EXAM 1 TMath 120 Summer 2011

NAME: Key

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be functions, and x, y, and z be non-zero real numbers.

   T F $\frac{2}{x} + \frac{1}{x+1} = \frac{2+1}{x+1} + \frac{1}{x+1} = \frac{4}{x+1}$ $\frac{2}{x} \neq \frac{2+1}{x+1}$

   T F $x^2 - y^2 = (x+y)(x-y)$ $x^2 - xy + xy - y^2$

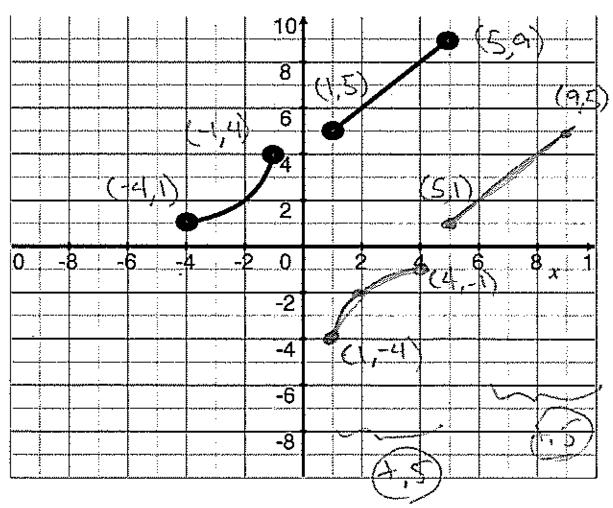
   T F A graph is a graph of a function if it passes the horizontal line test. (if it passes the vertical line test)

   T F The functions $f(x) = 2x - 5$ and $g(x) = \frac{x+5}{2}$ are inverses of each other.
   $(f(g)(x)$ $2\left(\frac{x+5}{2}\right) - 5 = x+5-5 = x$ ✓
   $g(f(x)) = \frac{(2x-5)+5}{2} = \frac{2x}{2} = x$ ✓

   T F The domain of $\frac{\sqrt{2+x}}{3-x}$ is $(-\infty, 3)$ and $(3, \infty)$.
   $2+x \geq 0$ and $3-x \neq 0$
   $x \geq -2$ => $x \neq 3$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. The graph on the right is a graph of the function f.



   (a) [1] (WebHW2#6) What is the domain of f?

   $[-4,-1]$ and $[1,5]$ +.5 +.5

   (b) [1] (WebHW2#6) What is the range of f?

   $[1,4]$ and $[5,9]$ +.5 +.5

   (c) [2] (§2.8 #69) Sketch the graph of f^{-1}

   Switch coords +1

3. [3] (§1.5 #17) Find all x so that:

stated (+.5)
clear den (+1)
alg (+1)

$$3x \cdot \frac{1}{x} = \left(\frac{4}{3x}+1\right)3x$$

$$3 = \frac{4 \cdot 3x}{3x} + 3x$$

$$3 = 4 + 3x$$

$$-4 \quad -4$$

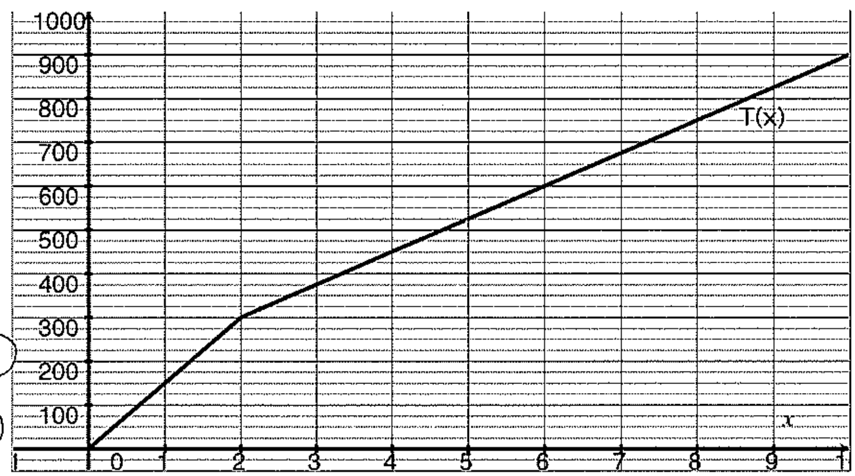
$$-1 = 3x$$

$$-\tfrac{1}{3} = x$$ (+.5)

4. The graph on the right is of T, the total cost of staying in a hotel for x nights.

(a) [1] (WebHW2 #6)
Estimate $T(6)$.

