Practice TQS 120 Final

NAME: This is a sample final to be used for practice. This is *not a template* for the Final that will be given in class. Many of the questions on the Final will look quite di?erent than those appearing here.

[10] Let f & g, be functions.

T F $(f \circ g)(x) = (g \circ f)(x)$ ex $f(x) = x+1$ $g(x) = 2x$

$(f \circ g)(x) = 2x+1$ but $(g \circ f)(x) = 2(x+1)$

T F $(\frac{f}{g})(x) = (\frac{g}{f})(x)$

T F $\sqrt{(x^2)} = x$ for all real numbers x. Let x be -1

T F If $h(x) = x^2 + 1$, then h is an even function.

T F $\ln \frac{x}{y} = \ln x - \ln y$ for all ~~non-negative~~ positive numbers x and y.

T F $\log(\log(10)) = 0$. $\log(1) = 0$

T F Just as every integer is either even or odd, every function is either an even function or odd function. hw problem ex $g(x) = (x+1)^3$

T F $\sin(\frac{\pi}{3} + x) = \sin \frac{\pi}{3} + x$

T F If $\sin\theta \geq 0$ and $\tan\theta \leq 0$, then $\cos\theta \leq 0$

so in quadrant II

T F The range of $\sin^{-1}$ is $[0, \pi]$

Right answers will *not* get credit without supporting work. Note "undefined" and "no solution" are possible answers.

1. Find all x such that

$$5x^{-2} - 7 = 0$$

$$\frac{5}{x^2} - 7 = 0$$

$$\frac{5}{x^2} = 7$$

$$5 = 7x^2$$

$$\frac{5}{7} = x^2 \quad \Rightarrow \quad x = \pm\sqrt{\frac{5}{7}}$$

2. [2] Explain what a function is.

A function is two sets (a domain & a range) and a rule between them such that every number in the domain has exactly one output.

3. Given $m(x) = x^2 - 5x$, and $n(x) = \sqrt{4x - 8}$,

(a) [4] If $p(x) = 3m(x + 1)$, find the domain and rule of p.

$3[(x+1)^2 - 5(x+1)]$

$3[x^2+2x+1-5x-5]$

$3[x^2-3x-4]$

$3x^2-9x-12$

Domain: all real #'s.

(b) [3] Find the domain and rule of $n \circ m$.

$n \circ m(x) = n(x^2-5x)$

$= \sqrt{4(x^2-5x)-8}$

$= \sqrt{4x^2-20x-8}$

#'s in the domain so that $4x^2-20x-8 \geq 0$

rule $4x^2-2x-8=0$

when $x = \dfrac{2 \pm \sqrt{2^2-4(4)(-8)}}{2(4)}$

$= \dfrac{2 \pm \sqrt{4+128}}{8} = \dfrac{2 \pm \sqrt{132}}{8}$

so $x < \dfrac{2-\sqrt{132}}{8}$

and $\dfrac{2-\sqrt{132}}{2} < x$

46
8/128
132 - 66 - 11
2 6

(c) [5] Find the domain and rule of $\dfrac{n}{m}$.

