \le x < 1 \\ (x-2)^2 & \text{if } 1 \le x < 4 \end{cases}$$

left line (+.5)

$y = mx + b$

$m = \frac{\Delta y}{\Delta x} = \frac{2}{3}$ (+.5)

or b/c thru $(-4,-1)$ & $(-1,1)$

$= \frac{1-(-1)}{-1-(-4)} = \frac{2}{3}$

passes thru $(-4,-1)$ so (+.5)

$-1 = 2/3(-4) + b$

$\Rightarrow -1 + 8/3 = 5/3 = b$

mid line (+.5)

$y = 1$ (+.5)

parabola (+.5)

shifted right 2 units

$(x-2)^2$ (+1)

— or —

vertex @ $(2,0)$

$(x-2)^2 + 0$

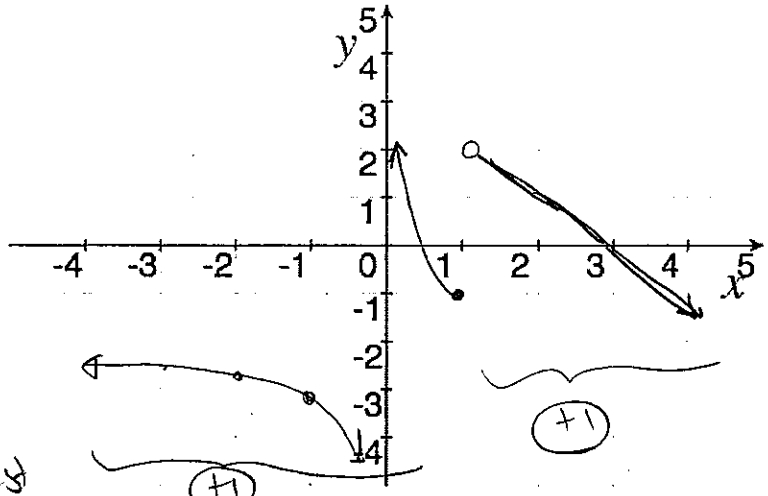
4. Let f be the *function* defined by

$$f(x) = \begin{cases} \frac{1}{x} - 2 & x \le 1 \\ -x + 3 & 1 < x \end{cases}$$

x	$f(x)$
-4	$\frac{1}{-4} - 2 = \frac{-10}{4}$
-3	$\frac{1}{-3} - 2 = \frac{-8}{3}$
-2	$\frac{1}{-2} - 2 = \frac{-5}{2}$
-1	$-1 - 2 = -3$
0	DNE
1	$1 - 2 = -1$

(a) [3] (Line Wks #6 & WebHW1 #21) Graph f.

endpoints (+1)



(+1)

(b) [2] Is f a function? Why or why not?

yes - passes (+1) vertical line test (+1)

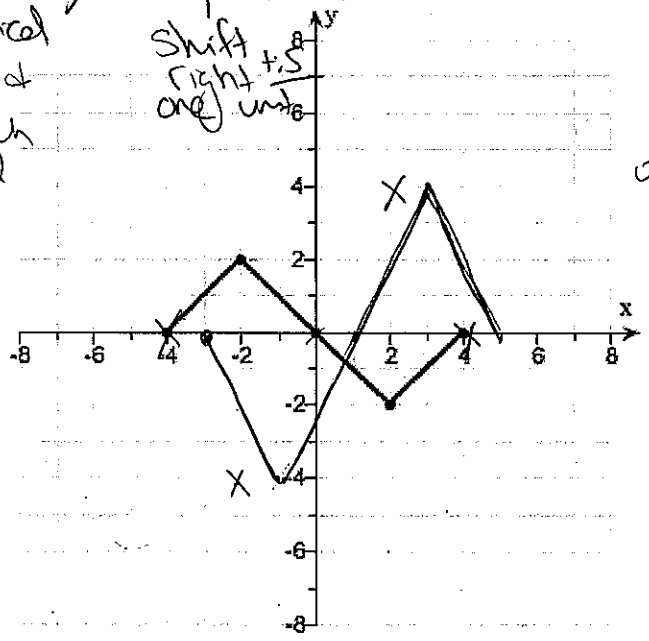
(c) [2] (Quiz1 #3b) Find all possible input(s) so that $f(x) = -1$.

from the graph when $x = 1$ (+1) and $x = 4$ (+1)

5. [3] (Quiz2 #2) Given the piece-wise defined function f shown below. Graph $g(x) = -2f(x-1)$.

vertical flip & stretch by 2

shift right 1 unit



(indicated with x's)

x	$x-1$	$f(x-1)$	$-2f(x-1)$
-3	-4	$f(-4) = 0$	0
-2	-3	$f(-3) = 1$	-2
-1	-2	$f(-2) = 2$	-4

6. Let $\alpha(x) = \sqrt{3x-7}$ and $\beta(x) = \frac{x-1}{x}$.

(a) [2] (§1.6 #39) What is the domain of α?

#'s under square root ≥ 0 (+1)

$3x - 7 \geq 0$

$+7 \quad +7$

$3x \geq 7$

$x \geq 7/3$ alg (+1)

(b) [4] (Quiz2 # 3b)
What is the rule of $\beta \circ \alpha$?

