NAME: *Key*

8:52
8:31
~~8~~

1. [3] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let b be a non-negative number, n be a positive integer, and x be a real number.

T (F) $b^{-2} = b^{\frac{1}{2}}$ $b^{-2} = \frac{1}{b^2}$

(T) F $\sqrt[n]{x} = x^{\frac{1}{n}}$ by def

T (F) $\sqrt{b+x} = \sqrt{b} + \sqrt{x}$ $\sqrt{4+4} = \sqrt{8}$ $\neq \sqrt{4} + \sqrt{4} = 2+2 = 4$

(T) F $\log_b 1 = 0$ $b^0 = 1$ as long as $b > 0$

T (F) $\log 8v^3 = 3\log 8v$ for all $v > 0$ $\log 8v^3 = \log 8 + \log v^3 = \log 8 + 3\log v$

(T) F $\log(\log 10) = 0$ $\log(1) = 0$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] Solve for r and simplify given: $\left(\text{from first exam} \right)$

$$\frac{1}{\frac{1}{r} + \frac{1}{s}} = t$$

$$1 = t\left(\frac{1}{r} + \frac{1}{s}\right)$$

$$\frac{1}{t} = \frac{1}{r} + \frac{1}{s}$$

$$\frac{1}{t} - \frac{1}{s} = \frac{1}{r}$$

$$r\left(\frac{1}{t} - \frac{1}{s}\right) = 1$$

$$r = \frac{1}{\frac{1}{t} - \frac{1}{s}} = \frac{1}{\frac{s-t}{ts}} = \frac{ts}{s-t}$$

(+1) fractions
(+1) legal alg
(+1) r on 1 side
(+1) simplified all the way

1

(based on zombie example in class)

3. Cyrano Jones gives one tribble to Uhura in the hopes to boost his sales. It turns out that tribbles are born pregnant and once fed they give birth. Dr. Spock noticed this phenomenon and began to track the population of tribbles when Uhura showed him her new family of twelve tribbles at 0800. The data is plotted below where the horizontal axis is recording the number of hours since 0800 and the vertical axis is recording the number of tribbles on the Enterprise. → Bar ↳ Bam .

⊗ (circled, top left margin)

(handwritten below graph x-axis): 0800   0900
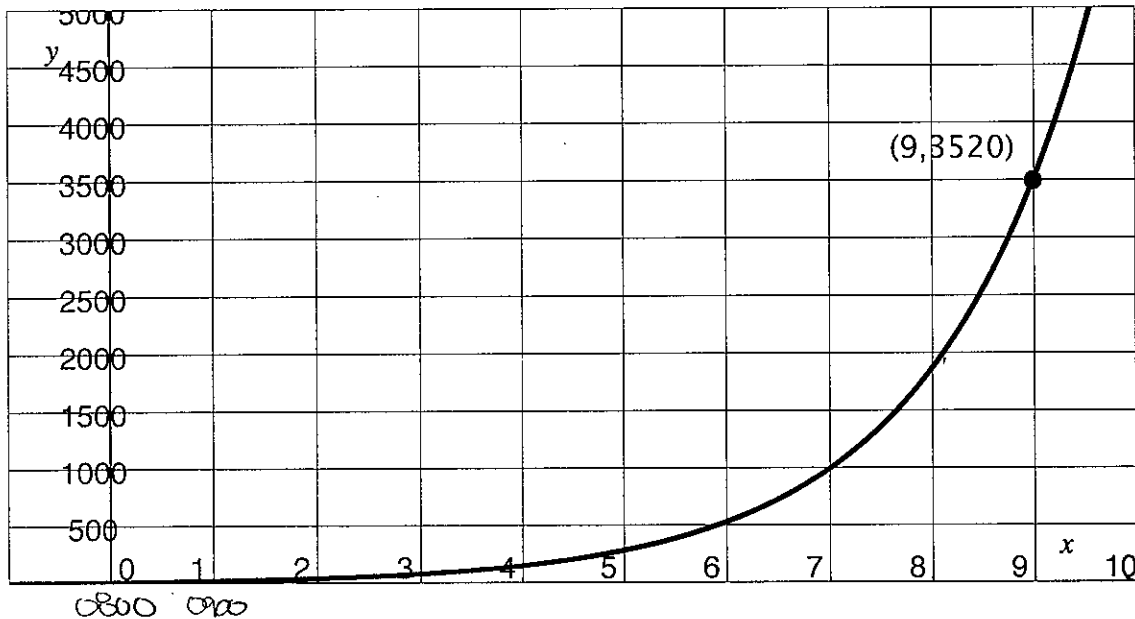
3 (a) ☒ Use the graph to estimate when there were 2000 tribbles on the Enterprise.

about 1610 hours

saw point (+1)   translated to time (+1)

(b) [2] Is the above a graph of a function? Why or why not?

yes (+1)   it passes the vert. line test (+1)

(c) [2] Reread the above discription. What is the initial population that Dr. Spock recorded?

