TMath 120

Key

NAME:

Show all your work. You are welcome to use a calculator but no notes, books, internet resources (Desmos is the exception!) or peers can be used. Reasonable supporting work must be shown to earn credit.

1. [2] Create a function whose range consists of colors.

So many possible answers for this?
Maybe inputs are days on the calender
whats are the color of the sky above my house @ noon

input (+.5)
whpt (+.5)
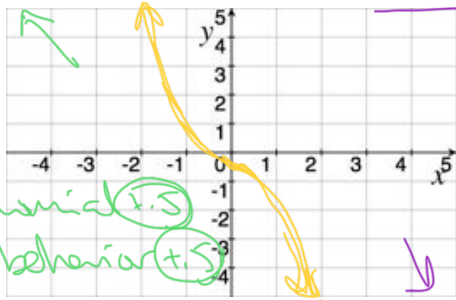*rule/clec (+.5)*
only 1 otpt (+.5)

(a) [1] Identify if/what input returns the color blue.

Yesterday April 21st 2020
The output would have been blue.

input (+.5)
otpt of blue (+.5)

2. Provide a graph AND an algebraic rule/expression for each of the functions described:
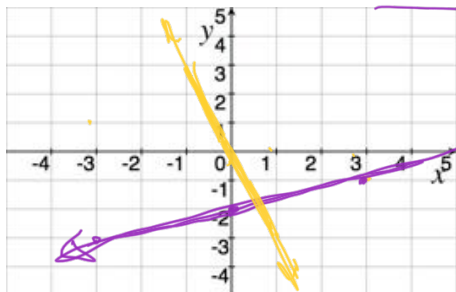
(a) [3] A polynomial with $f(x) \to -\infty$ as $x \to \infty$ and $f(x) \to \infty$ as $x \to -\infty$.

Polynomial (+.5)
end behavior (+.5)

$-x^3$ works
Polynomial (+.5)
leading coef (+.5)

so many correct answers here?
match (+.5)
start (+.5)

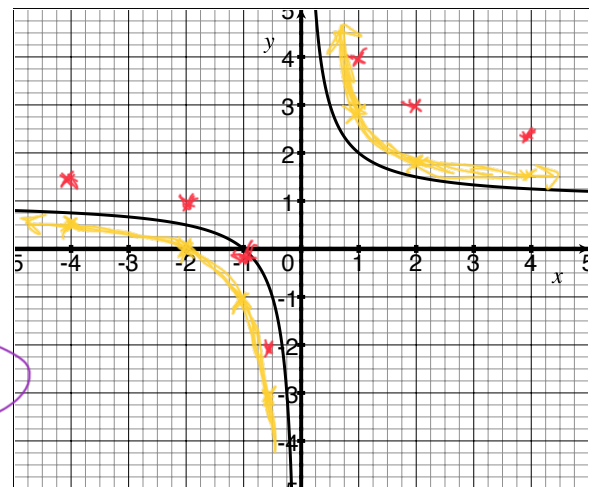(b) [3] A line perpendicular to $y = \frac{1}{3}x - 2$.

line (+.5)
⊥ line (+.5)
match (+.5)

⊥ slope is opp. reciprical
⇒ m = -3 (+1)
thru (any point) so
$y = -3x$ works (+.5)

3. Let f be the piece-wise defined graph comprised a line and a parabola shown below.

(a) Estimate the following *if* possible:

    i. [1] Is $(-3, 1)$ on the graph of f?

yes (+.5) $\underset{\text{input}}{x}$ $\underset{\text{output}}{y}$ (+.5)

    ii. [2] Is f a function? Why or why not?

start (+.5) *reason*   (+.5) *yes* each input (+1) ← has at most one output OR VLT

    iii. [1] $f(2)$

$\approx \frac{1}{2}$ (using graph) (.5)

    iv. [1] $(f - f)(2)$

(+.5) $= f(2) - f(2)$
(+.5) $= 0$ OR $f(2) - f(2) = \frac{1}{2} - \frac{1}{2} = 0$

    v. [2] $(f \circ f)(1)$

$= f(f(1)) = f(-1) = 0$
(+.5)   (+.5)   (+.5)   notation/graph reading (+.5)

    vi. [1] the y-intercept of f

$\approx -\frac{1}{2}$ (+.5)   an intercept (+.5)

    vii. [1] all possible x such that $f(x) = -1$.

all x when $y = -1$ ... $x = 1$ works
(+.5)   (+.5)

(b) [4] Find the formula for f in the indicated form:

$$f(x) = \begin{cases} -\frac{1}{2}x - \frac{1}{2} & \text{if } -4 \leq x \leq 1 \\ -\frac{1}{2}(x-3)^2 + 1 & \text{if } 1 < x \leq 4 \end{cases}$$

(.5)

(+.5) line: $y = mx + b$

$m = \frac{rise}{run} = \frac{-1}{2}$ (+.5)

thru $(-1, 0) \Rightarrow$ $0 = -\frac{1}{2}(-1) + b$
(+.5) $\Rightarrow 0 = \frac{1}{2} + b$
$-\frac{1}{2} = b$

(-.5) parabola: $a(x-h)^2 + k = y$
(+1) vertex @ $(3,1) \Rightarrow a(x-3)^2 + 1 = y$
(.5) thru $(1, -1) \Rightarrow a(1-3)^2 + 1 = -1$
$\Rightarrow a \cdot 4 + 1 = 1$ $\begin{cases} a = -\frac{2}{4} \\ \Rightarrow 4a = -2 \end{cases}$ $a = -\frac{1}{2}$

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4. Let $\alpha(x) = \dfrac{1}{x} + 1$. The graph of α is shown below.

