Note: post online

Final

TMath 120

Summer 2015

22
27
18
10
72

NAME: key

1. [8] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

T (F) $\frac{1}{x} + \frac{x}{y} = \frac{y+x}{xy}$ $y\frac{1}{x} + \frac{x}{y}x = \frac{y+x^2}{xy}$

T (F) $x^2 + x^3 = x^5$ $x^2 \cdot x^3 = (xx)(xxx) = x^5$

(T) F $(x+1)^2 = x^2 + 2x + 1$ $(x+1)(x+1) = x^2 + x + x + 1$

T (F) $i^3 = -1$ $i^3 = iii = (-1)i = -i$

(T) F $\log_2(16) = 4$ $\log_2(2^4) = 4$ or $\frac{\log 16}{\log 2} = 4$

(T) F $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 2\csc^2 x$ $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = \frac{1+\cancel{\cos x} + 1\cancel{-\cos x}}{1-\cos^2 x} = \frac{2}{1-\cos^2 x}$ $\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x$

(T) F $6x - 5$ divides $18x^2 + 15x - 25$ with no remainder.

$$\begin{array}{r} 3x+5 \\ 6x-5 \overline{)\,18x^2+15x-25} \\ -(18x^2-15x) \\ \hline 30x-25 \\ -(30x-25) \\ \hline 0 \end{array}$$

T (F) The vertex of $(x-2)^2 - 5$ is $(-2, -5)$ → $(2, -5)$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] (Exam2 #2) Find any real or imaginary x such that $6x^{-1} = 1 + 10x^{-2}$.

(5) start
(1) neg exp

$6x^{-1} = 1 + 10x^{-2}$

$\frac{6}{x} = 1 + \frac{10}{x^2}$

$x^2\left(\frac{6}{x}\right) = \left(1 + \frac{10}{x^2}\right)x^2$

(+5) alg

$6x = x^2 + 10$

$0 = x^2 - 6x + 10$

Solving Quadratic (+2)

$x^2 - 6x + 10 = 0$

(+1) $+\left(\frac{-6}{2}\right)^2 \quad +\left(\frac{-6}{2}\right)^2$

$x^2 - 6x + 9 + 10 = 9$

$(x-3)^2 + 10 = 9$

alg +5 $\quad -10 \quad -10$

$(x-3)^2 = -1$

1 $x - 3 = \pm\sqrt{-1}$

both sol +.5 $x = 3 \pm i$

(+1) quadratic formula

$x = \frac{6 \pm \sqrt{36 - 4(1)(10)}}{2 \cdot (1)}$

or alg +.5 $= \frac{6 \pm \sqrt{36 - 40}}{2}$

$= \frac{6 \pm 2i}{2} = 3 \pm i$

both sol +.5

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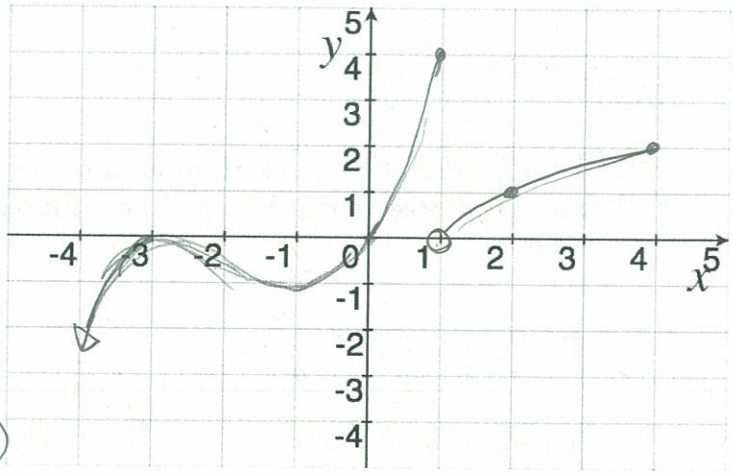
3. (Exam2 #7) Let h be piece-wise defined by:

$$h(x) = \begin{cases} \frac{1}{4}x(x+3)^2 & x \leq 1 \\ \log_2 x & 1 < x \leq 4 \end{cases}$$



(a) [1] (Quiz2 #3)
What is the domain of h?