600



(b) [2] (§2.7 #11)
Estimate $(T+T)(2)$.

$(T+T)(2) = T(2) + T(2)$ (+1)
$= 300 + 300$ (+1)
$= 600$

(c) [2] (§2.1 #68) What does the answer in part (a) represent?

stated (+.5)
sense (+.5)
correct (+1)

The total cost of staying at this particular hotel for 6 nights is \$600.

(d) [3] (§2.1 #68) Find a formula for the function T in the indicated form.

notation (+.5)

$$T(x) = \begin{cases} 150x & \text{if } 0 \le x < 2 \\ 75x + 150 & \text{if } 2 < x \end{cases}$$

both are lines: (+1) line on left: thru (0,0) & (2,300) +.5 slope = $\frac{300}{2} = 150$
y-intercept is (0,0) => $y = 150x$ +.5

(+1.5) line on right: thru (2,300) & (6,600)
slope = $\frac{600-300}{6-2} = \frac{300}{4} = 75$ +.5
passes thru (2,300) => $300 = 75(2) + b$ +.5
=> $b = 300 - 150 = 150$ +.5

5. Let $q(x) = x^2 + 4x - 2$.

(a) [3] (WebHW4 #5) Complete the square to write q in vertex form $(\alpha(x-h)^2+k)$.

$q(x) = x^2 + 4x - 2$ $+\left(\frac{b}{2}\right)^2$ or $\left(\frac{4}{2}\right)^2$ or 4 to both sides (+.5) (+.5) (+.5)

$q(x) + 4 = x^2 + 4x + 4 - 2$

$q(x) + 4 = (x+2)^2 - 2$ factor (+.5)

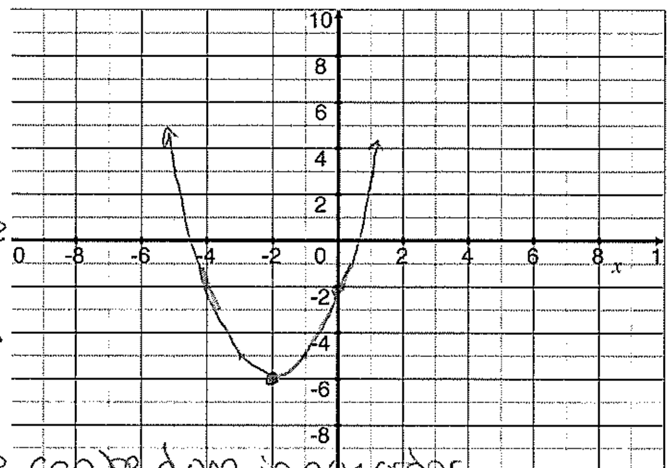
$q(x) = (x+2)^2 - 6$ clean up/form (+.5)   alg (+.5)

(b) [2] (§2.4 #25) List the graph transformations that would perform the graph of $f(x) = x^2$ into the graph of q. Be sure to list the transformations in order.

1) horiz. shift left by 2 units (+.5) (+.5)

1') vertical shift down 6 units (+.5) (+.5)

note: the transformations here can be done in any order.



(c) [2] (WebHW4 #5) Graph q.   shape (+.5) direction (+.5) vertex (+1)

(d) [3] (§2.7 #31) Find (but do *not* simplify) $(q \circ q)(x)$. (+.5)

$(q\circ q)(x) = q(q(x)) = q\left(\boxed{x^2+4x-2}\right)$

composition started (+.5) (+.5)

$= \boxed{x^2+4x-2}^2 + 4\,\boxed{x^2+4x-2} - 2$ (+.5) (+.5) (+.5)

$= (x^2+4x-2)^2 + 4(x^2+4x-2) - 2$

6. [3] (WebHW5 #11 & 14) Perform the indicated addition or division and write the result in the form $a + bi$