$\left(\dfrac{n}{m}\right)(x) = \dfrac{\sqrt{4x-8}}{x^2-5x}$

Domain

fed to the squareroot ≥ 0 and den $\neq 0$

$4x-8 \geq 0$

$4x \geq 8$

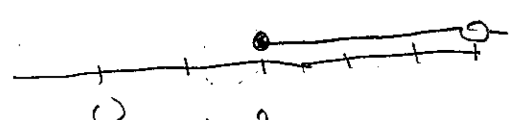
$x \geq 2$

$x^2-5x \neq 0$

$x(x-5) \neq 0$

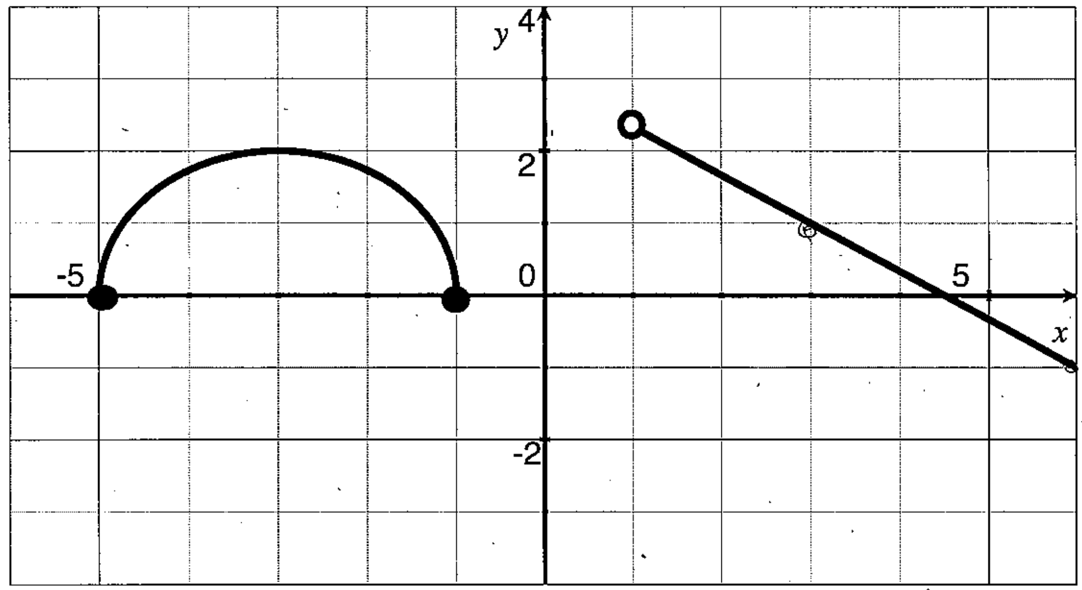
$x \neq 0$ or 5

So



$[2,5) \cup (5,\infty)$

4. [3] Let the following be the graph of g.



(a) What is the domain of g?

$[-5,-1] \cup (1,6]$

(b) The function g is a piecewise defined function consisting of a straight line and a semicircle. Write down the rule for g.

semicircle

center at $(-3,0)$

radius 2

$(x--3)^2+(y-0)^2=2^2$

$(x+3)^2+y^2=4$

$y=\sqrt{4-(x+3)^2}$

line

slope: $-2/3$

thru $(3,1)$

$1=-2/3(3)+b$

$\rightsquigarrow b=3$

$y=-2/3\,x+3$

So

$$g(x)=\begin{cases}\sqrt{4-(x+3)^2} & \text{if } -5\le x\le -\\ -2/3\,x+3 & \text{if } 1<x\le 6\end{cases}$$

(c) Find the exact x value(s) so that $g(x)=2$?

When

$2=\sqrt{4-(x+3)^2}$

$4=4-(x+3)^2$

$(x+3)^2=0$

$x+3=0$

$x=-3$

and when

$2=-2/3\,x+3$

$-1=-2/3\,x$

$-3=-2x$

$x=3/2$

(d) Find the equation for a line that is perpendicular to the line with endpoints $(3,1)$ and $(6,-1)$. (There are many right answers.)

