MIDTERM 2 **TQS 120** Spring 2010

NAME: Key #3d could use better phrasing.

1. [3] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

   Let u, v, a, and b be positive numbers.

   T (F) $\frac{x}{0} = 0$ 0 in den is not defined

   (T) F $\log_b a = x$ exactly when $b^x = a$

   T (F) $\frac{\log(u)}{\log(v)} = \log(u - v)$

   (T) F $\frac{\log u}{\log v} = \log_v(u)$

   (T) F The domain of b^x is all real numbers.

   (T) (F) $\log_b 0 = 1$ ✓ $b \neq 0$ $b^1 = b$

   .5 each

   LONG ANSWERS: Show all your work and circle you final answer. Correct answers will *not* get credit without supporting work.

2. [4] (Practice Exam) Given $-x = \frac{3y}{yx-6}$, solve for y and simplify as much as possible.

started (+1)
alg (+2)
solve for y (+1)

$-x(yx-6) = 3y$

$-x^2y + 6x = 3y$

$6x = 3y + x^2y$

$6x = y(3+x^2)$

$\frac{6x}{3+x^2} = y$

4. [8] Given that $\log_4 u = 3.2$ and $\log_4 v = 1.3$ find the following:

(§3.3 #30): $\log_4 \frac{u^2}{v^3}$

$= \log_4 u^2 - \log_4 v^3$ prop +/- (+1) prop power (+1)

$= 2\log_4 u - 3\log_4 v$

$= 2(3.2) - 3(1.3)$ sub (+1)

$= 6.4 - 3.9$ alg (+.5) stated (+.5)

$= 2.5$

(§3.3 #20): $\log_4 2uv$ prop +/- (+1) sub (+1)

$= \log_4 2u + \log_4 v$

$= \log_4 2 + \log_4 u + \log_4 v$

$= \log_4 4^{\frac{1}{2}} + 3.2 + 1.3$

$= .5 + 3.2 + 1.3$

$= 5$ (+.5)

stated (+.5) alg (+.5)

5. [7] Solve for x:

(§3.3 #33) $\log_7(x+5) = 2 + \log_7(x-1)$

$\log_7(x+5) - \log_7(x-1) = 2$ rule with log (+1) use exp (+1) write as log (+1) stated (+.5)

$\log_7 \frac{x+5}{x-1} = 7^2$

$\frac{x+5}{x-1} = 49$

$x + 5 = 49x - 49$

$54 = 48x$

$\frac{9}{8} = \frac{27}{24} = \frac{54}{48} = x$

(practice) $4^x - 3 * 2^x = 10$

$(2^x)^2 - 3(2^x) - 10 = 0$ sub (+1) quadratic (+1) finish solving (+1) stated (+.5)

$(2^x - 5)(2^x + 2) = 0$

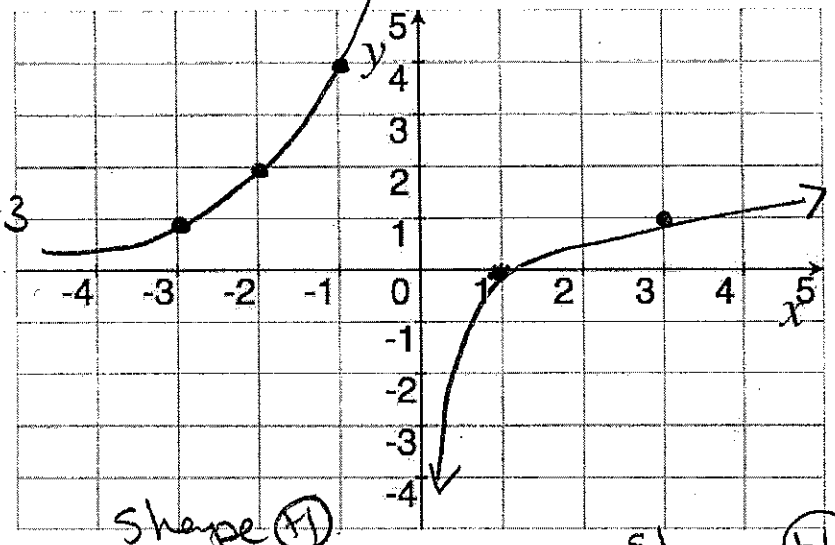
$2^x = 5$ or $2^x = -2$

$x = \log_2 5$

6. [4] Graph the function

$$f(x) = \begin{cases} 2^{x+3} & \text{if } x < 0 \\ \log_3 x & \text{if } 0 < x \end{cases}$$