$(-4 + 4i) + (5 - i)$

$-4 + 4i + 5 - i$

$1 + 3i$

(+.5) (+.5)

$\frac{100 + 75i}{4 - 3i} \cdot \frac{4+3i}{4+3i}$ (+.5)

foil (+.5) (+.5)

$\frac{400 + 300i + 300i + 225i^2}{16 + \cancel{12i} - \cancel{12i} - 9i^2}$

$= \frac{400 - 225 + 600i}{16+9} = \frac{175 + 600i}{25}$

$= 7 + 24i$ (+.5)

$x^2x^3 = (xx)(xxx) = x^5$

$(x^2)^3 = x^2x^2 \cdot x^2 = x^{2\cdot 3}$

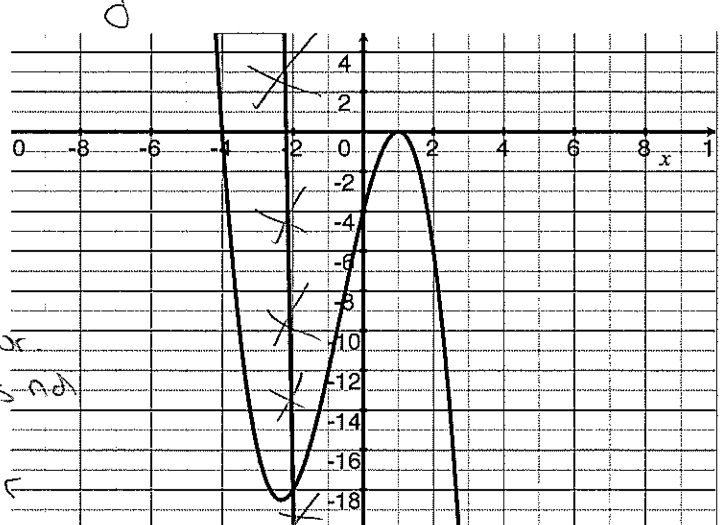
7. [4] (§1.2 #39) Simplify the given expression:

$$\frac{(x^2y^3)^4(xy^4)^{-3}}{x^2y}$$

(+1) dist. $\left\{ \frac{(x^2)^4(y^3)^4\ x^{-3}(y^4)^{-3}}{x^2y} \right.$ (+1) exp to exp $\left\{ = \frac{x^8\ y^{12}\ x^{-3}\ y^{-12}}{x^2y} \right.$ $= \frac{x^5}{x^2y}$ $\Big\}$ (+1) combine mult

$= \frac{x^3}{y}$ (+1) combine div.

8. [3] (WebHW6 #14) Find a degree ~~four~~ three polynomial whose graph is shown to the right.



correct roots factors (+.5)

(+.5) { note: 1 is a root
$\Rightarrow (x-1)$ is a factor

(+.5) { -4 is a root
$\Rightarrow (x - -4)$ is a factor

(+1) { The graph touches but does not cross at 1 so
$\Rightarrow (x-1)^2$ is a factor

Try $(x+4)(x-1)^2$ note the end behavior is backwards $\Rightarrow -(x+4)(x-1)^2$

& the point $(0,-4)$ is on the graph ✓

(+.5) sign

9. [3] (WebHW6 #11) Find the quotient and remainder using long division of

set up (+.5)

$$\frac{6x^3 + 2x^2 + 26x}{2x^2 + 3}$$

(+.5)

$$\begin{array}{r} 3x + 1 \\ 2x^2+3 \overline{\big)\, 6x^3 + 2x^2 + 26x + 0} \\ -\left(6x^3 \qquad + 9x\right) \\ \hline 2x^2 + 17x + 0 \\ -\left(2x^2 \qquad + 3\right) \\ \hline 17x - 3 \end{array}$$

} (+.5) subtract (+.5) algorithm (+.5)

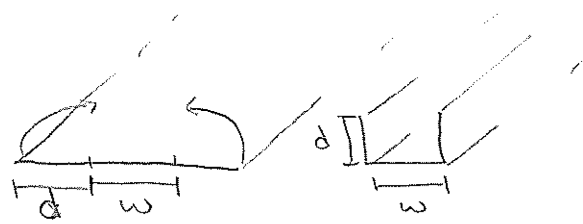
quotient: $3x+1$
remainder: $17x-3$ } got it (+.5)

10. [6] (Story Problem Worksheet) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

   (a) (Lecture) A gutter is to be made by bending up the edge of a 15 inch wide piece of aluminum. What depth should the gutter be to have the maximal possible cross sectional area?

