EXAM 1 TQS 120 Spring 2010

$\frac{4:38}{4:20}$
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NAME: Key

1. [2] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be non-zero real numbers.

T **F** $\frac{2}{x} + \frac{1}{x^2} = \frac{5}{x^2}$ $\frac{2}{x} + \frac{1}{x^2} = \frac{2x+1}{x^2}$

T **F** $x^{-1} + y^{-1} = \frac{1}{x+y}$ $x^{-1} + y^{-1} = \frac{1}{x} + \frac{1}{y}$

T F $(x+y)^2 = x^2 + 2xy + y^2$

T F The line $y = \frac{2}{3}x - \pi$ is perpendicular to the line $2y - 5 = -3x + 2$. $y - \frac{5}{2} = -\frac{3}{2}x + 1$

T **F** The graph of $-x^{15} + 6x^4 - 54x - 2.17$ is an even function.

T **F** All functions are either even or odd.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] Solve for r, writing it as a reduced fraction:

$$\frac{1}{\frac{1}{r} + \frac{1}{s}} = t$$

$1 = t\left(\frac{1}{r} + \frac{1}{s}\right)$

$\frac{1}{t} = \frac{1}{r} + \frac{1}{s}$

$\frac{1}{t} - \frac{1}{s} = \frac{1}{r}$

$\frac{s-t}{st} = \frac{1}{r}$

$\frac{st}{s-t} = r$

alg (+1)
frac add (+1)
solve for r on 1 side (+1)
started (+1)

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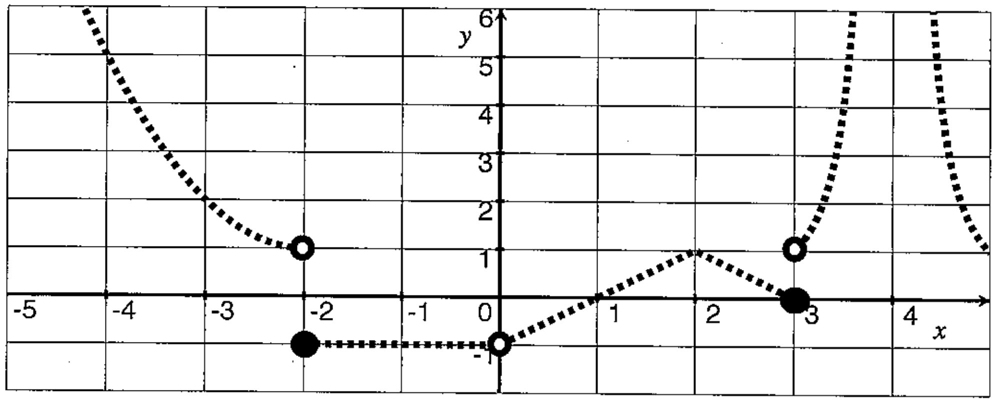
3. Let the following be the graph be a piece-wise defined graph of g.



(a) [2] (§1.2) Is g a function? Why or why not?

(+1) Yes passes the vertical line test (+1)

(b) [1] (§1.2 #21) Estimate the value of $g(-3+2)$.

$g(-1) = -1$

(c) [2] (§1.2) Estimate the value of $g(-3) + g(2)$.

$2 + 1 = 3$

(+1) (+1)

(d) [2] (§1.4) Estimate the value of $g \circ g(2)$.

(+1) (+1)

$g(g(2)) = g(1) = 0$

(e) [3] (§1.2 #19) What is the domain of g?

$(-\infty, 0) \cup (0, 4) \cup (4, \infty)$ notation (+1)

break @ 0 (+1) break at 4 (+1)

4. [3] (§2.1 #13) Find the number c so that $(c, 13)$ is on the line containing $(-4, -17)$ and $(6, 30)$.

Slope $\frac{30+17}{6+4} = \frac{47}{10} = 4.7$ (+1)

So $y = 4.7x + 1.8$ (+1)

$30 = 4.7 \cdot 6 + b$

$30 = 28.2 + b$

$30 - 28.2 = b$

$1.8 = b$

we want c so that

$13 = 4.7c + 1.8$ (+.5)

$13 - 1.8 = 4.7c$

$c = \frac{11.2}{4.7} \approx 2.38$ (+.5)

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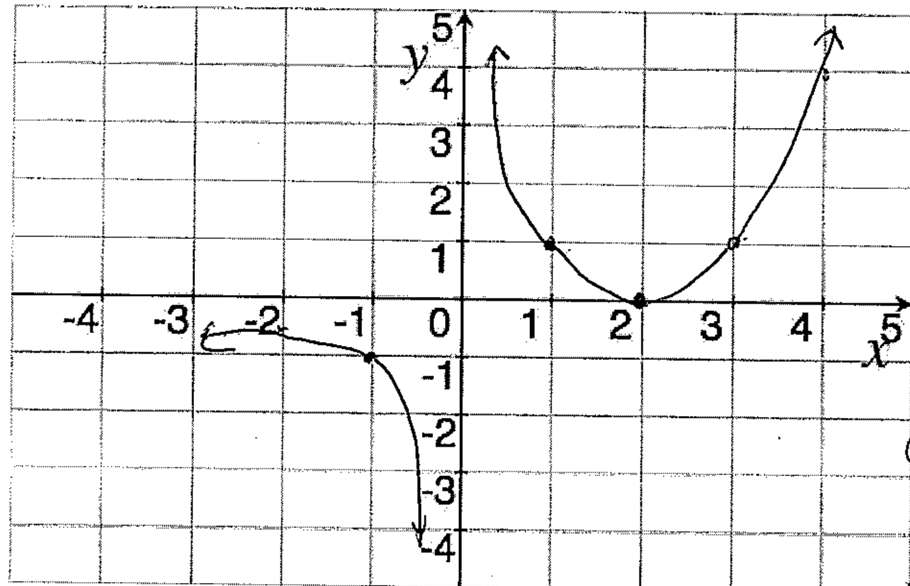
5. Let f be the *function* defined by

$$f(x) = \begin{cases} \frac{1}{x} & x \leq 1 \\ (x-2)^2 & 1 < x \end{cases}$$

shift to right 2 units.



