# Quiz 3

Key

Show *all* your work algebraically for each. No credit is given without supporting work. There are *two* sides to this quiz.

1. Let $s(x) = \frac{1}{x}$.

   (a) [2] (§2.5 #9) Write down the rule of $(3s - 2s)(x)$. $= 3s(x) - 2s(x)$ (+.5)

   stated (+.5) $3\frac{1}{x} - 2\frac{1}{x} = \frac{3}{x} - \frac{2}{x} = \frac{1}{x}$ (+.5) (+.5)

   (b) [2] (§2.5 #21) Write down the rule of

   $$\frac{s(3+x) - s(3)}{x}$$

   $s(3+x) = \frac{1}{3+x}$ (+1)

   $s(3) = \frac{1}{3}$ (+.5)

   (+.5)

   $$= 3\left(\frac{\frac{1}{3+x} - \frac{1}{3}}{x}\right)\frac{3+x}{3+x}$$

   $$= \frac{3 - (3+x)}{3(3+x)} \div \frac{x}{1}$$

   $$= \frac{-x}{3(3+x)} \cdot \frac{1}{x} = \frac{-1}{3(3+x)}$$

   (c) [1] (§2.5 #21) Simplify the rule in (b) as much as possible.

   oops I did this in b

   (+.5) common den

   (+.5) alg

Something to watch for: the expression in (1b) is known as the "difference quotient at 3" of the function $s$ and dominates first quarter calculus.

2. Let $m(x) = x^3 - \frac{1}{2}x^2 - \frac{13}{2}x - 3$ and $n(x) = x^2 - x - 6$.

(a) [3] (§2.5 #31) Use long division to write $\frac{m}{n}(x)$ as a 'mixed rational function'. Or more accurately, write $\frac{m}{n}(x)$ as an expression in the form $G(x) + \frac{R(x)}{n(x)}$, where $G$ and $R$ are polynomials with deg($R$) <deg($n$).

$$
\begin{array}{r}
x + \frac{1}{2} \\
x^2 - x - 6 \;\overline{)\; x^3 - \frac{1}{2}x^2 - \frac{13}{2}x + 3} \\
-(x^3 - x^2 - 6x) \\
\hline
\frac{1}{2}x^2 - \frac{1}{2}x - 3 \\
-\left(\frac{1}{2}x^2 - \frac{1}{2}x - 3\right) \\
\hline
0
\end{array}
$$

stacked (+.5)
set up (+.5)
algorithm (+1)
signs (+.5)
got it (+.5)

$$\frac{x^3 - \frac{1}{2}x^2 - \frac{13}{2}x - 3}{x^2 - x - 6} = x + \frac{1}{2} + \frac{0}{x^2 - x - 6}$$

(b) [1] Use the above result to factor $m$ completely.

$$x^3 - \frac{1}{2}x^2 - \frac{13}{2}x - 3 = \left(x + \frac{1}{2}\right)(x^2 - x - 6) \quad (+.5)$$
$$= \left(x + \frac{1}{2}\right)(x - 3)(x + 2) \quad (+.5)$$

(c) [1] Find the roots of $m$.

roots happen when $m(x) = 0$ (+.5) so

$$0 = x^3 - \frac{1}{2}x^2 - \frac{13}{2}x - 3 = \left(x + \frac{1}{2}\right)(x - 3)(x + 2) \quad (+.5)$$

$\Rightarrow$ $x + \frac{1}{2} = 0$ or $x - 3 = 0$ or $x + 2 = 0$

$\Rightarrow$ $x = -\frac{1}{2}$ or $x = 3$ or $x = -2$