Slope of line thru $(3,1)$ + $(6,-1)$ is $\frac{1--1}{3-6}=\frac{2}{-3}$

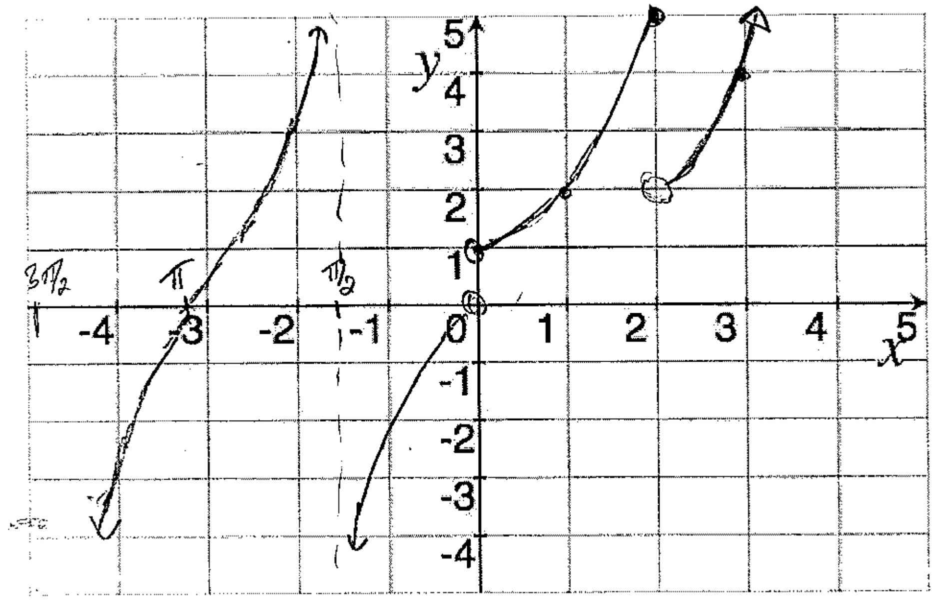
Slope of line $\perp$ to above is $3/2$

So $y=3/2\,x$ works.

5. Define f by

$$f(x) = \begin{cases} \tan x & \text{if } x < 0 \\ x^2 + 1 & \text{if } 0 < x \leq 2 \\ 2^{x-1} & \text{if } x \geq 2 \end{cases}$$

vert shift up one
horiz shift right



(a) [8] Graph f on the axes above.

(b) [9] Find the following if possible:

$f(1)$

$= 1^2 + 1 = 2$

$f(2) + f(3)$

$= (2^2+1) + (2^{3-1})$

$= 5 + 2^2$

$= 5 + 4 = 9$

$f(0)$

not defined

$f(\frac{-13\pi}{4})$

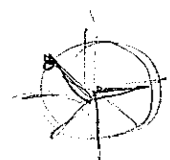
$= \tan(\frac{-13\pi}{4})$

$= \tan(-\frac{8\pi}{4} - \frac{5\pi}{4})$

$= \tan(-2\pi - 5\pi/4)$

$= \tan(-5\pi/4)$

$= \dfrac{\sin(-5\pi/4)}{\cos(-5\pi/4)} = \dfrac{1/\sqrt{2}}{-1/\sqrt{2}} = -1$



Range of f

$\mathbb{R}$

6. [3] If $f(x)$ is an even function, $f(2) = 6$, and $g(x) = \frac{1}{2}f(2x) - \frac{1}{3}$, what is $g(-1)$?

$g(-1) = \frac{1}{2}f(2(-1)) = \frac{1}{2}f(-2) = \frac{1}{2}f(2) = \frac{1}{2}\cdot 6 = 3$

b/c f is an even function

7. [6] Assuming that $\log_3 x = 5.3$ and $\log_3 y = 2.1$ find the following exactly:

$\log_3 \frac{27x^3}{y^2}$

$= \log_3 27x^3 - \log_3 y^2$

$= \log_3 27 + \log x^3 - \log_3 y^2$

$= \log_3 3^3 + 3\log x - 2\log_3 y$

$= 3 + 3\cdot 5.3 - 2\cdot 2.1$

$\log_9 3x = \log_9 3 + \log_9 x$

$= \log_9 9^{1/2} + \log_9 x$

$= \frac{1}{2} + \frac{\log_3 x}{\log_3 9}$

$= \frac{1}{2} + \frac{5.3}{2} = .5 + 2.65$

$= 3.15$

8. [4] Find all exact values for x that satisfy the following:

$\log(x - 16) = 2 - \log(x - 1)$

$\log(x-16) + \log(x-1) = 2$

$\cancel{10}^{\log(x-16)(x-1)} = 10^2$

$x^2 - 17x + 16 = 100$

$x^2 - 17x - 84 = 0$

$(x-21)(x+4) = 0$

$x = 21$ or $\cancel{-4}$ b/c domain problems.

84, 1 2, 42, 1 2, 2 21

$3^{5x}9^x = 27$

$3^{5x}(3^2)^x = 3^3$

$3^{5x}3^{2x} = 3^3$

$\log_3 3^{5x+2x}$ $\log_3 3^3$

$7x = 3$

$x = 3/7$

9. Simplify:

$\frac{\sqrt{c^2d^6}}{\sqrt{4c^3d^{-4}}} = \frac{(c^2d^6)^{1/2}}{(4c^3d^{-4})^{1/2}}$

$= \frac{(c^2)^{1/2}(d^6)^{1/2}}{4^{1/2}(c^3)^{1/2}(d^{-4})^{1/2}} = \frac{c\,d^3}{2c^{3/2}d^{-2}}$