$(-\infty, 4]$

(b) [2] (WebHW6 #9)
Find the real roots of h

ie. when $y=0$ } .5

$\frac{1}{4}x(x+3)^2 = 0$ } +.5

$\Rightarrow \frac{1}{4}x = 0$ or $(x+3)^2 = 0$

$x = 0$ or $x = -3$
(+.5) (+.5)

(c) [4] (Exam2 #7) Sketch a graph h

rats +1
bounce off of -3 +.5
end behavior +.5
endpoints/domain +.5

shape +.5
got it +.5
endparts +.5

4. [3] (PracticeFinal #3) Let $m(x) = \dfrac{x}{x+5}$. Given m passes the horizontal line test, find m^{-1}.

inverses swap x's & y's

+1 $x = \dfrac{y}{y+5}$

Solve for y

$x(y+5) = y$

$xy + 5x = y$

$5x = y - xy$

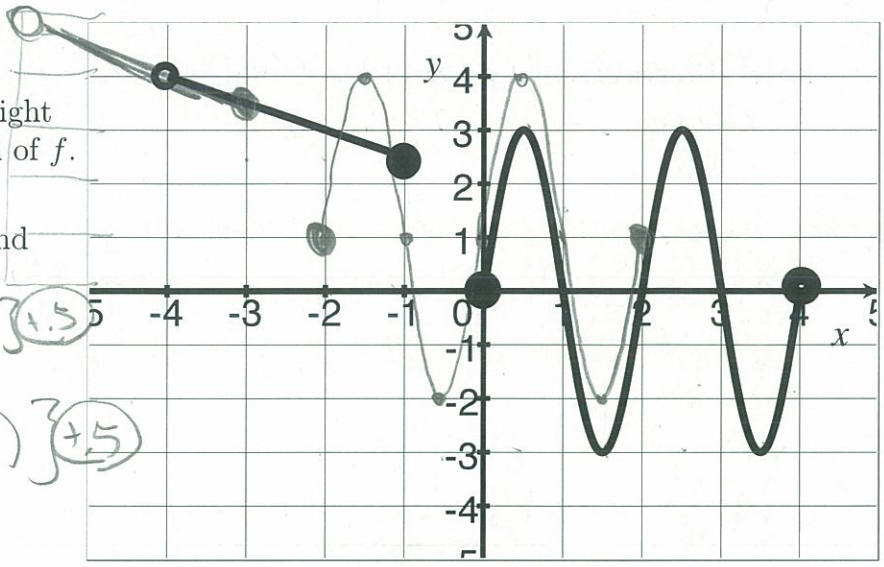
$5x = y(1-x)$

$\dfrac{5x}{1-x} = y$

alg +1
get y on one side +.5
get y alone on one side +.5

10

5. (PracticeFinal #5)
Let the graph to the right be the complete graph of f.



(a) [1] (Quiz1#2) Find the range of f.

y-values +.5

$[-3, 4)$ +.5

(b) [5] (WebHW12 #11 & LineWks #9)
The function f is a piecewise defined function consisting of a sinusoidal curve and a portion of a line. Find a formula for the function f in the indicated form:

$$f(x) = \begin{cases} -\frac{1}{2}x + 2 & \text{if } -4 < x \le -1 \\ 3\sin(\pi x) & \text{if } 0 \le x \le 4 \end{cases}$$

got +.5 +.5

+.5 $y = mx + b$ line

connects $(-2, 3)$ to $(-4, 4)$ +.5

slope $= \frac{rise}{run} = -\frac{1}{2}$ +.5

$y - 3 = -\frac{1}{2}(x - -2) \Rightarrow y = -\frac{1}{2}x + 2$ +.5

+.5 sine curve +.5 $A\sin(B(x+C))$

+.5 phase shift: none $\Rightarrow C = 0$ +.5

+.5 period $= 2 \Rightarrow 2 = 2\pi/B \Rightarrow B = \pi$

+.5 amplitude $= 3 \Rightarrow A = 3$

(c) [5] (§1.3 #56) Estimate the following *if* possible:

