EXAM 2 **TMath 120** Summer 2011

NAME: Key

1. [7] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

   T (F) $x^{-2} = x^{\frac{1}{2}}$ $x^{-2} = \frac{1}{x^2}$

   T (F) $\sqrt{b+x} = \sqrt{b} + \sqrt{x}$ b/c $b = 1-x$ $\sqrt{b+x} = \sqrt{3} \neq \sqrt{1}+\sqrt{1} = \sqrt{b}+\sqrt{x}$

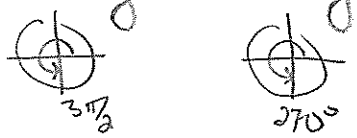
   T (F) $\log(x+y) = \log(x) \cdot \log(y)$ $\log(xy) = \log x + \log y$

   (T) F $\log_b 1 = 0$ for $b > 0$. b/c $b^0 = 1$ for $b > 0$

   T (F) $\log 8v^3 = 3\log 8v$ for all $v > 0$ but $3\log 8v = \log(8v)^3$

   (T) F $\frac{3\pi}{2}$ radians is the same angle as 270°

   (T) F For all x, $(\cos x)^2 + (\sin x)^2 = 1$. Pythagorean Theorem?



Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] (mini-quiz) Solve for r and simplify given:

$$\frac{1}{\frac{1}{r}+\frac{1}{s}} = t$$

started (+.5)
Sac add (+.5)
clear den (+.5)
alg/order of ops (+1)
r on one side (+.5)
simplified (+1)

$\left(\frac{1}{r}+\frac{1}{s}\right)\frac{1}{\left(\frac{1}{r}+\frac{1}{s}\right)} = t\left(\frac{1}{r}+\frac{1}{s}\right)$

$1 = t\left(\frac{1}{r}+\frac{1}{s}\right)$

$1 = \frac{t}{r} + \frac{t}{s}$

$-t/s \quad -t/s$

$1 - t/s = t/r \quad (r)$

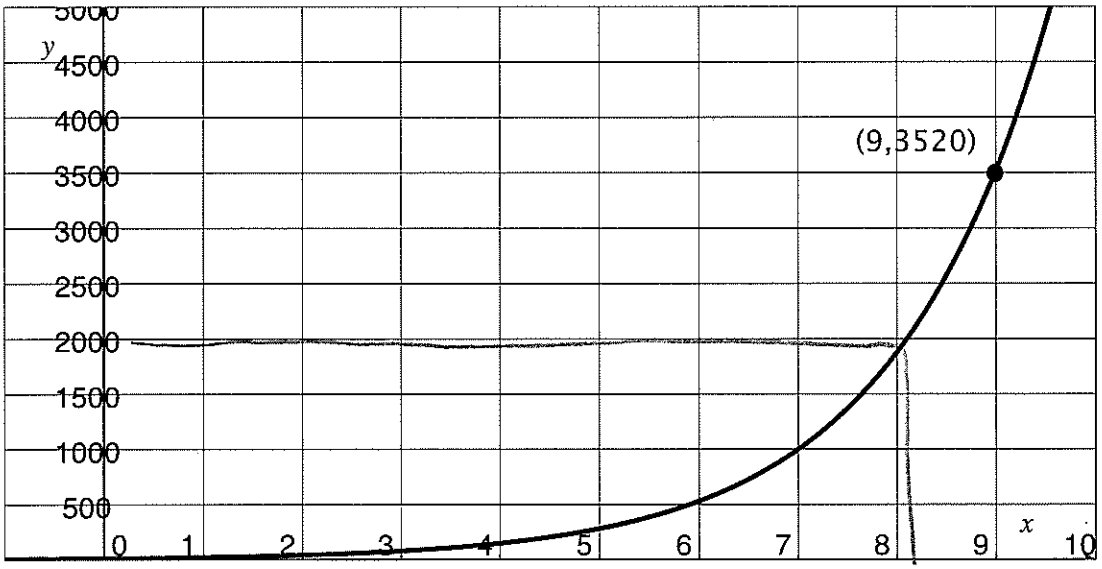
$r\frac{(1-t/s)}{1-t/s} = \frac{t}{1-t/s}$

$r = \frac{t}{1-t/s}$

$= \frac{t}{\frac{s-t}{s}} \quad s$

$= \frac{ts}{s-t}$

3. Cyrano Jones gives one tribble to Uhura in the hopes to boost his sales. It turns out that tribbles are born pregnant and once fed they give birth. Dr. Spock noticed this phenomenon and began to track the population of tribbles when Uhura showed him her new family of twelve tribbles at 0800 (8am). The data is plotted below where the horizontal axis is recording the number of hours since 0800 and the vertical axis is recording the number of tribbles on the Enterprise.