   (b) (§2.6 Example 6) A hockey team plays in an arena with a seating capacity of 15,000 spectators. Which the ticket price set at $14, average attendance at recent games has been 9500. A market survey indicates that for each dollar the ticket price is lowered, the average attendance increases by 1000. Find the price that maximizes revenue from ticket sales.

(a)



picture/variables (+1)

we want to maximize
Area $= w \cdot d$ (+.5)

notice $d + w + d = 15$

$\Rightarrow 2d + w = 15$

$\Rightarrow w = 15 - 2d$ (+.5)

sub (+.5)

so Area $= w \cdot d = (15-2d)\cdot d = 15d - 2d^2$ parabola?, opening down

to maximize we need to find the vertex. (+1)

(+1.5) Finding vertex: $-2d^2 + 15d = A \Rightarrow d^2 - \frac{15}{2}d = -\frac{1}{2}A$

$\Rightarrow d^2 - \frac{15}{2}d + \left(\frac{15}{4}\right)^2 = -\frac{1}{2}A + \left(\frac{15}{4}\right)^2$ +.5 factor +.5 alg +1

$\Rightarrow \left(d - \frac{15}{4}\right)^2 = -\frac{1}{2}A + \left(\frac{15}{4}\right)^2 \Rightarrow \left(d - \frac{15}{4}\right)^2 + \left(\frac{15}{4}\right)^2 = -\frac{1}{2}A$

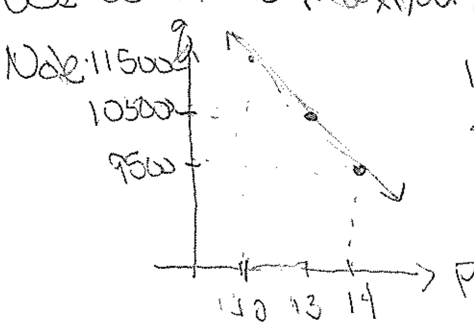
$\Rightarrow A = -2\left(d - \frac{15}{4}\right)^2 + \frac{15^2}{4^2}\cdot 2 = -2\left(d - \frac{15}{4}\right)^2 + \frac{15^2}{8}$

(+1) Thus if we set d to $15/4 \approx 3.75$ in, we will max the area (≈ 28 in^2)

(b) Let p be the price of tickets & q be the quantity sold. variables (+1)

We want to maximize revenue R or $R = p \cdot q$. (+.5)

Note: 11500



line connects p to q

slope: $\frac{-1000}{1} = -1000$

thru $(14, 9500)$ so

$9500 = -1000(14) + b$

$\Rightarrow b = 23500$ (+.5)

$q = -1000p + 23500$

So $R = p \cdot q$

sub (+.5) $= p(-1000p + 23500)$

$= -1000p^2 + 23500$

To maximize we need to find the vertex (+1)

+1.5

Finding vertex: $-1000p^2 + 23500p = R \Rightarrow p^2 - 23.5p = \frac{-1}{1000}R$

$$\Rightarrow p^2 - \frac{27}{2}p + \left(\frac{27}{4}\right)^2 = \frac{-1}{1000}R + \left(\frac{27}{4}\right)^2$$

$$\Rightarrow \left(p - \frac{27}{4}\right)^2 = \frac{-1}{1000}R + \left(\frac{27}{4}\right)^2$$

$$\Rightarrow (p - 6.75)^2 - 6.75^2 = \frac{-1}{1000}R$$

$$\Rightarrow -1000(p-6.75)^2 + 1000 \cdot 6.75^2 = R$$

+1

Thus we maximize revenue when the price is set to \$6.75 (we make about $1000 \cdot 6.75^2$ dollars)

$\cancel{9}$
$\cancel{11}$
$\cancel{13}$
10
$\cancel{6}$

20
19
10
49