Exam 2

TMath 120

Summer 2015

NAME: Key

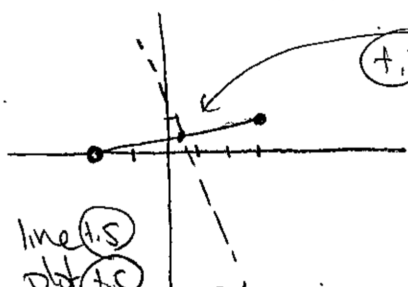
1. [2] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

T F $\quad \frac{x-1}{x} \cdot \frac{x+1}{2} = \frac{2(x-1)+x(x+1)}{2x}$ $\qquad \frac{x-1}{x} \cdot \frac{x+1}{2} = \frac{(x-1)(x+1)}{2x}$

T F $\quad$ 1 radian $\approx 57°$ $\qquad 1\,\cancel{rad} \cdot \frac{180°}{\pi\,\cancel{rad}} = \frac{180}{\pi} \approx \frac{180}{3} \approx 5$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] (§1.1 #78) Find the graph of a function α so that the points (x, y) on the graph of α if and only if (x, y) is the same distance from the two points $(3, 1)$ and $(-2, 0)$.



line (+.5)
plot (+.5)
right line (+.5)

(+.5) midpoint $= \left(\frac{3+-2}{2}, \frac{1+0}{2}\right) = \left(\frac12, \frac12\right)$ (+.5)

Slope of solid line $\frac{rise}{run} = \frac{1}{5}$ } (+.5)

α is $\perp$ to solid line so -5 } (+.5)

$y - \frac12 = -5(x - \frac12)$ or $y = -5x + 3$ } (+.5)

3. [4] (exponent wks #4) Find any real or imaginary x such that $4x^{-1} - 7x^{-2} = 1$.

$4x^{-1} - 7x^{-2} = 1$

neg exp (+1)

alg (+1)

$x^2\left(\frac{4}{x} - \frac{7}{x^2}\right) = (1)x^2$

$4x - 7 = x^2$

$0 = x^2 - 4x + 7$

solving quadratic

$x^2 - 4x + 7 = 0$
$\quad -7 \quad -7$

$x^2 - 4x = -7$
$+\left(\frac{4}{2}\right)^2 \quad +\left(\frac{4}{2}\right)^2$ (+.5)

$(x-2)^2 = -3$

$x - 2 = \pm\sqrt{-3}$ (+.5)

$x = 2 \pm \sqrt{-3}$

$x = 2 \pm i\sqrt{3}$

both sol (+1)

or

$x^2 - 4x + 7 = 0$

(+1) $x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(}}{2(1)}$

$= \frac{4 \pm \sqrt{16-28}}{2}$

$= \frac{4 \pm \sqrt{-12}}{2}$

$= \frac{4 \pm 2\sqrt{-3}}{2}$

$\left((x^2)^3 = x^2 x^2 x^2 = xxxxxx = x^6 \right)$

4. [3] (PracticeExam2 #4) Simplify $\dfrac{\sqrt[3]{x}(y^{-4})^{\frac{3}{2}}}{x^2y^{-5}}$

+.5 +.5 neg exp +1

$\frac{x^3}{x^4} = \frac{xxx}{xxxx} = \frac{1}{x} = x^{-1}$

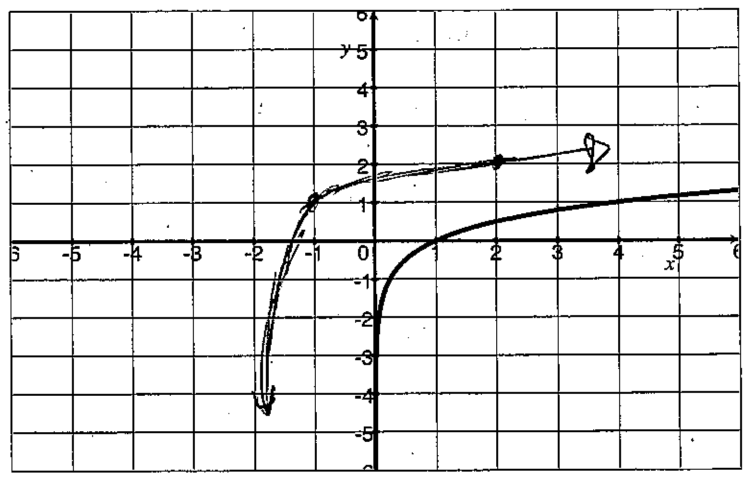
$$\frac{x^{\frac{1}{3}}\left(y^{-4}\right)^{3/2}}{x^2 y^{-5}} = \frac{x^{1/3} y^{-12/2}}{x^2 y^{-5}} = \frac{x^{1/3} y^{-6}}{x^2 y^{-5}} = \frac{x^{1/3} y^{5}}{x^2 y^{6}}$$

$$= x^{1/3 - 2} y^{5-6} = x^{-5/3} y^{-1} \text{ or } \frac{1}{x^{5/3} y}$$

alg/fractions +.5

+.5

5. Let f be the function whose graph is below:



(a) [2] (Quiz2 #2a)
What is the domain of f?