$= \frac{1}{2}c^{1-3/2}d^{3-2}$

$= \frac{1}{2}c^{-1/2}d^5$

$\log_2 \frac{1}{4}$

$= \log_2 \frac{1}{2^2}$

$= \log_2 2^{-2}$

$= -2$

10. Simplify:

$$\frac{(x^2)^{\frac{1}{3}}(8y^2)^{\frac{2}{3}}}{4x^{\frac{2}{3}}y^2} = \frac{\cancel{x^{2/3}}\,8^{2/3}(y^2)^{2/3}}{4\cancel{x^{2/3}}\,y^2}$$

$$= \frac{2y^{4/3}}{4y^2} = \frac{y^{4/3-2}}{2} = \frac{y^{-2/3}}{2} = \frac{1}{2y^{2/3}}$$

$2 - \log_5(25z)$

$\log_5 5^2 - \log_5 25z$

$\log_5 25 - \log_5 25z$

$\log_5 \frac{25}{25z} = \log_5 \frac{1}{z}$

$\sin^{-1}(\sin \frac{3\pi}{4})$

$\sin^{-1}(\frac{1}{\sqrt{2}}) = ?$

$\Rightarrow \sin ? = 1/\sqrt{2}$

$\Rightarrow ? = \pi/4$

$$\frac{\cos x}{1-\sin x} + \frac{1-\sin x}{\cos x}$$

$$\frac{\cos^2 x + (1-\sin x)^2}{\cos x(1-\sin x)}$$

$$\frac{\cos^2 x + 1 - 2\sin x + \sin^2 x}{\cos x(1-\sin x)}$$

$$= \frac{\boxed{\cos^2 x + \sin^2 x} + 1 - 2\sin x}{\cos x(1-\sin x)} = \frac{1+1-2\sin x}{\cos x(1-\sin x)}$$

$$= \frac{2-2\sin x}{\cos x(1-\sin x)} = \frac{2\cancel{(1-\sin x)}}{\cos x\cancel{(1-\sin x)}} = \frac{2}{\cos x}$$

11: [7] Given $f(3) = 0$, use the factor theorem to find the other roots of $x^4 - 3x^3 - 25x^2 + 75x = f(x)$

b/c $f(3) = 0$

$(x-3)$ is a factor of f

$$\begin{array}{r} x^3 - 25x \\ x-3 \overline{\smash{)}\, x^4 - 3x^3 - 25x^2 + 75x} \\ -(x^4 - 3x^3) \\ \hline 0 - 25x^2 + 75x \\ (-25x^2 + 75x) \\ 0 \end{array}$$

$\Rightarrow f(x) = x^4 - 3x^3 - 25x^2 + 75x$

$= (x-3)(x^3 - 25x)$

$= (x-3)\,x\,(x^2-25)$

$= (x-3)\,x\,(x+5)(x-5)$

the other roots are

$3, 0, -5 + 5.$

12. [5] Your given a 16 oz mocha that is a rather weak 3% espresso. You, knowing you'll be up late studying mathematics, would rather like a 30% espresso drink. Realizing this you purchase an espresso machine. How much weak mocha do you discard and replace with straight espresso to have a 16 oz mocha with the desired concentration?

let x be the mocha you keep
y be the amount of espresso

$$\begin{array}{r} 16 \\ \times\ 3 \\ \hline 48 \end{array}$$

total mocha $\Rightarrow 16 = x + y$

total espresso $\Rightarrow .3 \cdot 16 = .03x + y$

2 equations, 2 unknowns

$16 = x + y$ and $4.8 = .03x + y$

$\Rightarrow 16 - y = x$ sub into

to get $4.8 = .03(16 - y) + y$

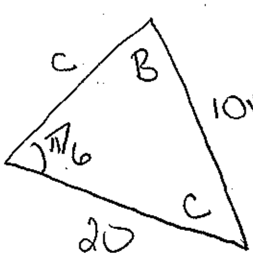
$4.8 = .48 - .03y + y$

$= .48 - .48$

$4.32 = .97y$

$\Rightarrow y = \frac{4.32}{.97} = \frac{432}{97}$ oz

13. [5] Use the conventions from the book and class and let A be measure of the angle opposite the side with length a. Given that $a = 10\sqrt{2}$, $b = 20$, and $A = \frac{\pi}{6}$ with the standard notation, determine if the information describes 0, 1, or 2 triangles and solve for them/it if they/it exist/s.



