NAME: This is a sample midterm to be used for practice. This is *not* a template for the midterm that will be given in class. Many of the questions on the Midterm will look quite different than those appearing here.

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

T F $\frac{3x+y}{3z} = \frac{x+y}{z}$ $\qquad \frac{x+y}{z} = \frac{3(x+y)}{3z}$

T F $(x+y)^2 = x^2 + y^2$ $\qquad (x+y)^2 = (x+y)(x+y) = x^2 + xy + yx + y^2$

T F $|x| = x$ $\qquad$ let $x = -2$

T F The function $\sqrt{(x-\sqrt{2})}$ has the domain $[\sqrt{2}, \infty)$ $\qquad x - \sqrt{2} \geq 0$, $x \geq \sqrt{2}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [3] Given $3(7+x)^{-2} - 4 = 2$, solve for x.

$$\frac{3}{(7+x)^2} - 4 = 2$$

$$\frac{3}{(7+x)^2} = 6$$

$$\frac{3}{6} = \frac{6(7+x)^2}{6}$$

$$\frac{1}{2} = (7+x)^2$$

$$\pm\sqrt{\frac{1}{2}} = 7 + x$$

$$-7 \pm \sqrt{\frac{1}{2}} = x$$

3. [4] Let the following describe the function α:

input:	$\bigcirc$	$\star$	Δ	$\star + \Delta$
output:	4	-2	3	-4

Find the following if possible:

$\alpha(\star) + \alpha(\Delta)$

$-2 + 3 = 1$

$\alpha(\star + \Delta)$

-4

$\alpha(\bigcirc) \times \alpha(\star + \Delta)$

$4 \cdot -4 = -16$

$\alpha(\Delta + \Delta)$

not defined

4. Consider $f(x) = \frac{x-1}{x}$ and $g(x) = 3x - 4$.

- [2] What is $f(z + \sqrt{2})$? Do *not* expand this.

$$f(z+\sqrt{2}) = \frac{z+\sqrt{2}-1}{z+\sqrt{2}}$$

- [3] Find the rule for $f \circ g$ and *simplify* as much as possible.

$$(f \circ g)(x) = f(g(x)) = f(3x-4) = \frac{3x-4-1}{3x-4}$$

- [2] The function f is one-to-one, find it's inverse.

$$x = \frac{y-1}{y}$$

$$xy = y - 1$$

$$xy - y = -1$$

$$y(x-1) = -1$$

$$y = \frac{-1}{x-1}$$

5. [4] Find the domain of f where $f(x) = \frac{2-\sqrt{5-2x}}{x+10}$.

den. can't be zero and square roots must be given nonneg #

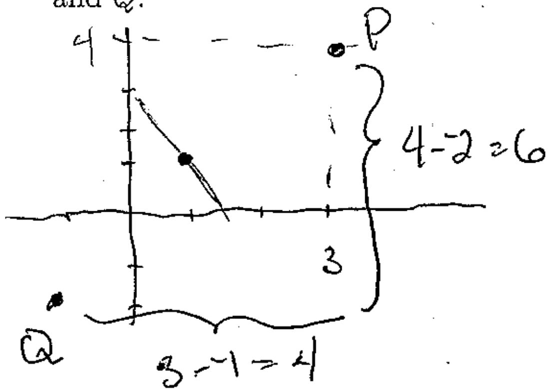
$x+10 \neq 0$ $\qquad$ $5-2x \geq 0$

$x \neq -10$ $\qquad$ $-2x \geq -5$

$x \leq \frac{5}{2}$

$(-\infty,-10) \cup (-10, \frac{5}{2}]$

6. [4] Consider the points $P = (3,4)$ and $Q = (-1,-2)$. Find the equation to a line that goes through the point $(1,1)$ and has a perpendicular slope to the line connecting P and Q.



slope of $\overline{PQ}$ is $\frac{6}{4} = \frac{3}{2}$

$\Rightarrow$ slope of $\perp$ to $\overline{PQ}$ is $\frac{-2}{3}$

must pass thru $(1,1)$ so

$1 = \frac{-2}{3}(1)+b$

$1+\frac{2}{3} = b$

$\frac{5}{3} = b$ $\Rightarrow$ $y = \frac{-2}{3}x + \frac{5}{3}$

- [1] What is the y intercept of the line you found?

when $x=0$

$\Rightarrow \frac{5}{3}$

- [1] Find the zeros of the line you found above.

when $y=0$

$0 = \frac{-2}{3}x + \frac{5}{3}$

$-\frac{3}{2} \cdot \quad -\frac{5}{3} = -\frac{2}{3}x \quad -\frac{3}{2}$

$\frac{5}{2} = x$

7. [4] Given that $f(x) = x^2 - 5x - 6$. Write f in vertex form.

want $a(x-h)^2+k$ form.