$(0, \infty)$

(b) [2] (§3.2 #62) Find the function of the form $f(x) = \log_b(x)$.

+.5 { passes thru $(4, 1)$

So $\log_b 4 = 1$ +1
$\Rightarrow b^1 = 4$

So $\log_4 x = f(x)$

got it +.5

(c) [3] (PracticeExam2 #9c) Notice that f passes the horizontal line test, so f^{-1} exists. Find f^{-1}.

$y = \log_4 x$

inverse swaps x's + y's

+1 { $\log_4 y = x$

$4^x = y$ +1 solve for y

start +.5
notation +.5

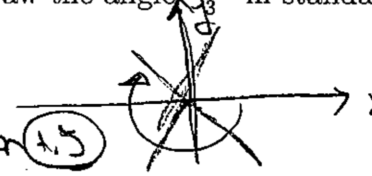
(d) [3] (WebHW7 #22) Sketch the graph of $f(x + 2) + 1$ on the axes above.

vertical shift up 1 +.5

horizontal shift left 2 units +1

drew
shape +1
horiz +1
vert +1

6. [3] (§4.1) Draw the angle $\frac{-4\pi}{3}$ in standard position.



standard position (+.5)

direction/neg angle (+1)

use $-\frac{4\pi}{3}\text{rad} \cdot \frac{180^\circ}{\pi\,\text{rad}} = -240^\circ$

got it (+1)

over π rad (+.5)

7. Let h be the piece-wise defined function:

$$h(x) = \begin{cases} x(x-1)(x+3)^2 & \text{if } x \le 1 \\ 2^x & \text{if } 1 < x \end{cases}$$

$\to$ 4th deg

(a) [3] (WebHW6 #9)
Identify each real zero.

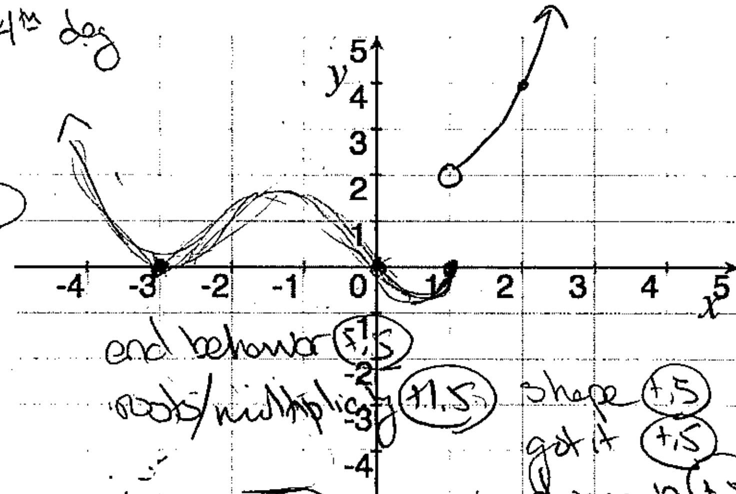
start (+.5)

i.e when $h(x) = 0$ (+.5)

(+.5) $0 = x(x-1)(x+3)^2$

$\Rightarrow x = 0, 1$ or -3 (+.5) (+.5) (+.5)

(b) [4] (§2.2#52 &WebHW7#14)
Graph h.



end behavior (+.5)

roots/multiplicity (+1.5)

domain (+.5)

shape (+.5)

got it (+.5)

domain (+.5)

8. Solve for x in the following:

[4] (§3.4 #68)

$\log_6(x+2) - \log_6(x-3) = 1$

$6^{\log_6 \frac{x+2}{x-3}} = 6^1$

$\frac{x+2}{x-3} = 6$

$x+2 = 6(x-3)$

$x+2 = 6x-18$

$-x \quad +18 \quad -x \quad +18$

$20 = 5x$

$4 = x$ ✓

start (+.5)

log prop (+.5)

exp (+.5)

exp rule (+.5)

order of op (+1)

notation (+.5)

algebra (+.5)