(a) [2] (§2.2 23) Use the graph to *estimate* when there were 2000 tribbles on the Enterprise.

about 8 hours after 8am so 16:00 hours or 4pm

graph reading (+1.5) interpret into time (+.5)

(b) [2] (§2.2 #55) Is the above a graph of a function? Why or why not?

yes (1.5) b/c it passes the vertical line test (+1) started (.5)

(c) [1] Reread the above description. What is the initial population that Dr. Spock recorded?

the new family was at 8am so 12 tribbles (+1)

(d) [4] (§4.1 #39) Given that the population of tribbles f is well approximated by an exponential function in the number of hours since 0800 x (of the form $f(t) = Pa^x$), use the data point on the graph to find the rule to describe the population at time x.

started (+1.5)

we know $(0,12)$ so $P \cdot a^0 = 12 \Rightarrow P = 12$ (+1.5)

we also know $(9,3520)$ is on the graph so (+1.5)

plug in (+.5) $\frac{12a^9}{12} = \frac{3520}{12}$

$a^9 = 293.\overline{3}$

alg (+1)

$a = \sqrt[9]{293.\overline{3}} \approx 1.87997$ report answer (+.5)

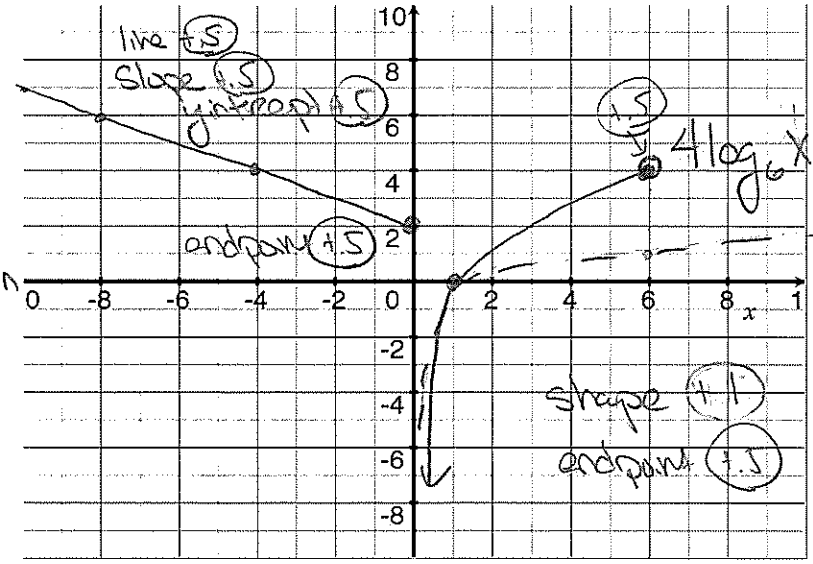
so $12 \cdot \left[(293.\overline{3})^{1/9}\right]^t = f(t)$

293
12 | 3520
-24
112
-108
40
-36

4. Let f be the *function* defined by:

$$f(x) = \begin{cases} \frac{-1}{2}x + 2 & x \le 0 \\ 4\log_6 x & 0 < x < 6 \end{cases}$$

$4\log_6 x$ looks like $\log_6 x$ but vertically stretched by a factor of 4 → point +1



(a) [4] (§4.2 #41) Graph f.

(b) [1] (WebHW7 #15) What is the domain of f?

$(-\infty, 6)$ know domain +.5 got it +.5

(c) [1] (§4.2 #53) What is the range of f?