$2f(-3)$

start +.5

reading values +.5

$2(3.5)$ +.5

7 pts +.5

$f(-3) - f(1)$

$3.5 - 0$ +.5 +.5

3.5 pts +.5

$f \circ f(-2)$

$f(f(-2))$ +.5

$f(3)$ +.5

0 +.5

(d) [2] (Quiz1 #2) Estimate the x value(s) so that $f(x) = 3$?

$x = .5$ or 2.5 or -2

+.5 +.5 +.5

no more +.5

(e) [3] (WebHW7 #22) Sketch the graph of $f(x + 2) + 1$

horiz shift left 2 units +1

vert shift up 1 unit +.5

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$(x^2)^3 = x^2 x^2 x^2$
$= xx\,xx\,xx$
$= x^6$

6. [4] (PracticeExam2 #4 & §4.6 #27) Simplify

$$\frac{\sqrt[3]{x^2}(y^2)^{\frac{3}{2}}}{x^{\frac{2}{3}}y^2}$$

+.5 $\frac{(x^2)^{1/3}(y^2)^{3/2}}{x^{2/3}y^2}$

+1 $\frac{x^{2/3}y^3}{x^{2/3}y^2} = \frac{y^3}{y^2} = y$ +.5 +.5

$\arcsin(\sin\frac{4\pi}{3})$

+.5

Not in range $\frac{4\pi}{3}$ +1

$\arcsin\left(\frac{-\sqrt{3}}{2}\right)$ +.5

$-\frac{\pi}{3}$ +.5

7. Solve for x in the following:

[4] (§3.4 #68)

$\log_6(x+2) - \log_6(x-3) = 1$

$\log_6 \frac{x+2}{x-3} = 1$

6

$\frac{x+2}{x-3} = 6$

$x+2 = 6(x-3)$

$x+2 = 6x-18$

$-x \quad +18 \quad -x+18$

$20 = 5x$

$4 = x$ ✓

sketch +.5
log prop +.5
exp +.5
exp right +.5
order of op +1
notation +.5
algebra +.5

order of op +1
log +.5
log prop +.5
alg after log +.5
notation +.5

[3] (WebHW8 #13)

$3 \cdot 2^{3x-2} + 4 = 15.$

$-4 \quad -4$

$\frac{3 \cdot 2^{3x-2}}{3} = \frac{11}{3}$

$2^{3x-2} = 11/3$

$\ln 2^{3x-2} = \ln 11/3$

$\frac{(3x-2)\ln 2}{\ln 2} = \frac{\ln 11/3}{\ln 2}$

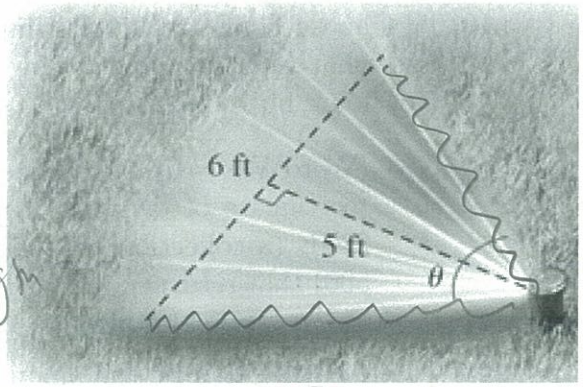
$3x-2 = \frac{\ln 11/3}{\ln 2}$

$+2 \quad +2$

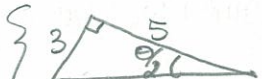
$\frac{3x}{3} = \frac{\frac{\ln 11/3}{\ln 2} + 2}{3}$

$x = \left[\frac{\ln 11/3}{\ln 2} + 2\right]/3 \approx 1.29$

8. [3] (§4.6 #87) A sprinkler rotates back and forth through the angle θ as shown to the right. At a distance of 5 feet from the sprinkler, the rays that form the sides of angle θ are 6 feet apart. Find θ.