[3] (WebHW8 #13)

$3 \cdot 2^{3x-2} + 4 = 15.$

$-4 \quad -4$

(+1) order of op

(+.5) log

(+.5) log prop

(+.5) alg after log

(+.5) notation

$\frac{3 \cdot 2^{3x-2}}{3} = \frac{11}{3}$

$2^{3x-2} = 11/3$

$\ln 2^{3x-2} = \ln 11/3$

$\frac{(3x-2)\ln 2}{\ln 2} = \frac{\ln 11/3}{\ln 2}$

$3x - 2 = \frac{\ln 11/3}{\ln 2}$

$+2 \qquad +2$

$\frac{3x}{3} = \frac{\frac{\ln 11/3}{\ln 2} + 2}{3}$

$x = \left[\frac{\ln 11/3}{\ln 2} + 2\right] / 3 \approx 1.29$

9. (WebHW9 #7) The pH of a solution is given by the formula pH$= -\log[H^+]$ where $[H^+]$ is the hydrogen ions concentration in moles per liter. The pH of solution A is 2.6 and the pH solution of B is 10.6.

   (a) [2] Find the hydrogen-ion concentration of solution B.

   (+.5) $\left\{ \frac{-\log[H^+]_B}{-1} = \frac{10.6}{-1} \right.$ find $[H^+]_B$ $\}$ (+.5)

   $\log[H^+]_B = -10.6$ $[H^+]_B = 10^{-10.6}$ log to exp (+1) $\approx 2.5 \cdot 10^{-11}$

   (b) [2] How many times less is the hydrogen-ion concentration of solution B to solution A?

   start (+.5)

   $[H^+]_B = ?\,[H^+]_A$ note $[H^+]_B = 10^{-10.6}$ we need $[H^+]_A$

   $\Rightarrow ? = \frac{[H^+]_B}{[H^+]_A}$ (+.5) ratio of concentrations (+.5) (+.5) $-\log[H^+]_A = 2.6 \Rightarrow [H^+]_A = 10^{-2.6}$

   So $\frac{10^{-10.6}}{10^{-2.6}} = 10^{-8}$ times less

10. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

    (a) (WebHW9 #5) The population of a town was 6,000 in 1990 and grew to 8,000 in 2000. Assume that the population will continue to grow exponentially.

       i. [4] What will the population be in 2015?

       ii. [2] When will the population hit 12,000?

    (b) (WordWks #4) Chad just graduated but does not have a job lined up yet. In his last year of graduate school he made ends meet by using his credit card and now has a balance of $9,000. His credit card compounds monthly with an annual interest rate of 14.9%.

       i. [3] Assume the worst and that Chad won't be able to make any payments on his credit card bill. How much will he owe on his credit card after two years (ignoring late fees)?

       ii. [3] Just before graduation, Chad received an ad for a State Farm Good Neighbor Visa Credit Card. The card will transfer his balance (with a 2.90% balance transfer fee) with an annual interest rate of 10.24%, compounded monthly. How much will Chad owe after two years if he transfers to the new card (ignoring late fees)?

next page

(a) start (+.5)

i)

(+.5)

t years since 1990	population
0	6000
10	8,000

(+.5) we need to find pop. when t = 25 ... we need to find the function

(+.5) Pe^{rt} = pop @ time t

(+.5) P = 6,000 so $6000e^{rt}$ = pop@t

(+.5) $\dfrac{8{,}000}{6000} = \dfrac{\cancel{6{,}000} e^{r \cdot 10}}{\cancel{6{,}000}}$

$4/3 = e^{r \cdot 10}$

$\dfrac{\ln 4/3}{10} = \dfrac{\cancel{10} r}{\cancel{10}}$ $\quad \underbrace{r = \frac{1}{10}\ln 4/3}_{(+.5)}$

So

pop @ time $t = 6{,}000\, e^{t(\frac{1}{10}\ln 4/3)}$

(+.5) pop in 2015 $= 6{,}000\, e^{25 \cdot \frac{1}{10}\ln 4/3}$

$\approx 12{,}316$

ii) find t so that

(+.5) $\dfrac{12000}{6000} = \dfrac{\cancel{6000}\, e^{t(\frac{1}{10}\ln 4/3)}}{\cancel{6000}}$

alg (+.5) $2 = e^{t(\frac{1}{10}\ln 4/3)}$

$\ln 2 = t(\frac{1}{10}\ln 4/3)$

sig + mpt (+1) $t = \dfrac{\ln 2}{\frac{1}{10}\ln 4/3} = \dfrac{10\ln 2}{\ln 4/3} \approx 24$ years

