EXAM 1 TQS 120 Summer 2010

NAME: Key

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be functions, and x, y, and z be non-zero real numbers.

$-3^2 = -(3^2) = -9$

T F $-3^2 = 9$ $(-3)^2 = (-3)(-3) = 9$

T F $(x+2)^2 = x^2 + 4$ $(x+2)^2 = (x+2)(x+2) = x^2+4x+4$

T F $\frac{3}{\frac{1}{x}} = 3x$ $3 \div \frac{1}{x} = 3 \cdot \frac{x}{1} = 3x$

T F All functions pass the vertical line test.

T F $f(x+y) = f(x) + f(y)$

T F $(f+g)(x) = f(x) + g(x)$

T F $\sqrt{2}x^2 + \pi x - 7$ is a polynomial.

T F The volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [3] (Exponent Wks #4) Find all x so that:

$$3x^{-2} - 7 = 0$$

started +.5

neg exp +1

both answers +.5

alg +1

$\frac{3}{x^2} - 7 = 0$

$\frac{3}{x^2} = 7$

$3 = 7x^2$

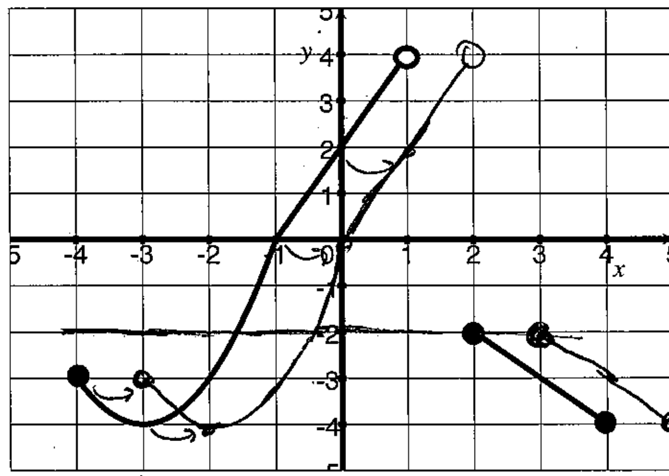
$3/7 = x^2$

$\pm\sqrt{3/7} = x$

3. The graph of a piece-wise defined function f is provided on the right.

(a) [1] (§1.2 #23) Estimate $f(-2)$.

-3



(b) [2] (§1.2 #19) What is the domain of f?

$[-4,1) \cup [2,4]$

#'s (+1)

end pts (+1)

(c) [3] (§1.2 #43) Estimate all values x so that $f(x) = -2$.

≈ -1.5 and 2

(+1) (+1)

Started (+.5)

got them (+.5)

(d) [2] (§1.3 #23) Let $n(x) = f(x-1)$. Carefully draw the graph of n on the axes above.

(+.5) Started. (+1) Shift right one unit. ex $n(0) = f(0-1) = f(-1) = 0$

4. [3] (Practice Exam $4) Let $h(x) = \frac{x-2}{x}$ The function h is one-to-one, find h^{-1}.

Started (+.5)

$x = \frac{y-2}{y}$ (+1)

$xy = y-2$ alg (+1.5)

$xy - y = -2$

$y(x-1) = -2$

$y = \frac{-2}{x-1}$

5. The graph of the function g is a straight line with slope $\frac{1}{3}$ that passes through (3,-1).

   (a) [2] (Line Wks #6) Find the rule of g.

+.5 $y = mx+b$

$y = \frac{1}{3}x + b$ +.5

thru (3,-1) so

$-1 = \frac{1}{3}(3) + b$ +.5

$-1 = 1 + b$

$-2 = b$ +.5 => $y = \frac{1}{3}x - 2$

   (b) [2] Find the rule of a line that is perpendicular to g. (There are many right answers.)

$\perp$ slope to $\frac{1}{3}$ is -3 +.5 +.5

opp

so +.5 $y = -3x$ works

got one +.5

   (c) [2] (§2.1 # 31) Find the intersection in the xy plane of the line g and the line described by $y = 2x$.

started +.5

$y = \frac{1}{3}x - 2$ intersects $y = 2x$ when they share the same x + y coord. so when

$\frac{1}{3}x - 2 = 2x$ +1

$-2 = \frac{5}{3}x$ => $-2 \cdot \frac{3}{5} = -\frac{6}{5} = x$ +.5

intersects at $\left(-\frac{6}{5}, -\frac{12}{5}\right)$

6. [3] (WebHW5 #16) Find a degree four polynomial with -5, 3, and 2 as it's *only* roots. (Note: there are many right answers here.)

$\underbrace{(x+5)(x-3)(x-2)^2}_{+1}$

started +.5

connection between factors + roots +.5

signs +.5

degree +.5

$x^2x^3 = (xx)(xxx) = x^5$

$(x^2)^3 = (x^2)(x^2)(x^2) = x^6$

7. [4] (§2.3 #31) Simplify the given expression

started (+.5)
dist exp (+1)
exp to exp (+1)
neg exp (+.5)
got it (+.5)
combine x's + y's (+1)

$$\frac{x(x^2y^{-5})^{-4}}{(x^5y^{-2})^{-3}y^2} = \frac{x\,(x^5y^{-2})^3}{(x^2x^{-5})^4\,y^2} = \frac{x(x^5)^3(y^{-2})^3}{(x^2)^4(x^{-5})^4y^2} = \frac{x\,x^{15}y^{-6}}{x^8y^{-20}y^2} = \frac{x^{16}y^{20}}{y^6x^8y^2} = x^8y^{12}$$

8. Let $m(x) = x^3 + x^2 - 14x + 6$ and $n(x) = x^2 + 4x - 2$.

(a) [3] Complete the square to find the vertex of n. started (+.5) vertex form (+.5)

$x^2+4x-2 = n(x)$

$+(4/2)^2 \quad +(4/2)^2$ (+.5)

add to both sides (+.5)

$x^2+4x+4-2 = n(x)+4$

$(x+2)^2-2 = n(x)+4$

$(x+2)^2-6 = n(x)$

vertex at

$(-2, -6)$ (+.5) (+.5)

(b) [2] (Practice Exam #4) Find the rule for $n(z+\sqrt{2})$. Do *not* simplify.