Note: we assume the spray is constant => sides are same length => Isosceles $\triangle$

(+1) 

$\tan\frac{\theta}{2} = \frac{3}{5}$ (+1)

$\Rightarrow \frac{\theta}{2} = \arctan\frac{3}{5}$ (+.5) so $\theta = 2\arctan\frac{3}{5}$ (+.5)

$\approx 61.8°$ or 1.08 rad

9. Given that $\sin\theta = \frac{-12}{13}$ and $\pi < \theta < \frac{3\pi}{2}$.

(a) [3] (§5.1 #7) Find $\cos\theta$ *exactly*

$\sin^2\theta + \cos^2\theta = 1$ (+.5)

$\left(\frac{-12}{13}\right)^2 + \cos^2\theta = 1$ (+.5)

$\cos^2\theta = 1 - \frac{144}{169}$

$\cos\theta = \pm\sqrt{\frac{25}{169}}$ alg (+.5)

$\cos\theta = -\frac{5}{13}$ b/c ~~(-)~~ sign (+.5)

Notation (+.5)

(+1)

SohCahToa (+.5)

$\sin\theta = \frac{opp}{hyp} = \frac{12}{13}$

or

$adj^2 + opp^2 = hyp^2$ (+.5)

$adj^2 + 12^2 = 13^2$

$\Rightarrow adj = 5$

$\cos\theta = \frac{adj}{hyp} = -\frac{5}{13}$ alg (+.5) b/c ~~(-)~~ sign (+.5)

[2] (b) ~~[5]~~ (WebHW14 #23) Find $\sin(2\theta)$ *exactly*

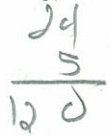
Notation (+.5)

$\sin(2\theta) = \sin(\theta+\theta) = \sin\theta\cos\theta + \sin\theta\cos\theta$

$= 2\sin\theta\cos\theta$ (+.5)

$= 2\left(\frac{-12}{13}\right)\left(\frac{-5}{13}\right)$ (+1)

$= \frac{24 \cdot 5}{169} = \frac{120}{169} \approx .71$



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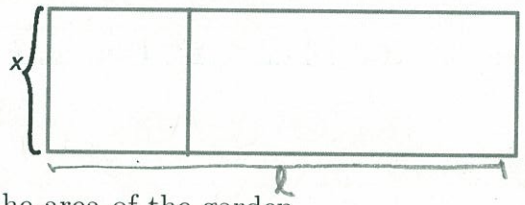
10. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (WordWks #6) You have \$5500 in a retirement fund. There are 5 year CDs (certificate of deposits) being offered with an annual rate of 2.03% and index funds (a collection of stocks from companies ~~included in measures like the S&P 500) that returned 8.2%~~ since the 1990's. You will invest all of your money ~~in one of these two options.~~

i. [3] Let x be the amount of money you invest in CD's. Find a function of x that represents the money earned in a year

ii. [2] How much money do you relegate to CD's and how much do you put in an index fund to get an annual return of 5%

(b) (Exam1 #7) A gardener wants to use 120 feet of fencing to enclose a rectangular garden and divide it into two plots, as shown in the figure to the right.



i. [3] Find a function of x that represents the area of the garden.

ii. [2] Algebraically determine the largest possible area for such a garden.