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[5](d) ☒ Given that the population of tribbles is well approximated by an exponential function, use the data point on the graph to find the rule to describe the population at time x.

(±.5 started) (circled)

use $P_0 a^x$ (+1)

Know $P_0 = 12$ so (+1)
we just need to find a

So $\boxed{12 \cdot 1.88^x}$ got it (+.5) (circled)

(+1) Since $(9, 3520)$ is on the graph

$$3520 = 12 \cdot a^9$$ alg (+1.5)
$$\Rightarrow 293.\overline{3} = a^9$$
$$\Rightarrow \sqrt[9]{293.\overline{3}} = a \approx 1.88$$

2

4. Let f be the *function* defined by

$$f(x) = \begin{cases} x^{\frac{1}{3}} & x \leq 0 \\ \log_3 x & 0 < x \end{cases}$$
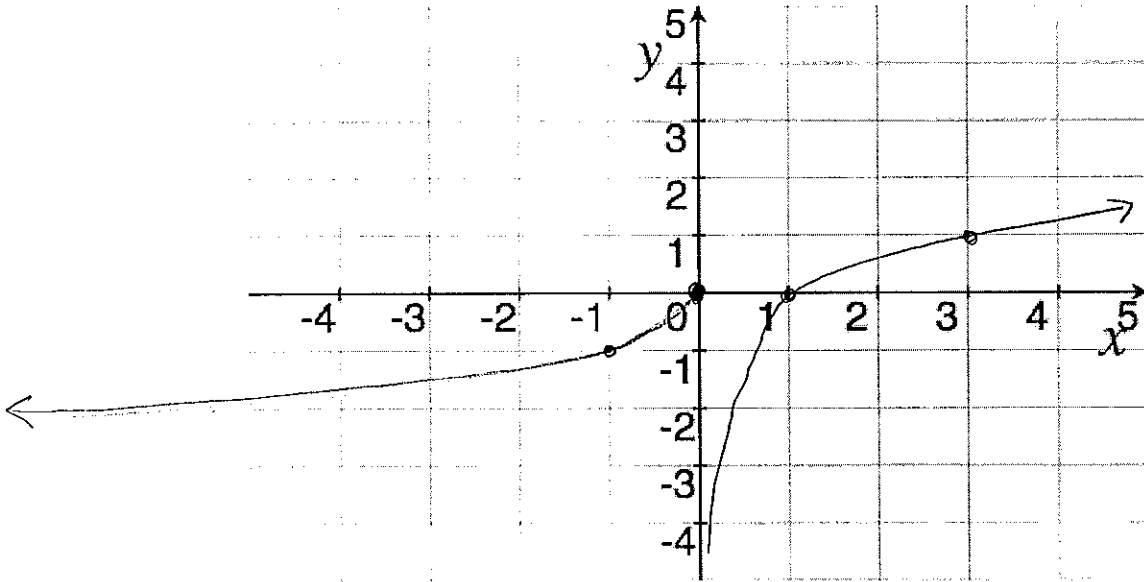
(a) [3] Graph f.

Shape (+1) each   end pts (+1)   (§3.1 & 3.2 lecture)

(b) [2] Find all possible input(s) so that $f(x) = 0$.

$x = 0$ and 1
(+1)   (+1)

5. [4] Simplify the following: (wooltw e #1)

$$\frac{(4y^{-2}\sqrt{x})^{\frac{3}{2}}}{x^{\frac{3}{4}}(\sqrt[3]{y8})^{-1}}$$

(+1) distribu mult
(·5) exp of exp
(+1) neg exp
(+1) divison
(+.5) numbers

$$\frac{4^{3/2}\, y^{-3}\, x^{3/4}}{x^{3/4}\, y^{-1/3}\, 8^{-1/3}} = \frac{2^3\, x^{3/4}\, y^{1/3}\, 8^{1/3}}{x^{3/4}\, y^3} = \frac{8\, y^{1/3}\, 2}{y^3}$$

$$= \frac{16}{y^{8/3}}$$

3

[7] 6. [X] Given that $\log_3 x = 5.3$ and $\log_3 y = 2.1$ find

$(\S 3.3 \ \#21 \& 17)$

$\log_3 \dfrac{x}{3y} = \log_3 x - \log_3 3y$

$\log_3 y^{100} = 100 \log_3 y$

prop 2 (+1)

