

NAME:

Key

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be functions, and x and y be positive real numbers.

T F $-3^2 = 9$ $-3^2 = -9 \neq 9$

T F For all real numbers z , $\sqrt{z^2} = z$ let $z = -2$ then $\sqrt{(-2)^2} \neq -2$

T F $\log(x+y) = \log(x) + \log(y)$ $\log(x) + \log(y) = \log(xy)$

F The radius of a circle described by $(x-2)^2 + (y+5.2)^2 = 5$ is $\sqrt{5}$

T F The area of a circle with radius r is $2\pi r$. area = πr^2

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] (Average algebra question faced by calculus students) Find all x so that:

$$\frac{x}{\sqrt{x^2-2}} + 4 = 0$$

$$\sqrt{\frac{x}{x^2-2}} = -4$$

$$x = -4\sqrt{x^2-2}$$

$$x^2 = (-4\sqrt{x^2-2})^2$$

$$x^2 = 16(x^2-2)$$

$$x^2 = 16x^2 - 32$$

$$32 = 15x^2$$

$$\frac{32}{15} = x^2$$

$$x = \pm \sqrt{\frac{32}{15}}$$

both $\left(\frac{32}{15}\right)$

Started $\left(\frac{32}{15}\right)$
 square to undo $\sqrt{\quad}$ $\left(\frac{32}{15}\right)$
 clear den $\left(\frac{32}{15}\right)$ $\left(\frac{32}{15}\right)$
 multiply $\times 15$ to one side $\left(\frac{32}{15}\right)$
 Result $\left(\frac{32}{15}\right)$

3. Let $f(x) = 4x^{3/7} - 1$.

(a) [3] (§3.1 #31) The function f has an inverse, find it.

started (+.5) $y = 4x^{3/7} - 1$ \rightarrow $\frac{x+1}{4} = y^{3/7}$ \rightarrow $\left(\frac{x+1}{4}\right)^{7/3} = y$

(+1) $x = 4y^{3/7} - 1$ $\frac{x+1}{4} = y^{3/7}$ \rightarrow alg (+.5)

(b) [3] (§3.1 #33) Let $g(x) = x^{1/2}$. Find a formula for $(f \circ g)(x)$ and simplify as much as possible. started (+.5)

$(f \circ g)(x) = f(g(x)) = f(x^{1/2}) = 4(x^{1/2})^{3/7} - 1$ comp (+1)
+ not mat.

$= 4x^{3/14} - 1$ simplify (+1)

4. [3] (Mini Quiz & WebHW6 #1) Simplify as much as possible:

started (+.5) $x^{2/3} = x^{1/3} x^{1/3}$ (+.5)

factored (+1) $\frac{3\sqrt[3]{x} + 9yx^{2/3}}{3x^{1/3}} = \frac{3x^{1/3} + 9yx^{2/3}}{3x^{1/3}}$

Alg (+1) $= \frac{3x^{1/3}(1 + 3yx^{1/3})}{3x^{1/3}}$

$= 1 + 3yx^{1/3}$

5. [6] Find x that satisfy the following:

