Exam 2 **TMath 120** Spring 2020

Key

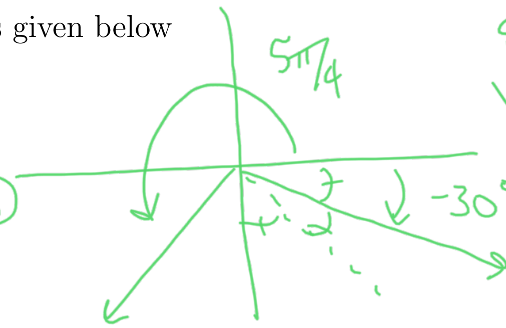
NAME:

Show all your work. You are welcome to use a calculator but no notes, books, internet resources (Desmos is the exception!) or peers can be used. Reasonable supporting work must be shown to earn credit.

Activity 3 #7

1. Sketch the angles with the measures given below

Start (+.5)
begin @ (+.5) pos x-axis



(a) [2] -30°

direction (+.5)
space of angle (+.5)
terminal side (+.5)

WebWork 10 #6

(b) [2] $\frac{5\pi}{4}$ radians

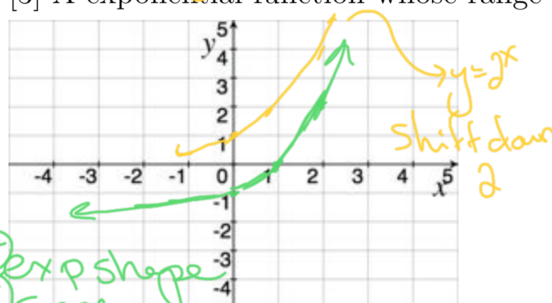
direction (+.5)
space of angle (+.5)
terminal side (+.5)

2. Provide a graph AND an algebraic rule/expression that which is described:

(a) [3] A exponential function whose range (outputs) is $(-2, \infty)$.

$y = b^x$ shift down

§ 3.2 #56 and WebWork 7 #8

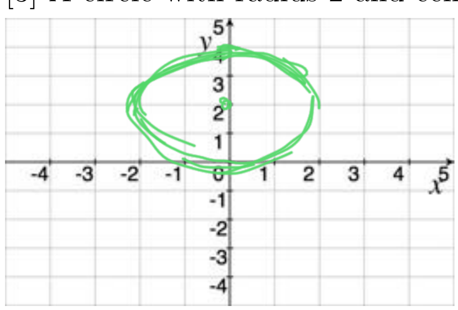


$y = 2^x - 2$ works

exponential (+.5) match (+.5)
shifts (+.5) start (+)

(+.5) exp shape
(+.5) range

(b) [3] A circle with radius 2 and centered at $(0, 2)$



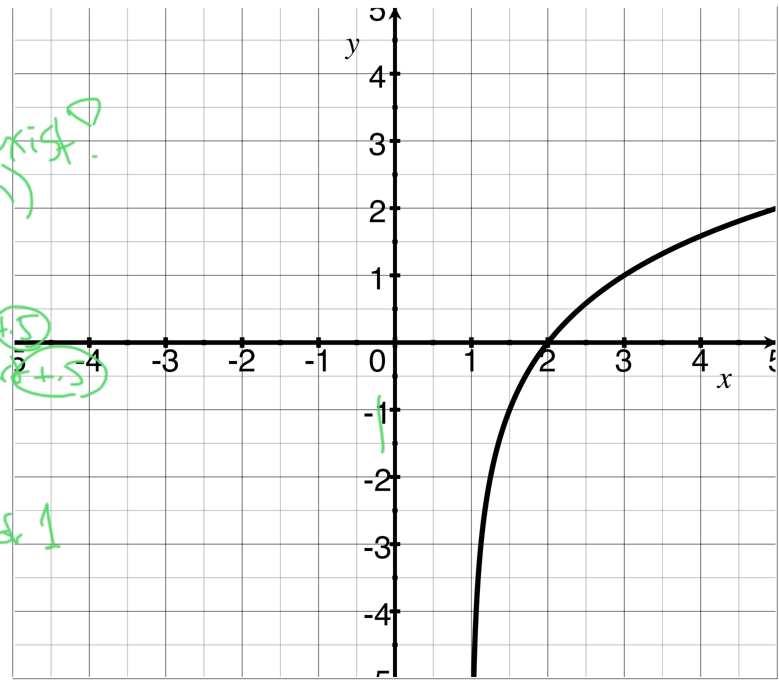
note: looks like an oval b/c the scales on y axis are dif. than those on x-axis

$$(y-2)^2 + (x-0)^2 = 2^2$$

eq of circle (+.5)
center (+.5) match (+.5)
radius (+.5)

center/radius (+.5)
shape (+.5)

3. Let f be a logarithm function (whose base is unknown!!!) that has been horizontally shifted & graphed below.



Webwork7 #15

Practice Exam2 #9

(a) Estimate the following *if* possible:

i. [1] $f(0)$ Does not exist? (trick question) +1

ii. [2] the domain of f

$(1, \infty)$ inf +.5 start +.5

x-values +.5
not inc 1 +.5

iii. [1] all a so that $f(a) = 1$.

