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Exam 1 TMath 120 Summer 2015

NAME: Key

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

T F $\frac{\frac{1}{2}}{a} + \frac{1}{2a} = a$ $\quad \frac{2}{2}\frac{\frac{1}{2}}{a} + \frac{1}{2a} \Rightarrow \frac{1}{2a} + \frac{1}{2a} = \frac{\cancel{2}}{\cancel{2}a} = \frac{1}{a}$

T F $(x+2)^2 = x^2 + 4$ $\quad (x+2)^2 = (x+2)(x+2) = x^2 + 2x + 2x + 4$

T F $f(x-1) = f(x) - 1$ $\quad$ Let $f(x) = x^2$ note $f(x-1) = (x-1)^2 \neq f(x) - 1 = x^2 -$

T F $(2+3i)(1-i) = 2*1 + 3*(-1)i = 2 - 3i$
$2 - 2i + 3i - 3i^2 = 2 + i - 3(-1) = 2 + i + 3 = 5 + i$

T F A fourth degree polynomial always has four complex roots. ?? Fundamental Theorem of Algebra??

T F If f has an inverse and $f(1) = -2$ then $f^{-1}(-2) = 1$
b/c $f(f^{-1}(-2)) = -2$ and $f^{-1}(f(1)) = 1$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] (Practice Exam #3) Find any real or imaginary x such that $\frac{2x}{2x-2} - \frac{x+1}{2x-2} = \frac{1}{2x}$.

start (+.5)
fractions (+1.5)
algebra (+1.5)
notation (+.5)

$\frac{2x - (x+1)}{2x - 2} = \frac{1}{2x}$

$\frac{2x - x - 1}{2(x-1)} = \frac{1}{2x}$ x.5

$\frac{\cancel{x-1}}{2(\cancel{x-1})} = \frac{1}{2x}$ $\cancel{2x}$ x.5

$\cancel{2}\frac{1}{\cancel{2}} = \frac{1}{\cancel{2x}}$ .2

$\Rightarrow 1 = 1/x \Rightarrow x = 1$

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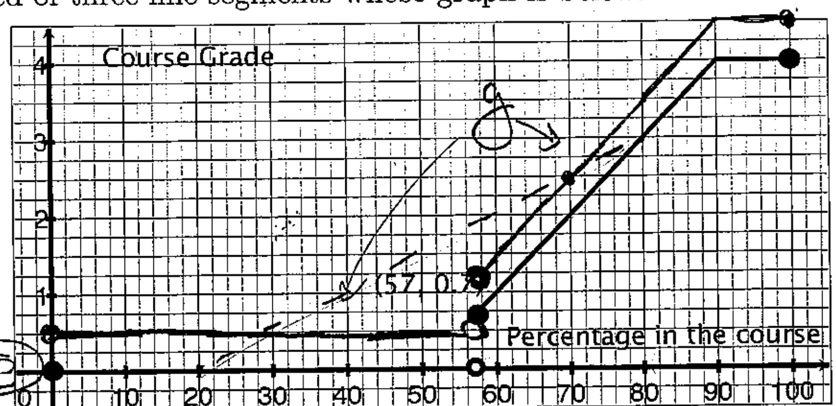
3. Let f be the function comprised of three line segments whose graph is below:



(a) [4] (Quiz1 #2) Estimate the following *if* possible:

i. $f(85)$

≈ 3.5 (+1)

ii. $6f(70)+3$

$6(2)+3 = 12+3$ (+.5)

$= 15$ (+.5)

iii. $(f \circ f)(90)$

$f(f(90)) = f(4) = 0$ (+1)

comp (+.5)

$90 \xrightarrow{f} 4 \xrightarrow{f} 0$ (+.5)

(b) [2] (§1.5 #18) Draw the graph of g if $g(x) = f(x) + \frac{1}{2}$.

got it (+2)

each pt +.5

+1 ↳ vertical shift up $\frac{1}{2}$

(c) [1] (Quiz1 #2) Identify the range of f.