(WebHW6 #13) $\log_x 4 = 2$

started (+.5) $\log_x 4 = 2$

Alg (+1) $\frac{\log 4}{\log x} = 2$

get it (+.5) $\log 4 = 2 \log x$

$\frac{1}{2} \log 4 = \log x$

$\log 2 = \log x$

$2 = x$

(§3.2 #33) $\frac{10^x + 1}{10^x + 2} = 0.8$

change of base (+1) $10^x + 1 = .8(10^x + 2)$

Alg (+1) $10^x + 1 = .8 \cdot 10^x + 1.6$

get it (+.5) $.8 \cdot 10^x - 1 = .8 \cdot 10^x - 1$

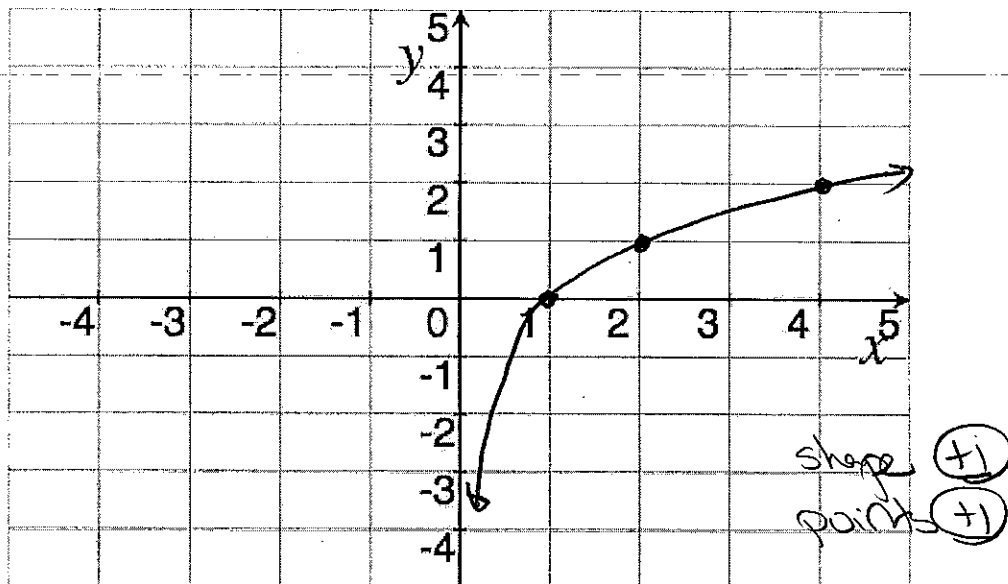
started (+.5) $.2 \cdot 10^x = .6$

used logs (+1) $10^x = \frac{.6}{.2} = 3$

Alg (+1) $x = \log 3 \approx .477$

6. (Quiz 4) Let $h(x) = \log_2 x$.

(a) [2] (pg 239) Carefully draw the graph of h on the set of axes below.



(b) [1] (§3.2 #63) Find a decimal approximation for $h(13)$. Show work.

$$h(13) = \log_2(13) = \frac{\log 13}{\log 2} \approx 3.70044$$

(+1.5) (+1.5)

(c) [1] (§3.2 #47) Find all x so that $h(x) = 13$.

$$2 \log_2 x = 26$$

$$x = 2^{13} = 8192$$

(+1.5) (+1.5)

7. [7] Find x that satisfy the following:

(§3.3 Lecture) $2 \ln(-x) = \ln 36$

$$\ln(-x)^2 = \ln 36$$

$$x^2 = 36$$

$$\Rightarrow x = \cancel{6} \text{ or } -6$$

↳ b/c domain restrictions

check answers (+1.5)

started (+1.5)
used (+1)
alg (+1)
got both (+1.5)

(§3.3 #33) $\log(x+1) + \log(x-1) = \log 1$

$$\log(x+1)(x-1) = \log 1$$

$$(x+1)(x-1) = 1$$

$$x^2 - 1 = 1$$

$$x^2 = 2$$

$$x = \pm\sqrt{2} \text{ or } \cancel{-\sqrt{2}}$$

b/c domain rest

check answers (+1.5)

8. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

(a) (Word Problems Worksheet) Jim's grandparents started a "life-insurance with cash value" policy when he was born in 1990 with an initial investment of \$10,000. Now that Jim is leaving the house his grandparents have transferred the funds into his name. In 2010, the investment has a cash value of \$24,121.51 (so if Jim decides to cash out he will be given the cash value of the investment). What effective annual interest rate would his grandparents had to acquire (through Cash Deposits (CDs) or a bank's savings account) to have the same return?