inputs w/ output of 1

$a = 3$ +1

iv. [1] the x-intercept(s)

$x = 2$ +1

(b) [2] Does f have an inverse? Why or why not?

yes? +.5

Webwork7 #21

(c) [2] We know from above that the graph of f is shifted horizontally. Describe precisely how much and in what direction $f(x)$ can be obtained from the graph of a basic logarithmic function (whose base is unknown!!!).

start +.5 $\log_b(x) = y$ passes thru $(1,0)$ b/c $\log_b(1) = 0$ } So the log graph has been shifted right by 1 unit +.5 +.5

+.5 horiz shift

Quiz3 #3

(d) [3] Find the algebraic rule for the function f.

$y = \log_b(x-h)$ logarithmic equation +.5 horiz shift +.5

graphs thru $(2,0)$ +.5

so $0 = \log_b(2-h)$

$\Rightarrow b^0 = 2-h$

$\Rightarrow 1 = 2-h$

$\Rightarrow h+1 = 2$

$h = 1$ +.5

OR horiz shift right by 1 unit

$\Rightarrow y = \log_b(x-1)$

+.5 graph goes thru $(3,1)$

so $1 = \log_b(3-1)$ +.5

$\Rightarrow b^1 = 3-1$ explained +.5

$\Rightarrow b = 2$ +.5

so $y = \log_2(x-1)$

4. The temperature T (in C°) of coffee at time t minutes after its removal from the microwave is given by the equation $T = 25 + 73e^{-0.28t}$.

start (+.5)

Activity 11 #5 Week 5 #1

(a) [2] Find the temperature when after a half hour has passed.

half an hour = 30 min = t (+.5) So $25+73e^{-.28(30)}$ plug in (+.5) $\approx 25°C$ (+.5)

(b) [3] When will the temperature reach 30°C?

find t when (+.5) $30 = 25 + 73e^{-0.28t}$

$-25 \quad -25$

$\frac{5}{73} = \frac{73e^{-.28t}}{73}$

$5/73 = e^{-.28t}$

$\ln(5/73) = -.28t$

$t = \frac{-1}{.28}\ln(5/73)$

≈ 9.6 min (+.5)

(+1) order of op (+.5) use log's (+.5) use log rules right

§3.3 #16

5. Let x and y be defined so that $\ln(x) = 2$ and $\ln(y) = 5$. Compute the following:

(a) [1]

$$\ln(x) = 2$$

:)

(b) [1]

$$(\ln(x))(\ln(y))$$

$2 \cdot 5$

$= 10$

(c) [2]

$$\ln(xy) = \ln(x) + \ln(y) = 2 + 5 = 7$$

(+1) property
(+1) plug in

(d) [2]

$$x$$

(+.5) since $\ln(x) = 2$

$\Rightarrow e^{\cancel{\ln(x)}} = e^2$

$x = e^2$

$x \approx 7.39$ (+.5)

use exp (+.5)
use right (+.5)

Word Problems 2 #3

6. Entropy S is a function of the number of possible states W, that are accessible to a system with a given amount of energy. We can explicitly compute entropy by

$$S = k\ln(W)$$

where k is Boltzmann's constant which is approximately $1.38065 \cdot 10^{-23}\mathrm{m}^2\mathrm{kg}\ \mathrm{s}^{-2}\mathrm{K}^{-1}$.

(a) [3] If a gas has entropy 2, about how many possible states does the gas have?

start (+.5) $\quad S$ (+.5) $\quad$ want to find W (+.5)

(+.5) $\dfrac{2}{K} = \dfrac{\cancel{K}\ln(W)}{\cancel{K}}$

$2/K = \ln W$

$e^{2/K} = W$] (+1)

$W \approx$ BIG Number?

(b) [4] If liquid A has 1,000,000,000,000 (so $1 \cdot 10^{12}$) times more possible states than liquid B, which liquid has a higher entropy and what is the difference?

let W_A be # of states of liquid A $\quad$ define variables (+1)

let W_B be # of states of liquid B

$W_A = 1 \cdot 10^{12} W_B$ $\quad$ (+.5)

entropy for A $\quad$ entropy for B

$K\ln(W_A)$ $\quad$ $K\ln(W_B)$

$= K\ln(1 \cdot 10^{12} \cdot W_B)$

$= K[\ln(10^{12}) + \ln W_B]$ $\quad$ (+1) algebra/log prop

$= K\ln(10^{12}) + K\ln W_B$

$= K\ln(10^{12}) + \text{entropy of B}$

(+1) find relation between entropy of A & entropy of B

higher entropy by $\quad$ difference (+.5)

$K \cdot \ln(10^{12}) = K \cdot 12\ln(10$

$= 1.38065 \cdot 10^{-23} \cdot 12 \cdot \ln(10)$

$\approx$ small #