$x^2-5x-6=y$

$+(-\frac{5}{2})^2 \quad +(-\frac{5}{2})^2$

$[x^2-5x+(-\frac{5}{2})^2]-6=y+(\frac{-5}{2})^2$

$[x-\frac{5}{2}]^2-6=y+\frac{25}{4}$

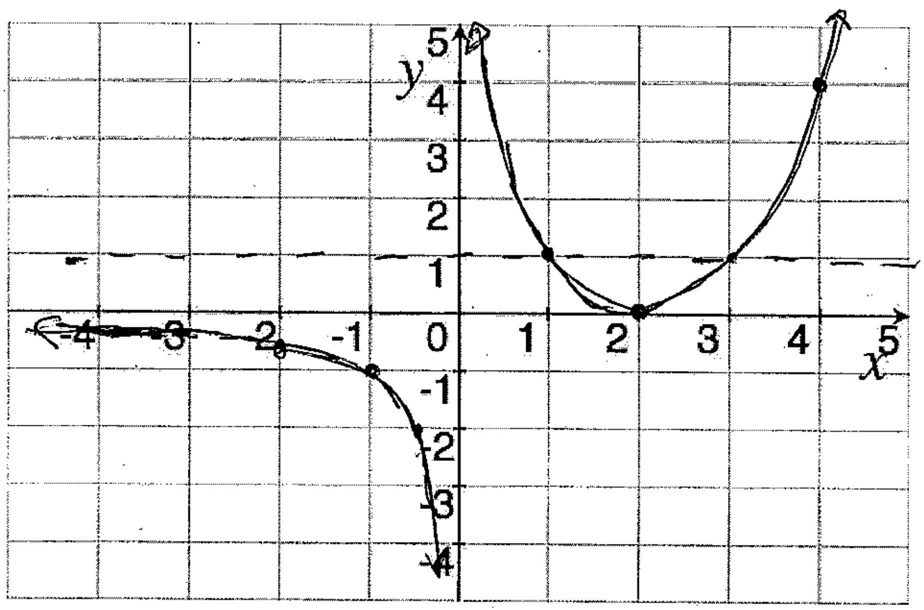
$(x-\frac{5}{2})^2-6-\frac{25}{4}=y$

$(x-\frac{5}{2})-\frac{49}{4}=y$

8. Let f be the *function* defined by

$$f(x) = \begin{cases} \frac{1}{x} & x \leq 1 \\ (x-2)^2 & 1 < x. \end{cases}$$

parabola shifted right 2 units



(a) [3] (§1.3) Graph f. (Explaining graph transformations is worth partial credit.)

(b) [2] (§1.2 #43) Find all possible input(s) so that $f(x) = 1$.

by the graph $x=1$ and 3

or with the rule:

$1 = 1/x$ or $(x-2)^2 = 1$

$\Rightarrow x = 1$

$x-2 = \pm 1$

$\Rightarrow x = +1$ or 3

9. [4] Simplify the following as much as possible:

$$\frac{(2x^4y^3)^3(6xy^3)^{-3}}{x^4y^4}$$

$(x^2)^3 = x^2 x^2 x^2$
$= (xx)(x \cdot x)(xx)$
$= x^6$

$x^2 x^3 = (x\,x)(xx\,x)$
$= x^5$

$\frac{x^3}{x^2} = \frac{x\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}} = x$

$$\frac{2^3(x^4)^3(y^3)^3\ 6^{-3}x^{-3}(y^3)^{-3}}{x^4y^4}$$

$$= \frac{2^3x^{12}y^9 6^{-3}x^{-3}y^{-9}}{x^4y^4}$$

$$\frac{2^3 x^9 6^{-3}}{x^4y^4}$$

$$= \frac{2^3x^{9-4}}{6^3y^4} = \frac{2\cdot2\cdot2\,x^5}{6\cdot6\cdot6\,y^4} = \frac{x^5}{3\cdot3\cdot3y^4} = \frac{x^5}{27y^4}$$