$\mathbb{R}$

5. [5] (§1.2 #39) Simplify the following:

$\log_3 \sqrt{27}$ prop +.5 got it +.5

$= \log_3 (27)^{\frac{1}{2}}$

$= \log_3 (3^3)^{\frac{1}{2}}$

$= \log_3 3^{\frac{3}{2}}$

$= \frac{3}{2}$

$$\frac{(4y^{-2}\sqrt{x})^{\frac{3}{2}}}{x^{\frac{3}{4}}(\sqrt[3]{y8})^{-1}} = \frac{4^{\frac{3}{2}}(y^{-2})^{\frac{3}{2}}(x^{\frac{1}{2}})^{\frac{3}{2}}}{x^{\frac{3}{4}}\left[(y8)^{\frac{1}{3}}\right]^{-1}}$$

roots $\Rightarrow \frac{1}{n}$ (+1) dist exp (+1) order of op (+1) coef (+.5)

$$= \frac{4^{\frac{3}{2}} y^{-3} x^{\frac{3}{4}}}{x^{\frac{3}{4}}\left(y^{\frac{1}{3}} 8^{\frac{1}{3}}\right)^{-1}}$$

combine x's +.5 combine y's +.5

$$= \frac{8\, x^{\frac{3}{4}}}{x^{\frac{3}{4}} y^3 \left(y^{\frac{1}{3}}\right)^{-1} (2)^{-1}}$$

$$= \frac{8 \cdot 2^1 y^{\frac{1}{3}}}{y^3} = 16 y^{\frac{1}{3}-3}$$

$$= 16 y^{-\frac{8}{3}}$$

$$= 16y$$

6. [5] (Quiz 5 #4 & WebHW9 #2) Find all x that satisfy the following:

Typo? →

$\log_6(4) - \log_6(x) = \log_6(x-4) - \log_6(9)$

$6^{\log_6 \frac{4}{x}} = 6^{\log_6 \frac{x-4}{9}}$

$9 \cdot \frac{4}{x} = \frac{x-4}{9} \cdot 9$

$x \cdot \frac{36}{x} = (x-4) \cdot x$

$36 = x^2 - 4x$

$+4 \quad\quad +4$

$40 = (x-2)^2$

$\pm\sqrt{40} = x-2$

$x = 2 \pm \sqrt{40}$

but $2-\sqrt{40}$ doesn't check

so

$x = 2+\sqrt{40}$

≈ 8.325

used prop (+1)
alg/order of op (+1)
quadratic (+.5)
check answer (+.5)

$(1+e^{-x})\frac{18}{1+e^{-x}} = 2\,(1+e^{-x})$

$18 = 2 + 2e^{-x}$

$-2 \quad -2$

$\frac{16}{2} = \frac{2e^{-x}}{2}$

$8 = e^{-x}$

$\ln 8 = -x$

$-\ln 8 = x$ ✓ it checks

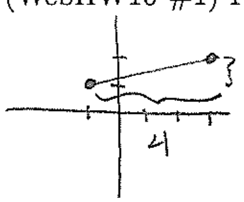
$-2.079 \approx$

alg/order op (+.5)
used ln (+.5)
check (+.5)

7. Let A and B be points $(3, 2)$ and $(-1, 1)$ respectively.

(a) [2] (WebHW10 #1) Find the distance between A and B.

started/drew (+.5)



$1^2 + 4^2 = \text{dist}^2$ (+1)

$\Rightarrow$ distance $= \sqrt{1+16} = \sqrt{17}$ got it (+.5)

(b) [2] (WebHW10 #5) Find the equation of a circle that is centered at A and passes through B.

$(x-h)^2 + (y-k)^2 = r^2$ (+.5)

(+.5) (+.5)

$(x-3)^2 + (y-2)^2 = r^2$

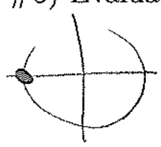
but we found the radius in part (a) so

$(x-3)^2 + (y-2)^2 = (\sqrt{17})^2 \Rightarrow (x-3)^2 + (y-2)^2 = 17$

(+.5)

8. [4] (§5.2 #3) Evaluate the following *exactly*:

$\cos\pi$



-1

(+.5)

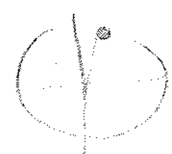
$\sin\frac{-2\pi}{3}$

(+.5) y-coord

$-\frac{\sqrt{3}}{2}$

(+.5) (+.5)

$\tan\frac{\pi}{3} = \frac{\sin \pi/3}{\cos \pi/3}$



(+.5)

$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$ (+.5) (+.5)

simplify +.5

9. [6] (Story Problem Worksheet) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

(a) (Worksheet) Chad just graduated but does not have a job lined up yet. In his last year of graduate school he made ends meet by using his credit card and now has a balance of \$7,000. His credit card compounds monthly with an annual interest rate of 19.9%. Assume the worst and that Chad won't find a job for the next two years and will have to move in with his parents. He won't be able to make any payments on his credit card bill.