$\frac{\sin \pi/6}{10\sqrt{2}} = \frac{\sin B}{20}$

$\frac{20 \sin \pi/6}{10\sqrt{2}} = \sin B$

$\frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{1}{2} = \sin B$

$\Rightarrow B = \pi/4$ or $3\pi/4$

Recall
$\sin(\theta + \varphi) = \sin\theta\cos\varphi + \cos\theta\sin\varphi$

$\boxed{\text{if } B = \pi/4}$

$C = \pi - \pi/4 - \pi/6$

$= \frac{12\pi - 3\pi - 2\pi}{12}$

$\therefore \boxed{C = 7\pi/12}$

$\Rightarrow \frac{\sin \pi/6}{10\sqrt{2}} = \frac{\sin \frac{7\pi}{12}}{c}$

$\frac{1}{20\sqrt{2}} = \frac{\sin \frac{7\pi}{12}}{c}$

$c = 20\sqrt{2} \sin \frac{7\pi}{12}$

$c = 20\sqrt{2} \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$c = 20\sqrt{2}\left[\sin \pi/4 \cos \pi/3 + \cos \pi/4 \sin \pi/3\right]$

$\boxed{c = 20\sqrt{2}\left[\frac{1}{\sqrt{2}}\frac{1}{2} + \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}\right] = 10(1+\sqrt{3})}$

$\boxed{\text{if } B = 3\pi/4}$

$C = \pi - 3\pi/4 - \pi/6$

$= \frac{12\pi - 9\pi - 2\pi}{12}$

$\boxed{\therefore C = \pi/12}$

$\frac{\sin \pi/6}{10\sqrt{2}} = \frac{\sin \pi/12}{c}$

$\frac{1}{20\sqrt{2}} = \frac{\sin \pi/12}{c}$

$c = 20\sqrt{2} \sin \pi/12$

$c = 20\sqrt{2} \sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$c = 20\sqrt{2}\left[\sin \pi/3 \cos \pi/4 + \cos \pi/3 \sin\right.$

$\boxed{c = 10(\sqrt{3} - 1)}$

14. [5] Suppose a radioactive isotope is such that one-fifth of the atoms in a sample decay after three years. Find the half-life of this isotope

use $P_0 2^{-t/h} = P(t)$

start with P_0 & end with $4/5 P_0$ when $t=3$.

$$\frac{4/5\,\cancel{P_0}}{\cancel{P_0}} = \frac{\cancel{P_0}\,2^{-3/h}}{\cancel{P_0}}$$ solve for h.

$$4/5 = 2^{-3/h}$$

$$\ln 4/5 = \frac{-3}{h}\ln 2$$

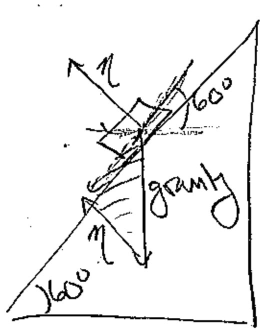
$$\frac{\ln 4/5}{\ln 2} = \frac{-3}{h}$$

$$h\,\frac{\ln 4/5}{\ln 2} = -3$$

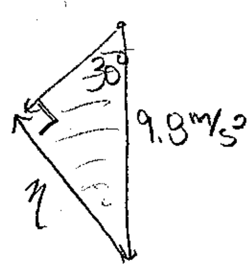
$$\Rightarrow h = \frac{-3\ln 2}{\ln 4/5}$$

15. [5] The force of friction is sometimes calculated by multiplying the normal force (the force holding the object up) by the mass of the object and by a 'coefficient of friction'. The coefficient of friction is a dimensionless number that depends on the two surfaces being pressed together.

A 10kg block is sliding down a dry glass ramp with angle of elevation of 60° and with a coefficient of friction of .94. Find the force of friction acting on the block.



need to find η & then compute $.94\eta$ (mass of obj)
ie $.94\eta \cdot 10$kg



Sohcahtoa

$$\sin 30° = \frac{\eta}{9.8}$$

$$\Rightarrow \eta = 9.8 \sin 30° = 9.8 \cdot \tfrac{1}{2}$$

So the force of friction is $.94 \cdot 9.8 \cdot .5 \cdot 10$ m/s²·kg