(a) start (+.5)

(+.5) { Money earned in a year = CD interest + index interest
$= .032y + .0203x$
(+.5) { where y is the left over \$

So $x + y = 5500$
$\Rightarrow y = 5500 - x$ } (+1)

sub (+.5)

So \$ earned in a year $= .032(5500 - x) + .0203x$
$= 451 - .0617x$

ii) we want $.05 \cdot 5500$ returned so
\$275 (+.5)

$275 = 451 - .0617x$
$-176 = -.0617x$ } (+1)

\$2,852.51 $= x$ into CDs } (+.5)
so then \$2647.49 into index funds

(b) i) Area $= x \cdot \ell$ } (+.5) start (+.5)

(+.5) { need to shoot w/ x's somehow

note fence = 120ft
$x + x + x + \ell + \ell = 120$ } (+1)
$3x + 2\ell = 120$
$\Rightarrow 2\ell = 120 - 3x$
$\Rightarrow \ell = 60 - \frac{3}{2}x$

So Area $= x \cdot \ell$
$= x(60 - \frac{3}{2}x)$ sub (+.5)
$= -\frac{3}{2}x^2 + 60x$

ii) (+.5) parabola opening down $\Rightarrow$ max @ vert

Area $= -\frac{3}{2}x^2 + 60x$
$-\frac{2}{3}\text{Area} = x^2 - 40x$
$+400 \quad +(400)$ (+1)
$-\frac{2}{3}\text{Area} + 400 = (x-20)^2$
$-\frac{2}{3}\text{Area} = (x-20)^2 - 400$
$\text{Area} = -\frac{3}{2}(x-20)^2 + 600$

vertex (20, 600)
so
Max area is (+.5) 600 ft^2

or $-\frac{b}{2a} = \frac{-60}{2(-\frac{3}{2})} = 20$ (+1)
Put 20 into Area eq

5

11. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (WordProblemWks2 #3) You have four ten-year subsidized loans you took out to pay for college. Below is a table of the loans taken and their respective effective annual interest rates (AIR):

loan ($)	8,000	9,000	10,000	12,000
AIR (%)	3.49	4.22	5.01	6.29

Assume all loans are continuously compounded once interest start accruing. You do not have a job lined up yet so you doubt you will be able be to make any payments for the three years,

i. [2] Assuming you defer your loans so that you don't have any additional fees beyond interest. What will the total balance you owe in three years without making any payments?

ii. [3] After graduation you are given the option of consolidating (that is take out one loan to pay off *all* the balances on your current loans). What rate would you need to consolidate your loans at to be in a better position three years from now?

(b) A sound with intensity x has $10 \log \frac{x}{I_0}$ decibels,
where $I_0 = 10^{-12}$ watts per square meter (W/m^2).

i. [2] France passed a law limiting iPods and other MP3 players to a maximum possible volume of 100 decibels. Find the maximum intensity (in W/m^2) an iPod is legally allowed to output in France.

ii. [3] Normal conversation has a sound level of about 65 decibels. How many more times intense than normal conversation is the sound an iPod operating at the French maximum of 100 decibels?

(a) i: Pe^{rt} (+.5) start (+.5) $t=3$ (+.5)

$8000e^{.0349 \cdot 3} + 9000e^{.0422 \cdot 3}$

$+10{,}000e^{.0501 \cdot 3} + 12{,}000e^{.0629 \cdot 3}$

$= 45{,}211.66$ } get it (+.5)

ii) i.e. find r so that (+.5)

$45{,}211.66 > (8000+9000+10{,}000+12{,}000)e^{r \cdot 3}$ (+.5) (+.5)

$\frac{45{,}211.66}{39000} > \frac{39{,}000e^{3r}}{39000}$

$1.15927 > e^{3r}$ (+.5) $\ln 1.15927 > 3r$ (+.5) $.04926 > r$ (+.5) so less than 4.9% (+.5)

(b) i) start (+.5)

100 decibels $= 10 \log \frac{\text{intensity}}{10^{-12}}$ (+.5) (+.5)

$10 = \log \frac{\text{intensity}}{10^{-12}}$ alg (+.5)

$\Rightarrow 10^{10} = \frac{\text{intensity}}{10^{-12}} \Rightarrow \text{intensity} = 10^{-2} = .01$

ii) $65 = 10 \log \frac{\text{conversation intense}}{10^{-12}}$ (+1.5)