(b) (Word Problems Worksheet) You have four ten-year loans you took out to pay for college. You got the first loan 4 years ago, the second loan 3 years ago, the third loan 2 years ago, and the last loan is only one year old. All the loans are federally subsidized loans so no interest has accrued. Below is a table for the loans taken and their respective effective annual percentage rates (APR):

loan (\$)	8,000	9,000	10,000	12,000
APR (%)	3.51	4.22	5.01	6.31

After graduation you are given the option of consolidating (that is take out one loan to pay off *all* the balances on your current loans). What rate would you need to be offered to make this worth doing?

started +.5

a) find r given (+1)

formula (+1)
used right (+1)

$$10,000(1+r)^{20} = 24,121.51$$

algebra (+2)

$$(1+r)^{20} = 2.412151$$

$$1+r = \sqrt[20]{2.412151}$$

$$r = \sqrt[20]{2.412151} - 1$$

$$\approx .045009$$

got it (+.5)

so about 4.5%

into 1 ten year loan

started +.5
formula +.5

b) Current deal:

$$\begin{aligned} & 8,000(1+.0351)^{10} + 9,000(1+.0422)^{10} \\ & + 10,000(1+.0501)^{10} + 12,000(1+.0631)^1 \\ & \approx \$63,333.87 \end{aligned}$$

(+1) we need to find r so that

$$29,000(1+r)^{10} < 63,333.87$$

alg (+2)

$$(1+r)^{10} < 2.183927$$

$$1+r < 1.081244$$

$$r < .081244$$

so a rate less than 8.12%
got it (+.5)

9. [6] Choose ONE of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

(a) (WebHW9 #8) The 1906 San Francisco earthquake had a magnitude of 8.3 on the Richter scale. At the same time in South America there was an earthquake with magnitude 3.9 that caused only minor damage. How many times more intense was the San Francisco earthquake than the South American one?

(b) (WebHW8 #5) You go to the doctor and he gives you 10 milligrams of radioactive dye. After 12 minutes, 7 milligrams of dye remain in your system. To leave the doctor's office, you must pass through a radiation detector without sounding the alarm. If the detector will sound the alarm if more than 2 milligrams of the dye are in your system, how long will your visit to the doctor take, assuming you were given the dye as soon as you arrived? Give your answer to the nearest minute.

a) Let S_F be the intensity of the San Fran. earthquake and S_A be " " " South American " " .

started (+.5)
notation (+.5)

use 8.3-3.9
w/0 killing me
dominates 3.9

Then we know $\log \frac{S_F}{S_0} = 8.3$ and $\log \frac{S_A}{S_0} = 3.9$ (+1)

We want to find $\frac{S_F}{S_A}$ (+1)

(+1) Use $\log \frac{S_F}{S_0} = 8.3 \Rightarrow \frac{S_F}{S_0} = 10^{8.3} \Rightarrow S_F = S_0 \cdot 10^{8.3}$
 (+1) & similarly $\log \frac{S_A}{S_0} = 3.9 \Rightarrow S_A = S_0 \cdot 10^{3.9}$
 (+1) Thus $\frac{S_F}{S_A} = \frac{S_0 \cdot 10^{8.3}}{S_0 \cdot 10^{3.9}} = 10^{4.4} \approx 25,118$

b) We need to find a formula that tells us the amount of dye in our blood at time t . Use $P_0 e^{rt}$. formula (+.5)

Use $P_0 = 10$ and $P_0 e^{r \cdot 12} = 7$ Know (+1)
 So $10 e^{12r} = 7 \Rightarrow e^{12r} = .7 \Rightarrow 12r = \ln .7 \Rightarrow r = \frac{\ln .7}{12}$

Thus $P(t) = 10 \cdot e^{\frac{\ln .7}{12} t}$ (+1)

We need to find t so that $P(t) = 2$ (+1)

(+1) $2 = 10 \cdot e^{\frac{\ln .7}{12} t} \Rightarrow .2 = e^{\frac{\ln .7}{12} t} \Rightarrow .2 = .7^{t/12} \Rightarrow \frac{\ln .2}{\ln .7} = \frac{t}{12}$
 so $t \approx 54.15$ so 55 min