EXAM 1 **TQS 120** Winter 2010

NAME: Key

1. [2] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers.

   T (F) $5 + 4(2^3 - 3^2) = -9$ $5+4(8-9) = 5+4(-1)$

   T (F) $x \cdot x = 2x$ $x \cdot x = x^2$ $2x = x+x$

   T (F) All functions pass the horizontal line test. parabola

   T (F) $\sqrt{x^2} = x$ $\sqrt{(-1)^2} \neq -1$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] Solve for r and simplify given:

$$\frac{1}{\frac{1}{r} + \frac{1}{s}} = t$$

(+1) fractions
(+1) legal alg.
(+1) r on 1 side
(+1) simplified all the way

$$1 = t\left(\frac{1}{r} + \frac{1}{s}\right)$$

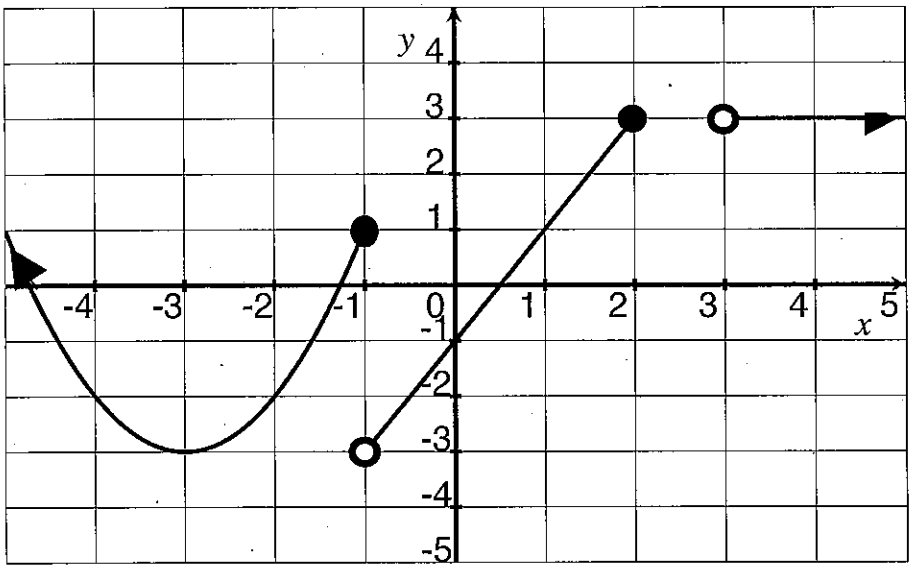
$$1 = t\frac{1}{r} + t\frac{1}{s}$$

$$1 - t/s = t/r$$

$$r\left(1 - t/s\right) = t$$

$$r = \frac{t}{1 - t/s} = \frac{t}{\frac{s-t}{s}} = \frac{st}{s-t}$$

3. Let the following be the graph of g.



(a) [2] Is g a function? Why or why not?

yup (+1) it passes the vertical line test (+1)

(b) [1] Find $g(-1)$.

1 (+1)

(c) [2] Find $g(-2) + g(4)$. (+1) (+1)

$-2 + 3 = 1$

(d) [3] What is the domain of g?

$(-\infty, 2] \cup (3, \infty)$ sense (+1)

endpts (+1)

idea of 2 & 3 (+1)

4. [4] Find a number t so that the line containing the points $(1, t)$ and $(3, \frac{2}{3})$ has slope $\frac{3}{5}$.

want slope (+1) $\frac{2/3 - t}{3 - 1} = \frac{3}{5}$ (+1)

alg (+1)

get it (+1)

$\Rightarrow \frac{2/3 - t}{2} = \frac{3}{5}$

$\Rightarrow 5(2/3 - t) = 6$

$\Rightarrow 2/3 - t = 6/5$

$2/3 - 6/5 = t$

$\frac{10 - 18}{15} = t$

$-8/15 = t = -.5\overline{3}$

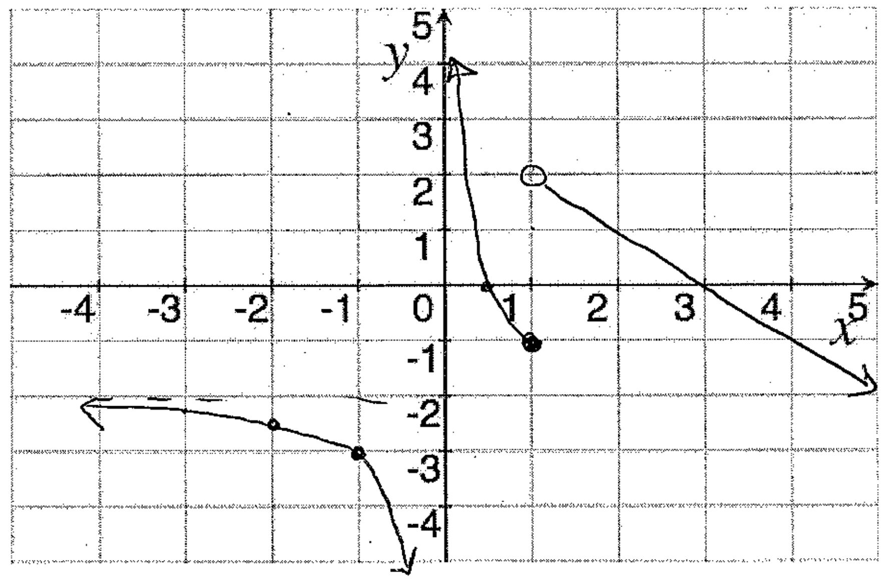
or slope find b plug in 1 get it.