(+.5) 0 and between .7 and 4 inclusive (+.5)

or $\{0\} \cup [.7, 4]$

(d) [2] (WebHW2 #14) Find the average rate of change of f from $x = 20$ to $x = 80$

i.e. slope of dashed line (+.5)

$= \frac{\text{rise}}{\text{run}}$ (+.5) $= \frac{3}{60}$ (+1) $= \frac{1}{20}$

or $\frac{f(80)-f(20)}{80-20}$ (+.5) (+1)

(+.5) $\frac{3-0}{60} = \frac{1}{20}$

(e) [4] (PracticeExam#4) Find the piece-wise defined rule of f in the indicated form.

$$f(x) = \begin{cases} 0 & \text{if } 0 \le x < 57 \\ \frac{1}{10}x - 5 & \text{if } 57 \le x < 90 \\ 4.0 & \text{if } 90 \le x \le 100 \end{cases}$$

(+.5) (+.5)

line segment in the middle $y = mx + b$ (+.5)

passes thru $(70, 2)$ and $(80, 3)$ (+.5)

slope $= \frac{\text{rise}}{\text{run}} = \frac{3-2}{80-70} = \frac{1}{10} = m$ (+1)

b/c passes thru $(70, 2)$

$2 = \frac{1}{10}(70) + b$

$\Rightarrow b = -5$ (+1)

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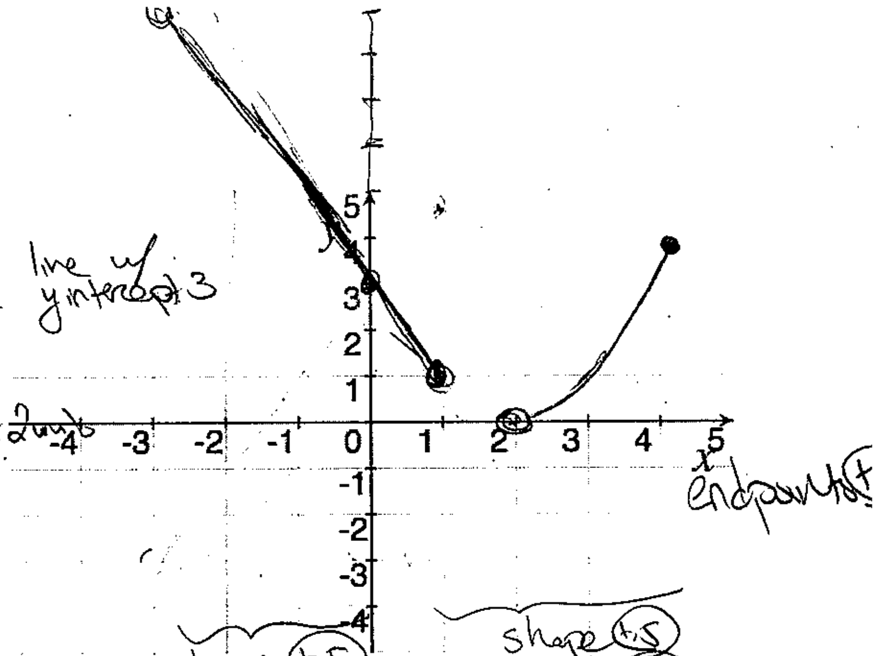
4. Let h be piece-wise defined by:

slope

$$h(x) = \begin{cases} -2x+3 & -3 < x \le 1 \\ (x-2)^2 & 2 < x \le 4 \end{cases}$$

line w/ y intercept 3

(a) [3] (WebHW1 #19) Graph h. → shift right 2 units



(b) [2] (WebHW2 #11) Is h a function? Why or why not?

yes? +1

it passes the vertical line test +1

(c) [2] (§1.1 #48) Identify the y-intercept.

3 or (0, 3) +1 i.e. where passes thru the y-axis +1

(d) [1] (Quiz1 #2) Find all possible input(s) so that $h(x) = 1$.

looking at graph +.5

x = +1 and 3 +.5

5. Let $\alpha(x) = \dfrac{x+1}{x+2}$ and $\beta(x) = 2 - \sqrt{x+1}$. Both α and β have inverses that exist.

(a)]2] Find the domain of β.

#'s below sq. root ≥ 0 +.5

x + 1 ≥ 0 +.5

x ≥ -1 +.5 or [-1, ∞)

inclusive +.5

(b) [2] (§1.6 #38) $(\alpha \circ \beta)(x)$.

$\alpha(\beta(x)) = \alpha\left(\boxed{2-\sqrt{x+1}}\right) = \dfrac{\boxed{2-\sqrt{x+1}}+1}{\boxed{2-\sqrt{x+1}}+2}$ +.5 +.5