$6.5 = \log \frac{\text{conversation intense}}{10^{-12}}$

$10^{6.5} = \frac{\text{conversation}}{10^{-12}}$ notation (+.5)

$\Rightarrow$ intensity of conversation $10^{-5.5}$

So $\frac{10^{-2}}{10^{-5.5}} = 10^{3.5} \approx 3{,}162$ times (+.5)

12. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (§6.2 #63) A laser beam with an angle of elevation of 42° is reflected from a target and received 1200 yards from the point of origin (assuming the ground is horizontal). Assume the trajectory of the beam forms and isosceles triangle.

i. [1] Draw a diagram of the situation described.

ii. [2] Find the total distance that the beam travels.

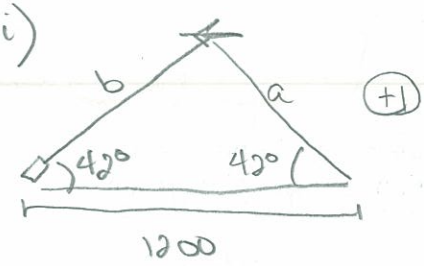
iii. [2] Find the height of the target.

(b) (PracticeFinal #16) Points A and B are separated by a lake. To find the distance between them, a surveyor locates a point on land such that $\angle CAB = 30°$. She also measures CA as 475ft and CB as 345ft.

i. [2] Draw a picture of the situation. Do you have enough information to find the distance between A and B? Justify yourself.

ii. [3] If you can, find the distance between A and B. If you can't, what two distances are possible?

(a) i)



+1

ii) beam travels top two legs of $\Delta \Rightarrow a+b$

(+.5) Law of Sines $\frac{\sin 42°}{a} = \frac{\sin 96°}{1200}$ (+.5)

(+.5) $180 - 84 = 96°$

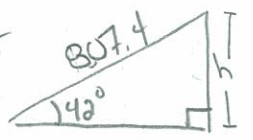
$\Rightarrow a = \left(\frac{\sin 42°}{\sin 96°}\right) 1200$

≈ 807.4

b/c isosceles (+.5)

$a+b = a+a = 2a = 2(807.4) \approx 1614.8$ yd

iii) consider

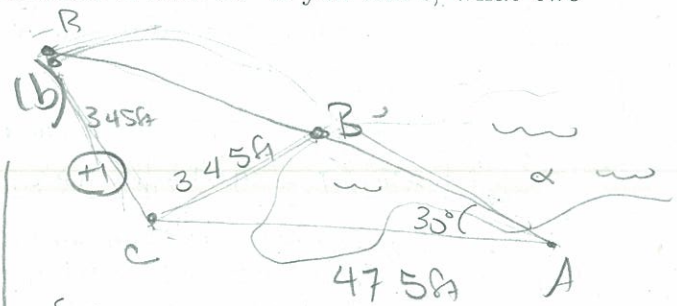


(+.5) SohCahToa notation (+.5)

(+.5) $\sin 42° = h/807.4$

(+.5) $h = 807.4 \sin 42° \approx 540.3$ yds

(b)



(+1)

(+.5) This is a SSA or ASS Δ

(+.5) we may not have enough information because there might be 2 Δ's?

ii) Law of Sines (+.5) $\frac{\sin 30°}{345} = \frac{\sin B}{475}$

$\Rightarrow .688 = \sin B$

(+.5) $\Rightarrow B = .759$ rad or $43.5°$

or

(+.5) $B = \pi - .759$ or $180 - 43.5°$

$= 2.383$ rad $= 136.5°$

So there are two cases

if $\angle B = 43.5°$

$\Rightarrow \angle C = 106°$ (+.5)

$\frac{\sin 106°}{c} = \frac{\sin 30°}{345}$

$\Rightarrow c = 663.3$ ft (+.5)

if $\angle B = 136.5°$

$\Rightarrow \angle C = 13.5°$

$\frac{\sin 13.5°}{c} = \frac{\sin 30}{345}$

$\Rightarrow c = 161.1$ ft