(a) [2] Find $\alpha(x+h) - \alpha(x)$ and simplify.

$\left(\dfrac{1}{x+h} + 1\right) - \left(\dfrac{1}{x} + 1\right) = \dfrac{1}{x+h} + \cancel{1} - \dfrac{1}{x} - \cancel{1}$

$= \dfrac{x}{x \cdot x+h} - \dfrac{1}{x} \cdot \dfrac{x+h}{x+h} = \dfrac{x}{x(x+h)} - \dfrac{x+h}{x(x+h)}$

(common denominator) $= \dfrac{-h}{x(x+h)}$

re-sign (+.5)
fractions (+.5)

(b) [2] Find the algebraic rule/expression for $\alpha^{-1}(x)$.

↳ inverse to α s

order of op (+1)

$x = \dfrac{1}{y} + 1$ ⟶ $y(x-1) = \dfrac{1}{\cancel{y}} \cdot \cancel{y}$ ⟶ $y = \dfrac{1}{x-1}$

$x - 1 = \dfrac{1}{y}$ $y(x-1) = 1$

(c) [2] Write the graph transformations to transform $\alpha(x)$ into $\beta = 2\alpha(x) - 1$.

order (+.5)

① multiply y-coord by 2 (+.5)

② vert. shift down by 1 (+1)

(d) [2] Sketch a graph of $\beta = 2\alpha(x) - 1$.

shape (+.5)   step 1 (+.5)   step 2 (+.5)   sketch (+.5)

5. [3] The area of a rectangle is $3x^4 - 6x^3 + 14x^2 - 4x + 8$ square centimeters. The width is $x^2 - 2x + 4$ cm. Find its length (as a function of x!).

area of rectangle = width · length (+.5)

$3x^4 - 6x^3 + 14x^2 - 4x + 8 = x^2 - 2x + 4 \cdot ?$

$\Rightarrow ? = \dfrac{3x^4 - 6x^3 + 14x^2 - 4x + 8}{x^2 - 2x + 4}$

long division? (+.5)

set up (+.5)

algorithm (+.5)

$$? = 3x^2 + 2$$

$3x^2 + 2$ quotient:

$x^2 - 2x + 4 \,\overline{)\, 3x^4 - 6x^3 + 14x^2 - 4x + 8}$

$-(3x^4 - 6x^3 + 12x^2)$

$ 2x^2 - 4x + 8$

$-(2x^2 - 4x + 8)$

$ 0$

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6. [2] Explain how to multiply two complex numbers of the form $a + bi$ and $c + di$.

$$(a+bi)(c+di) = ac + adi + bci + bidi = ac + bdi^2 + (adi + bci)$$

$$\underbrace{F}_{} \quad \underbrace{O}_{} \quad \underbrace{I}_{} \quad \underbrace{L}_{}$$

$$= ac - bd + (ad+bc)i$$

Essentially distribute twice & remember $i^2 = -1$

got ✓ +1.5

(+1)   (+.5)

7. Tony Stark (Iron Man) is in this TMath 120 course. After seven weeks, Tony is getting a bit curious about his grade since he knows Dr. Vanderpool hasn't figured out how to get Canvas "computed" grades to make any sense. He has looked at the gradebook on Canvas and has computed the averages listed below. In case you don't remember, the weights specified in the syllabus and the graph of the function f that takes your class percentage p and returns your final course score on a 4. scale are also provided.

	weight	Tony's ave
Final Exm	25%	X
2 Exams	20%	100%
Quizzes	15%	100%
Participation	10%	0%
WebHW	15%	50%
WrittenHW	15%	30%

$$f(p) = \begin{cases} 4.0 & 90 < p \\ .1p - 5 & 57 \le p \le 90 \\ 0.0 & p < 57 \end{cases}$$

$\uparrow^X$

(a) [3] Write a function whose input is Tony's final exam percentage and returns his course percentage marks. Be sure to clearly define your variables!!!

(b) [2] Determine what Tony has to get on his final in order to earn a 2.0 in the class, if that is still possibl.e

sta/ +1.5   (+.5) looking for a function of inputs/outputs

a) Let X be Tony's final exam (+.5)

$$P = \text{Course \%} = \underset{\text{final}}{\text{\% from}} + \underset{\text{Exams}}{\text{\% from}} + \underset{\text{Quizzes}}{\text{\% from}} + \underset{\text{Participatn}}{\text{\% from}} + \underset{\text{webHw}}{\text{\% from}} + \underset{\text{writtenHw}}{\text{\% from}}$$

$$= .25x + .20 \times 100 + .15 \times 100 + .10 \cdot 0 + .15 \cdot 50 + .15 \cdot 30$$

$$= .25x + 20 + 15 + 0 + 7.5 + 4.5 \qquad \uparrow \text{ mult.} \quad \text{weights (+1)}$$

$$= .25x + 47 \quad (+.5)$$

So $P = $ Course % marks $= .25x + 47$

b) In order to get a 2.0 we need to know the Curse % (P)

(+.5) $2.0 = .1p - 5$ } we need to find x so that $p = $ Course % = 70

$+5 \qquad +5$ } using (a) (+.5) $70 = .25x + 47$

$\Rightarrow 7.0 = .1p$ $-47 \qquad -47$ (+.5)

$\cdot 1$

$\Rightarrow 70 = p$ (+.5) ✓ $\Rightarrow \frac{23}{.25} = \frac{.25x}{.25} \Rightarrow \boxed{X = 92}$

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