$n(z+\sqrt{2}) = (z+\sqrt{2})^2 + 4(z+\sqrt{2}) - 2$ (+1) (+1)

(c) [2] (Practice Exam #4) Find the rule for $(m \circ n)(x)$. Do *not* simplify. composition (+.5)

$(m\circ n)(x) = m(x^2+4x-2) = (x^2+4x-2)^3 + (x^2+4x-2)^2 - 14(x^2+4x-2) + 6$ (+.5) (+.5) (+.5)

(d) [2] (Lecture 7/6) Write down the rule of $(m+n)(x)$. *Simplify.*

$(m+n)(x) = m(x)+n(x) = (x^3+x^2-14x+6)+(x^2+4x-2) = x^3+2x^2-10x+4$ (+1) (+1)

(e) [4] (Lecture 7/6) Is n a factor of m? That is, does $x^2 + 4x - 2$ divide into $x^3 + x^2 - 14x + 6$ with no remainder? Justify your answer.

$$\begin{array}{r} x-3 \quad \text{R}0 \\ x^2+4x-2 \,\big)\, x^3+x^2-14x+6 \\ -(x^3+4x^2-2x) \\ \hline -3x^2-12x+6 \\ (-3x^2-12x+6) \\ \hline 0 \end{array}$$

started (+.5)
algorithm (+1)
signs (+1)
got it (+.5)
answered consistently (+1)

Yes?

$$\frac{x^3+x^2-14x+6}{x^2+4x-2} = x-3$$

$\Rightarrow$

$x^3+x^2-14x+6 = (x-3)(x^2+4x-2)$

4

$$\begin{array}{r} 360 \\ 360 \\ \hline 180 \\ \hline 900 \end{array}$$

9. [6] (Story Problem Worksheet) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

   (a) (Word Problems Worksheet) An airplane flew with the wind for 2.5 hours and returned on the same route against the wind in 3.5 hours. If the cruising speed of the plane was a constant 360 mph in air, how fast was the wind blowing?

   (b) (Word Problems Worksheet) A radiator contains 8 quarts of fluid, 40% of which is antifreeze. How much fluid should be drained and replaced with pure antifreeze so that the new mixture is 60% antifreeze?

$$\begin{array}{r} 300 \\ 3.5 \\ \hline 150 \\ 900 \\ \hline 1050 \end{array} \qquad \begin{array}{r} 420 \\ 420 \\ 210 \\ \hline 1050 \end{array}$$

a) dist = rate $\cdot$ time.

Let w be the velocity of the wind

$\text{dist}_{\text{going}} = \text{rate}_{\text{going}} \cdot 2.5$ where $\text{dist}_{\text{going}}$ is the dist. traveled with the wind at $\text{rate}_{\text{going}}$ mph.

started (+.5)
variables (+.5)
d=r·t or antifreeze (+.5)
2nd equation (+1)
alg/solo (+1)
got it (+1)

$\text{dist}_{\text{coming}} = \text{rate}_{\text{coming}} \cdot 3.5$ where $\text{dist}_{\text{coming}}$ is dist traveled against the wind at $\text{rate}_{\text{coming}}$ mph

Note $\text{dist}_{\text{going}} = \text{dist}_{\text{coming}}$

$\Rightarrow \text{rate}_{\text{going}}\, 2.5 = \text{rate}_{\text{coming}} \cdot 3.5 \quad (*)$

Also note

$\text{rate}_{\text{going}} = \text{rate of airplane} + w \doteq 360 + w$

$\text{rate}_{\text{coming}} = \text{rate of airplane} - w \doteq 360 - w$

Let v be the rate of the airplane (which is 360)

Thus we have from $(*)$

$(360+w)\,2.5 = (360-w)\,3.5$

$900 + 2.5w = 1260 - 3.5w$

$6w = 360 \Rightarrow w = \frac{360}{6} = 60 \text{ mph}$

Check: $(360+60)(2.5) = 1050 = (360-60)(3.5)$ ✓

b) Let x be the amount of fluid drained

$\Rightarrow$ we keep $8-x$ qts.

We want the mixture to contain $.60 \cdot 8$ qts of antifreeze

If we keep $8-x$ qts we'll have $.40(8-x)$ qts of antifreeze

then we'll add x qts of straight $1 \cdot x$ qts of antifreeze

Thus, we want

$.6 \cdot 8 = .4(8-x) + x$ solve for x

$4.8 = 3.2 - .4x + x$

$-3.2 \quad -3.2$

$1.6 = .6x$

$16/6 = x$ So dump out $16/6$ qts.

Check: Keep $.4(8 - 16/6)$ qts of antifreeze

add $16/6$ qts of antifreeze