or $y = 3/5\,x - 17/15$

5. Let f be the *function* defined by

$$f(x) = \begin{cases} \frac{1}{x} - 2 & x \leq 1 \\ -x+3 & 1 < x \end{cases}$$

shape (+1) .5 each
transformations (+1)
end pts (+1)
↓ functions integer spot



(a) [3] Graph f. (Explaining graph transformations is worth partial credit.)

$\frac{1}{x}$ shifted down 2 units

(b) [2] Find all possible input(s) so that $f(x) = -1$.

1 (+1) and 4 (+1) work

next time find the exact x-intercepts.

6. Let $g(x) = x^2 + 7x - 12$.

(a) [3] Find the x-intercepts.

when $y=0$ (+1) $0 = x^2 + 7x - 12$

stated quad (+1) $x = \frac{-7 \pm \sqrt{49-4(-12}}{2(1)}$

$= \frac{-7 \pm \sqrt{97}}{2}$ got it (+1)

$\approx 1.4244 \quad -8.42$

(b) [3] Put g into vertex form.

$y = x^2 + 7x - 12$

$= x^2 + 7x + (7/2)^2 - (7/2)^2 - 12$ (+1) remembered to add (+1)

$= (x + 7/2)^2 - 49/4 - \frac{48}{4}$

$(x + 7/2)^2 - \frac{97}{4}$ (+1)

7. Let $\alpha(x) = \sqrt{3x-7}$ and $\beta(x) = \frac{x-1}{x}$.

(a) [4] What is the domain of α? (+2) +1 partial

$3x - 7 \geq 0$

$3x \geq 7$

$x \geq 7/3$ (+1) (+2)

What is the domain of β?

$x \neq 0$ (+1)

(b) [4] What is the rule of $\beta \circ \alpha$? Do not simplify. (+1)

$$\frac{\sqrt{3x-7} - 1}{\sqrt{3x-7}}$$

(+1)

What is the domain of $\beta \circ \alpha$? (+1) (+1)

$\sqrt{3x-7} \neq 0$ and $3x-7 \geq 0$

$x \neq 7/3$ $\Rightarrow x \geq 7/3$

so $(7/3, \infty)$ -1 notation if no work.

(c) [3] Given that β has an inverse, find β^{-1}.

Want $g(x)$ so that $\beta(g(x)) = x$

so

(+1) $\frac{g(x) - 1}{g(x)} = x$ (+1)

alg (+1)

$g(x) - 1 = xg(x)$

$-1 = xg(x) - g(x)$

$-1 = g(x)(x-1)$

$\frac{-1}{x-1} = g(x)$

(d) [3] What is the range of β? Justify yourself. (+1)

range of β = domain of β^{-1} (which we found in (c))

Knows what range is / x.5

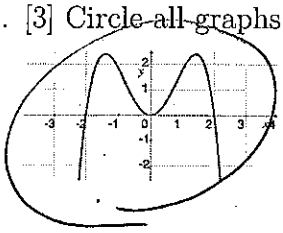
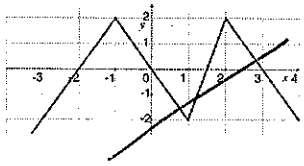
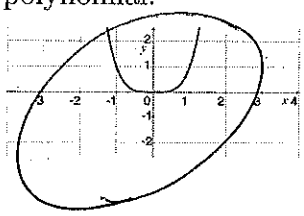
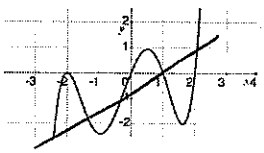
so all # so that $x - 1 \neq 0$

got it (+1)

ie all x so that $x \neq 1$

or $(-\infty, 1) \cup (1, \infty)$

8. [3] Circle all graphs that could be of a 4^{th} degree polynomial.

9. [4] Simplify the following as much as possible (remember to show your work):

$$(4a^5b^4c^3)^{-2}(2a^5b^2c^2)^3$$

$$\frac{2^3(a^5)^3(b^2)^3(c^2)^3}{(4a^5b^4c^3)^2} = \frac{2^3a^{15}b^6c^6}{4^2(a^5)^2(b^4)^2(c^3)^2}$$

$$= \frac{2^3a^{15}b^6c^6}{4\cdot4\,a^{10}b^8c^6}$$

neg exp (+1)
exp of exp (+1)
dist over mult (+1)

$$= \frac{4\cdot2\,a^{15-10}b^{6-8}}{4\cdot4}$$

$$= \frac{1}{2}a^5b^{-2}$$ got it (+1)