10. [3] Find a cubic polynomial whose graph passes through the points $(-2, 0)$ and $(1, 0)$ and has a root at 6. Note: there are many correct answers possible here.

roots: $-2, 1 + 6$ so

$(x - {-2})(x-1)(x-6)$ works

so does $14(x+2)(x-1)(x-6)$ or $-3(x+2)(x-1)(x-6)$ etc

11. (Lecture 4/15) Let $m(x) = x^3 + x^2 - \frac{39}{4}x + 9$ and $n(x) = x + 4$. Use long division to find $G(x)$ and $R(x)$ so that $\frac{m(x)}{n(x)} = G(x) + \frac{R(x)}{n(x)}$

$$\begin{array}{r l}
 & x^2 + 5x - \frac{119}{4} \\
x+4 & \overline{)\,x^3 + x^2 - \frac{39}{4}x + 9} \\
 & -(x^3 - 4x^2) \\
 & \overline{\quad 5x^2 - 39/4\,x + 9} \\
 & -(5x^2 + 20x) \\
 & \overline{\quad -\frac{119}{4}x + 9} \\
 & -\left(-\frac{119}{4}x - 119\right) \\
 & \overline{\quad 128}
\end{array}$$

$-\frac{39}{4} - \frac{80}{4} = -\frac{119}{4}$

so

$$\frac{x^3+x^2-\frac{39}{4}x+9}{x+4} = x^2+5x-119/4 + \frac{128}{x+4}$$

$G(x) = x^2 + 5x - 119/4$

and

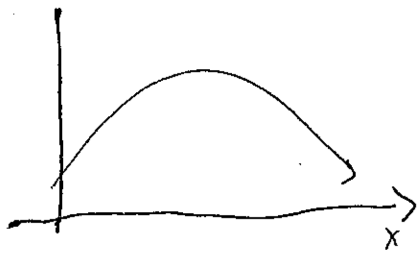
$R(x) = 128$

12. The height y (in feet) of a ball thrown by a child on the planet Gethen is

$$y = -x^2 + 15x + 3$$

where x is the horizontal distance in feet from the point at which the ball is thrown. Answer the following questions.

(a) [2] How high is the ball when it leaves the child's hand?



ie when $x=0$ how high is the ball?

$-0^2 + 15 \cdot 0 + 3 = 3\text{ft}$

(b) [2] How far from the child does the ball hit the ground?

$$\begin{array}{r} 15 \\ \times 15 \\ \hline 75 \\ 150 \\ \hline 225 \end{array}$$

ie when does the height equal 0?

$0 = -x^2 + 15x + 3$

$-1 \cdot \quad 0 = -x^2 + 15x + 3 \quad \cdot -1$

$0 = x^2 - 15x - 3$

$\left(-\frac{15}{2}\right)^2 \qquad +\left(-\frac{15}{2}\right)^2$

$\frac{15^2}{4} = \left[x^2 - 15x + \left(-\frac{15}{2}\right)^2\right] - 3$

$\frac{15^2}{4} = \left(x - \frac{15}{2}\right)^2 - 3$

$\frac{15^2}{4} + 3 = \left(x - \frac{15}{2}\right)^2$

$\pm\sqrt{\frac{15^2+12}{4}} = x - \frac{15}{2}$

$x = \frac{15}{2} \pm \sqrt{\frac{15^2+12}{4}}$

$= -.19$ or 15.2ft

or

$x = \frac{-15 \pm \sqrt{15^2 - 4(-1)(3)}}{2(-1)}$

$= \frac{-15 \pm \sqrt{15^2 + 12}}{-2}$

$= \frac{-15 \pm \sqrt{225 + 12}}{-2}$

$= \frac{-15 \pm \sqrt{237}}{-2}$

$= -.19$ or 15.2ft