$(\beta \circ \alpha)(x) = \beta(\alpha(x))$

$= \beta(\sqrt{3x-7})$ notation (+.5)

$= \frac{\sqrt{3x-7} - 1}{\sqrt{3x-7}}$ (+.5) (+.5)

composition (+.5)

(WebHW3 #24)
What is the domain of $\beta \circ \alpha$? (+1.5)

(+1.5) denominators $\neq 0$ AND #'s under $\sqrt{} \geq 0$

$\sqrt{3x-7} \neq 0$ AND $3x - 7 \geq 0$

$3x - 7 \neq 0$ $3x \geq 7$

$3x \neq 7$ $x \geq 7/3$

$x \neq 7/3$ alg (+1)

So $x > 7/3$

(c) [3] (§1.7 #56) Given that β has an inverse, find β^{-1}.

$y = \frac{x-1}{x}$

(+1) $x = \frac{y-1}{y}$

$xy = y - 1$

$xy - y = -1$

$y(x-1) = -1$

$\frac{y(x-1)}{x-1} = \frac{-1}{x-1}$

$y = \frac{-1}{x-1}$ or $\frac{1}{1-x}$

order of operations (+.5)
solve for y (+.5)
alg (+1)

(d) (Inverse Wks) [2] What is the range of β? Justify yourself.

range of β = domain of β^{-1} (+1)

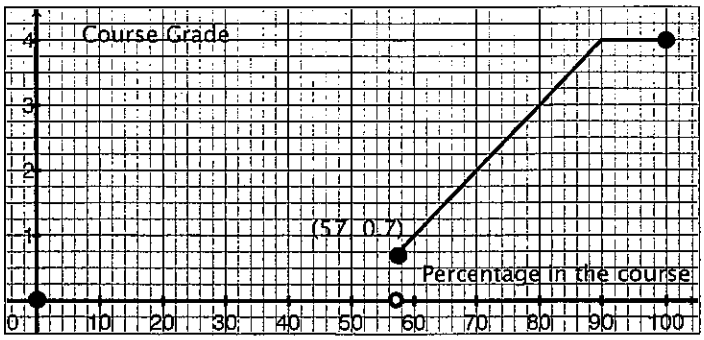
= all x so that $x \neq 1$ (+1)

7. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (Word Problems #9) Leonard McCoy is in this course and during the seventh week he logged into MyMathLab and computed the average of his WebHW's, WrittenHW's, Quizzes, and Exams. The numbers are reported below.

McCoy's work did not change drastically in the remaining 3 weeks of the course and ultimately he earned a 3.0 in the course. What grade did he earn on the final exam to earn a 3.0 for the course? In case you don't remember, the weights specified in the syllabus and the graph of the function f that takes your class percentage x and returns your score on a 4. scale are also provided.

	weight	McCoy's ave
Mini-Quizzes	5%	95%
WebAssign	10%	100%
WrittenHW	15%	95%
Quizzes	15%	65%
2 Exams	30%	70%
Final	25%	?



(b) (Practice Exam #12) A manufacturer of soft drinks advertises their orange soda as "naturally flavored", although it contains only 6% orange juice. A new federal regulation stipulates that to be called "natural" a drink must contain at least 15% fruit juice. The manufacturer mixes their juices in closed 900 gallon containers (to avoid contamination). How much juice must they remove from the 900 gallon container and replace with pure orange juice to conform to the new regulation?

a) let ? be the score on McCoy's final } (+.5) Std (+.5)

Course % $= .95 \cdot 5 + 1.00 \cdot 10 + .95 \cdot 15 + .65 \cdot 15 + .70 \cdot 30 + ? \cdot 25$ } (+1.5)

$= 4.75 + 10 + 14.25 + 9.75 + 21 + ? \cdot 25$

$= 59.75 + ? \cdot 25$

to earn a 3.0 in the course McCoy needs an 80% } (+.5)

according to the graph so

(+.5) } $80 = 59.75 + ? \cdot 25$ arithmetic (+1)

$\Rightarrow 20.25 = ? \cdot 25$ pathway/plan (+.5)

$\Rightarrow ? = \frac{20.25}{25} = \frac{81}{100}$ or 81%

b) let x = amount of pure orange juice to add
y = the amount of juice to keep } +5 +5 STA

note x also equals the amount of juice we have to remove.

$$\begin{cases} \text{total juice volume} = 900 = x + y \\ \text{pure orange juice we want} = .15 \cdot 900 = x + .06y \end{cases}$$ +15

(+5) (+1)

or $\begin{cases} 900 = x + y \\ 135 = x + .06y \end{cases}$

$\Rightarrow x = 900 - y$

sub into 2nd equation

$135 = (900 - y) + .06y$

$\Rightarrow 135 = 900 - .94y$

$\Rightarrow \frac{-765}{-.94} = \frac{-.94y}{-.94}$

$\Rightarrow 813.8 = y$

algebra +1
pathway/plan +.5

So $900 = x + 813.8$

$\Rightarrow 900 - 813.8 = x$

$\Rightarrow x = 86.2$

So remove 86 gallons & replace with pure orange juice