$= 5.5 - (\log_3 3 + \log_3 y)$

$= 100 \cdot 2.1$

prop 1 (+.5)

$= 210$

plug in (+1) $= 5.5 - (1 + 2.1)$

dist neg (+.5) $= 5.5 - 3.1$

prop 3 (+1)
plug in (+1)
Started (+.5)
alg (+.5)

(+.5) started

$= 2.4$

(+.5) alg

[5] 7. [X] Find all x that satisfy the following:

$(\S 3.3 \ \#33)$

$\log_7(x+5) = 2 + \log_7(x-1)$

$\log_7(x+5) - \log_7(x-1) = 2$

use exp (+1)
prop of log (+1)
alg (+1) checked op.

$3^{5x} 3^{2x^2} = 27$

$\log_7\left(\dfrac{x+5}{x-1}\right) = 2$

(+.5) started

$3^{5x+2x^2} = 3^{3}$

$5x+2x^2 = 3$

$\dfrac{x+5}{x-1} = 49$

(+1) alg in normal

$2x^2 + 5x - 3 = 0$

$x+5 = 49x - 49$

$x = \dfrac{-5 \pm \sqrt{25 + 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \dfrac{-5 \pm \sqrt{25 + 24}}{4}$

$54 = 48x$

$\dfrac{54}{48} = x \approx 1.125$ (+.5) got it

$= \dfrac{-5 \pm 7}{4} = \dfrac{1}{2}$ or -3

8. [5] Suppose the ratio of carbon-14 to carbon-12 in a mummified cat is 64% of the corresponding ratio for living organisms. About how long ago did the cat die? Recall that the half-life of carbon-14 is 5730 years.

use $a(t) = a_0 2^{t/h}$ (+5)

started (+.5)

$(\S 3.5 \ \#7)$

So (+1)

$.64 a_0 = a_0 2^{-t/5730}$ (+.5)

$-5730 \dfrac{\ln .64}{\ln 2} = t$

order of operations (+1)

$.64 = 2^{-t/5730}$

(+.5) 3689 $\approx t$
years ago.

log to get at (+1)

$\ln .64 = \dfrac{-t}{5730} \ln 2$

$\dfrac{\ln .64}{\ln 2} = \dfrac{-t}{5730}$

4

9. [5] James T. Kirk is in this course and would like to know if it is still possible to earn a 2.5 now that he's taken two exams. He has looked at the gradebook on catalyst and has computed the averages listed below. ~~Percentage does~~

Assuming James' work does not drastically change in the remaining 3 weeks and his averages remain about the same, find what ~~grade~~ he needs to get on the final to receive a 2.5 in the course. In case you don't remember, the weights specified in the syllabus and the function f that takes your class percentage x and returns your score on a 4. scale are also provided below.

	weight	Paul's ave
Mini-Quizzes (top score)	5%	100%
WeBWork assignments	10%	21%
Handwritten assignments	15%	32%
Quizzes	15%	89%
2 Midterms	30%	100%
Final	25%	

$$f(x) = \begin{cases} 4.0 & \text{if } 95 < x \\ .1x - 5.5 & \text{if } 62 \le x \le 95 \\ 0 & \text{if } x < 62 \end{cases}$$

$(\ast)$

$$.05 \cdot 100 + .1 \cdot 21 + .15 \cdot 32 + .15 \cdot 89 + .3 \cdot 100 + .25 x = \text{Class \%}$$

to get a 2.5 in the course we need x so that

$$f(x) = 2.5 \quad (+1)$$

$$\Rightarrow .1x - 5.5 = 2.5$$

$$.1x = 8.0$$

$$\Rightarrow x = 80 \quad (+1)$$

so need the class % to be 80. $(\ast)$ reduces to:

create variable $(+1)$ weighted ave $(+1)$

$$55.25 + .25x = 80$$

$$\tfrac{1}{4} x = 24.75$$

alg $(+.5)$ got it $(+.5)$

$$\Rightarrow x = 99$$

James T Kirk needs to get a 99% on the final exam.