(b) start (+.5)

i) (+.5) $P(1+\frac{r}{n})^{nt}$

$\underbrace{9000}_{(+.5)}\left(1+\underbrace{\frac{.149}{12}}_{(+.5)}\right)^{\underbrace{12 \cdot 2}_{(+.5)}}$

$= 12{,}102.23$ (+.5)

ii) $\underbrace{9000(1.029)}_{(+1)}$ = new principal

notation (+.5)

So

$[9000(1.029)]\left(1+\underbrace{\frac{.1024}{12}}_{(+.5)}\right)^{\underbrace{12 \cdot 2}_{(+.5)}}$

= (+.5)

Mini-Quiz 10

[10] Leave answers in factored form as reduced fractions. No credit will be given for non-reduced answers or mixed numbers. Assume all letters are real numbers and that no combination of symbols equal zero in the denominator.

$\frac{1}{6a}+\frac{1}{8b}$ $\qquad$ $\frac{-1}{\frac{1}{2}f(x)}+\frac{1}{f(x)}$ $\qquad$ $-a+\frac{1}{a^2}$ $\qquad$ $\frac{1}{\frac{1}{2}f(x)}+\frac{1}{f(x)}$

$\frac{4b+3a}{24ab}$ $\qquad$ $\frac{-2}{f(x)}+\frac{1}{f(x)}$, $\frac{-1}{f(x)}$ $\qquad$ $\frac{-a^3+1}{a^2}$ $\qquad$ $\frac{2}{f(x)}+\frac{1}{f(x)}$, $3/f(x)$

$\frac{1}{b^2}+\frac{1}{b(f(x))^2}$ $\qquad$ $\frac{4}{3a}+\frac{-1}{\frac{3}{5}a}$ $\qquad$ $\frac{3}{f(x)}+\frac{1}{\frac{1}{3}a}$ $\qquad$ $\frac{2}{b+1}+\frac{2}{b+2}$

$\frac{[f(x)]^2+b}{b^2[f(x)]^2}$ $\qquad$ $\frac{4}{3a}-\frac{5}{3a}$, $\frac{-1}{3a}$ $\qquad$ $\frac{3}{f(x)}+\frac{3}{a}$, $\frac{3a+3f(x)}{af(x)}$ $\qquad$ $\frac{2(b+2)+2(b+1)}{(b+1)(b+2)}$

$a+\frac{a}{7}$ $\qquad$ $\frac{3}{10}+\frac{5}{6}$ $\qquad$ $\frac{1}{\frac{2}{3}a}+\frac{1}{\frac{2}{7}a}$ $\qquad$ $\frac{2}{f(x)}+\frac{1}{\frac{1}{2}a}$

$\frac{8a}{7}$ $\qquad$ $\frac{9+25}{30}=\frac{34}{30}$, $17/15$ $\qquad$ $\frac{3}{2a}+\frac{7}{2a}$, $\frac{10}{2a}=\frac{5}{a}$ $\qquad$ $\frac{2}{f(x)}+\frac{2}{a}$, $\frac{2a+2f(x)}{af(x)}$

$\frac{2}{a+2}+\frac{1}{(a+2)^2}$ $\qquad$ $\frac{a-1}{a+1}+\frac{-1}{a}$ $\qquad$ $f(x)+\frac{1}{a-1}$ $\qquad$ $\frac{-1}{f(x)}+\frac{f(x)}{x-2}$

$\frac{2(a+2)+1}{(a+2)^2}$ $\qquad$ $\frac{a(a-1)-(a+1)}{a(a+1)}$ $\qquad$ $\frac{f(x)(a-1)+1}{a-1}$ $\qquad$ $\frac{2-x+[f(x)]^2}{f(x)(x-2)}$

$\frac{1}{(x+h)^2}+\frac{-1}{x+h}$ $\qquad$ $\frac{x-h}{x+h}+\frac{1}{x}$ $\qquad$ $\frac{1}{f(x+h)}+\frac{1}{f(x)}$ $\qquad$ $\frac{2}{x^2}+\frac{2}{x-1}$

$\frac{1-x-h}{(x+h)^2}$ $\qquad$ $\frac{x(x-h)+x+h}{x(x+h)}$ $\qquad$ $\frac{f(x)+f(x+h)}{f(x)f(x+h)}$ $\qquad$ $\frac{2(x-1)+2x^2}{x^2(x-1)}$