Just before graduation, Chad received an ad for a State Farm Good Neighbor Visa Credit Card. The card will transfer his balance (with a 2.90% balance transfer fee) and then give him a lower annual interest rate of 10.24% that is still compounded monthly. Should he take this option instead of staying with his first credit card?

(b) (§4.5 Example 10) In 1906 San Francisco had an estimated magnitude of 8.3 on the Richter scale. In 1989 Loma Prieta earthquake shook San Francisco again at a magnitude of 7.1 on the Richter scale. Recall that the Richter scale defined the magnitude M of an earthquake to be $M = \log\left(\frac{I}{S}\right)$, where I is the intensity of the earthquake and S is the "standard earthquake".

i. How many more times more intense was the 1906 earthquake than the 1989 one?

ii. If an earthquake hit San Francisco with an intensity of the 1906 earthquake *and* that of the 1989 earthquake, what would this 'mega-earthquake"s magnitude be on the Richter scale?

iii. If an earthquake was 8 times more intense than the 1906 San Francisco earthquake, what would its magnitude be on the Richter scale?

1) Stated
2) Use $P(1+\frac{r}{n})^{nt}$
3) Know $n=12$
4) Know $t=2$
5) understood rate/percent

(a) $P = 7{,}000$ original credit card: 19.9% annual rate compounded monthly
time is 2 years
Know P (+.5)
Know r (+.5)
→ after 2 years with the original credit card Chad will owe $7000\left(1+\frac{.199}{12}\right)^{12\cdot 2}$
$\approx \$10{,}387.95$

new credit card will have a new principal b/c it's charging 2.9% of 7000 to move the balance so
Know P (+.1)
Know r (+.5)
$P = 7000 + .029 \cdot 7000 = 7203$
Compounded monthly with an annual rate of 10.24% means after 2 years Chad will owe $7203\left(1+\frac{.1024}{12}\right)^{12\cdot 2}$
$\approx \$8{,}832.42$
he should take the new credit card!

(b) i) Let I_{1906} be the intensity of the 1906 earthquake
& I_{1989} be the intensity of the 1989 earthquake.
We want to know K so that $I_{1906} = K I_{1989}$
or $\frac{I_{1906}}{I_{1989}}$. } knew what wanted (+1.5)

(1.5 started +1 use def of Richter) Because the Richter magnitude:
of the 1906 earthquake was 8.3 ; of the 1989 earthquake was 7.1

$\Rightarrow \log\left(\frac{I_{1906}}{S}\right) = 8.3$ $\Rightarrow \log\left(\frac{I_{1989}}{S}\right) = 7.1$

prop/comp (+1)
$\Rightarrow \frac{I_{1906}}{S} = 10^{8.3}$ $\Rightarrow \frac{I_{1989}}{S} = 10^{7.1}$

$\Rightarrow I_{1906} = S\,10^{8.3}$ $\Rightarrow I_{1989} = S\,10^{7.1}$

Thus $\frac{I_{1906}}{I_{1989}} = \frac{S\,10^{8.3}}{S\,10^{7.1}} = 10^{8.3-7.1} = 10^{1.2} \approx 16$ times

ii) If the earthquake had intensity $I_{1906} + I_{1989}$, the magnitude on the Richter scale would be (knew what wanted +1.5)

prop/comp (+1) $\log\left(\frac{I_{1906}+I_{1989}}{S}\right) = \log\left(\frac{S\cdot 10^{8.3} + S\cdot 10^{7.1}}{S}\right) = \log\left(10^{8.3}+10^{7.1}\right)$
≈ 8.30 (knew what wanted +1.5)

iii) If an earthquake were 8 times I_{1906}, the magnitude on the Richter scale would be

prop/comp (+1) $\log\left(\frac{8I_{1906}}{S}\right) = \log\left(8\cdot\left(\frac{I_{1906}}{S}\right)\right) = \log 8 + \log\left(\frac{I_{1906}}{S}\right)$
$= \log 8 + 8.3 \approx .90 + 8.3 = 9.2$

H 9 20 H 13 30 $\Rightarrow$ 50
6