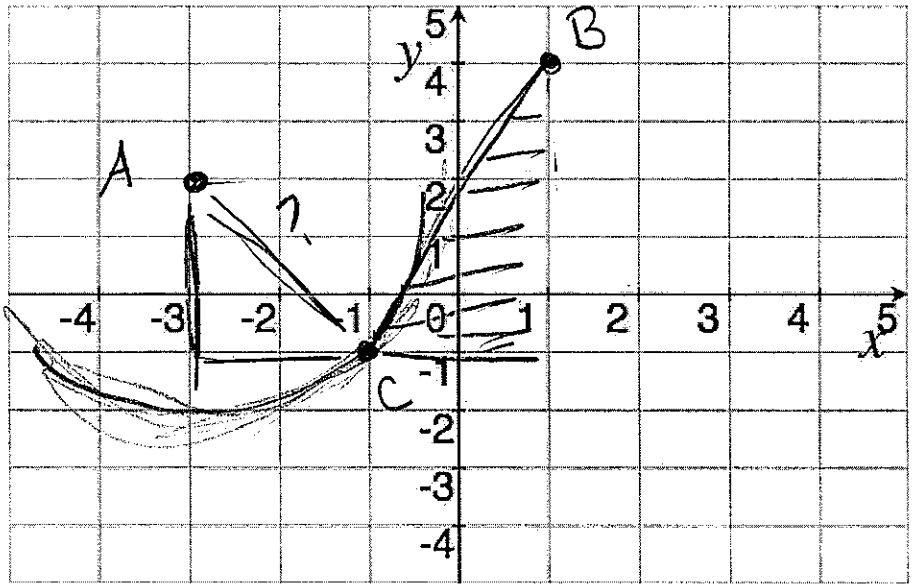
shift left 3



boundary (+.5)

shape (+1) shift (+1) VA (+.5) shape (+1)

3. Let $A = (-3, 2)$, $B = (1, 4)$, and $C(-1, -1)$.



(a) [1] Plot the point A, B and C on the above graph.

(b) [2] (WebHW10 #4) Find the distance between A and C.

$\sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$

≈ 3.606

+.5 saw dist on graph +.5

+1 dist formula +.5

+.5 got 4

(c) [4] (WebHW10 #10) Find the equation of a circle centered at A that passes through C.

$(x-h)^2 + (x-k)^2 = r^2$

$(x-(-3))^2 + (x-2)^2 = r^2$

$(x+3)^2 + (x-2)^2 = (\sqrt{13})^2$

$(x+3)^2 + (x-2)^2 = 13$

eq +1

center +1

radius +1

signs +1 from (b)

(d) [3] (§4.2 #13) Find the area region in the xy-plane under the line segment that connects B and C.

and above the line $y = -1$ +1

Drew/Knew scale +1

$\frac{1}{2} \cdot \text{base} \cdot \text{height}$

$\frac{1}{2}\, 2 \cdot 5 = 5$

+.5 +.5

7. [4] Simplify the following:

(§3.1 #35): $3 + (x^{\frac{2}{7}})^{\frac{5}{4}}$

Powers intact +1

adg +.5 stated +.5

$3 + x^{2/7 \cdot 5/4}$

$3 + x^{5/4}$

$5/4 \approx .35714$

(WebHW8 #14): $\log_3 \frac{1}{27}$

$\log_3 27^{-1}$ power of 3 stated +.5 adg prop +.5

$\log_3 3^{-3}$

-3

change of base +1

8. [4] You are shopping for a home loan to buy a modest \$120,000 home and would like a fixed interest rate since you've heard how variable rates have treated other home owners.

A Credit Union approves your request for \$96,000 with 4.5% effective annual interest on a 25 year loan. You will have to pay the 20% down payment of \$24,000 out of pocket.

A bank offers you a 20 year loan for \$80,000 with an effective annual interest rate of 4.5% *and* a second loan to act as a down payment. The second loan is a 10 year loan for \$40,000 has an effective interest rate of 15%.

Which deal will reduce your overall financial obligations? How much overall money would you save making your choice over the other?

Credit Union

$96000(1+.045)^{25} + 24000$

+1 +.5

vs

Bank

$80,000(1+.045)^{20} + 40,000(1+.15)^{10}$

+1 +1

stated +.5

\$ 312,521.71

better by

42,237.72

\$ 354,759.43

9. [6] (Story Problem Worksheet) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

(a) (Class Worksheet) Thomas Malthus lived in Britain in the late 18th century. Malthus observed the following statistics about the population of England:

year	1801	1826	1851
population (in millions)	8.3	11.9	16.8

The data fits reasonably well to an exponential curve which Malthus made use of. When would Malthus have predicted the population of England to be ~~33~~ 61

(b) (WebHW10 #6) You go to the doctor and he gives you 10 milligrams of radioactive dye. After 12 minutes, 7 milligrams of dye remain in your system. To leave the doctor's office, you must pass through a radiation detector without sounding the alarm. If the detector will sound the alarm if more than 2 milligrams of the dye are in your system, how long will your visit to the doctor take, assuming you were given the dye as soon as you arrived? Give your answer to the nearest minute.

a) P_0a^x or Pe^{rt} (+1) let t be measured in years since 1801 started (+.5)

$\Rightarrow P_0 = 8.3$ (+1)

$8.3e^{r25} = 11.9 \Rightarrow \frac{11.9}{8.3} = e^{r25}$ (+1) looking for r

$\Rightarrow \ln\frac{11.9}{8.3} = r25$ got it r (+.5)

$\Rightarrow r = \frac{1}{25}\ln\frac{11.9}{8.3} \approx .014$

Want to find t so that $61 = 8.3e^{.014t}$ (+1)

alg used (+.5) ln (+.5) $\ln\frac{61}{8.3} = t\,.014$

started (+.5) $142 = t$ so 1943

b) P_0a^x or P_0e^{rt} (+1) let t be when Dr gave you dye

$\Rightarrow P_0 = 10$ (+1) looking for r (+1)

$7 = 10e^{r\cdot12} \Rightarrow \ln\frac{7}{10} = r12 \Rightarrow r = \frac{1}{12}\ln\frac{7}{10}$ got it (+.5)

Want to find t so that $2 = 10e^{rt}$ (+1)

alg (+.5) used ln (+.5) $\frac{5}{1}\ln\frac{1}{5} = t$ 2 decimals