$x \xrightarrow{\beta} \boxed{2-\sqrt{x+1}} \xrightarrow{\alpha}$

composition +.5

got it +.5

(c) [4] (§1.7 #55) Find $(\alpha^{-1})(x)$

+1.5 we want $\alpha(\alpha^{-1}(x)) = x$

so $\dfrac{\alpha^{-1}(x)+1}{\alpha^{-1}(x)+2} = x$ or $\dfrac{y+1}{y+2} = x$

y by itself +.5

alg +1.5

order of op +.5

$\cancel{y+2}\ \dfrac{y+1}{\cancel{y+2}} = x\ y+2$ 1.5

$y+1 = xy+2x$

$y+1 = xy+2x$

$-xy \quad -xy$ +.5

$y - xy + 1 = 2x$

$y - xy + 1 = 2x$

$\quad -1 \quad -1$

+.5 $y(1-x) = 2x-1$

$y = \dfrac{2x-1}{1-x}$

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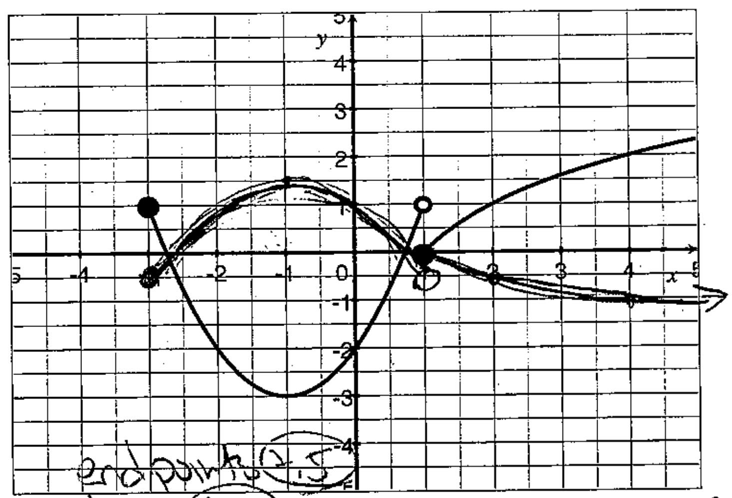
6. [3] (WebHW3 #8) Let f be the function whose graph is shown to the right. Graph g where $g(x) = -\frac{1}{2}f(x)$.

vertical flip and stretch by 1/2

— or —

mult the y-coord by $-1/2$

+1



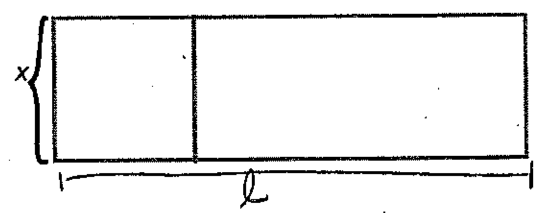
endpoints (+.5)

shape (+.5)

(+1)

(+1)

7. [8] (WordProblem #5) A gardener wants to use 130 feet of fencing to enclose a rectangular garden and divide it into two plots, as shown in the figure to the right.



start (+.5)

define variable (+.5)

(a) [5] Find a function of x that represents the area of the garden.

(+1) $\text{Area} = x \cdot l$

(+.5) need to sub out l w/ x's somehow...

note fence = 130 ft

(+1) $x + x + x + l + l = 130$

$3x + 2l = 130$

(+.5) $\Rightarrow$ $2l = 130 - 3x$

$l = 65 - 1.5x$

So $\text{Area} = x \cdot l$

$= x(65 - 1.5x)$

sub (+1)

(b) [3] Algebraically determine the largest possible area for such a garden.

$\text{Area} = x(65 - 1.5x)$

$= 65x - 1.5x^2$

$= -1.5x^2 + 65x$

(+1) parabola opening down $\Rightarrow$ maximum is @ the vertex?

$\frac{\text{Area}}{-1.5} = \frac{-1.5x^2 + 65x}{-1.5}$

got it (+1)

method (+1)

$-\frac{2}{3}\text{Area} = x^2 - \frac{130}{3}x$

$+\left(\frac{130/3}{2}\right)^2 \qquad + \left(\frac{130/3}{2}\right)^2$

$-\frac{2}{3}\text{Area} + \left(\frac{130}{6}\right)^2 = \left(x - \frac{130}{6}\right)^2$

$-\left(\frac{130}{6}\right)^2 \qquad -\left(\frac{130}{6}\right)^2$

$-\frac{2}{3}\text{Area} = \left(x - \frac{130}{6}\right)^2 - \left(\frac{130}{6}\right)^2$

$\text{Area} = -\frac{3}{2}\left(x - \frac{130}{6}\right)^2 + \frac{3}{2} \cdot \frac{130^2}{6^2}$

$= -\frac{3}{2}\left(x - \frac{130}{6}\right)^2 + \frac{130^2}{2 \cdot 2 \cdot 3}$

so max @ $\left(\frac{130}{6}, \frac{130^2}{24}\right)$

largest area is $\frac{130^2}{24}$