end pts (+1)

(+1) (+1)

(a) [3] (§1.3) Graph f. (Explaining graph transformations is worth partial credit.)

(b) [2] (§1.2 #43) Find all possible input(s) so that $f(x) = 1$.

at $x = 1$ and 3 (+1) (+1)

6. (Practice Exam) Let $g(x) = x^2 + 5x - 6$.

(a) [2] Find the roots of g.

roots happen when (+1) $0 = x^2 + 5x - 6$

$0 = (x+6)(x-1)$

$\Rightarrow x = -6$ or $x = 1$ (+1)

(b) [3] Put g into vertex form.

vertex form (+.5)

$y = x^2 + 5x - 6$ (+1)

$\left(\frac{5}{2}\right)^2 + y = x^2 + 5x + \left(\frac{5}{2}\right)^2 - 6$

$\frac{25}{4} + y = \left(x + \frac{5}{2}\right)^2 - 6$

$y = \left(x + \frac{5}{2}\right)^2 - 6 - \frac{25}{4}$

$= \left(x + \frac{5}{2}\right)^2 - \frac{24}{4} - \frac{25}{4}$

$= \left(x + \frac{5}{2}\right)^2 - \frac{49}{4}$

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balanced (+.5) solved (+.5) ~~algebra (+.5)~~ got it (+.5)

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7. Let $\alpha(x) = \dfrac{x-1}{-2x+7}$.

(a) [1] What is the domain of α?

$-2x+7 \neq 0$ (+.5)

$-2x \neq -7 \Rightarrow x \neq 7/2$ (+.5)

(b) [2] Given that α has an inverse, find α^{-1}.

$x = \dfrac{y-1}{-2y+7}$ (+1)

$-2xy + 7x = y - 1$

$7x+1 = y+2xy$

$7x+1 = y(1+2x)$

$y = \dfrac{7x+1}{1+2x}$

alg (+1)

(c) [2] What is the range of α? Justify yourself.

range of α = domain of α^{-1} so all $x \ni 1+2x \neq 0$

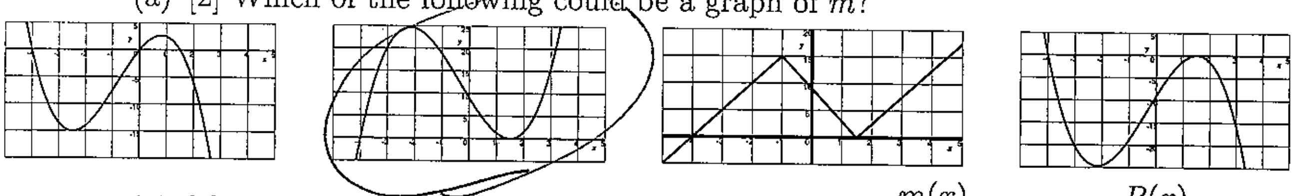
(+1)

ie $x \neq -1/2$ (+1)

started .5

8. (Lecture 4/15) Let $m(x) = x^3 + x^2 - \frac{39}{4}x + 9$ and $n(x) = x + 4$.

(a) [2] Which of the following could be a graph of m?



(b) [4] Use long division to find $G(x)$ and $R(x)$ so that $\dfrac{m(x)}{n(x)} = G(x) + \dfrac{R(x)}{n(x)}$

$-\frac{39}{4} + \frac{48}{4} = \frac{9}{4}$

$$\begin{array}{r l}
 & x^2 - 3x + 9/4 \\
x+4 \,\big)\! & x^3 + x^2 - \frac{39}{4}x + 9 \\
 & -(x^3 + 4x^2) \\
 & -3x^2 - \frac{39}{4}x + 9 \\
 & -(-3x^2 - 12x) \\
 & \frac{9}{4}x + 9 \\
 & -(\frac{9}{4}x + 9) \\
 & 0
\end{array}$$

(+.5) started

(+1) set up

(+1) algorithm

(+1) sign probs

(+.5) interpreted answer

$\dfrac{x^3 + x^2 - \frac{39}{4}x + 9}{x+4} = x^2 - 3x + 9/4$

with no remainder.

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9. [4] (WebHW6 #8) Simplify the following as much as possible (remember to show your work):

$$(3a^2b^3c^5)^2(\tfrac{1}{3}b^{-1}a)^3$$

$\underbrace{9(a^2)^2(b^3)^2(c^5)^2}_{\text{dist power } (+.5)}$ $\underbrace{\tfrac{1}{3}^3(b^{-1}a)^3}_{\text{dist power } (+.5)}$

$9a^4b^6c^{10}$ $\tfrac{1}{3}^3 b^{-3}a^3$ powers to powers (+1)

$\cancel{1}9\, a^{4+3}c^{10}b^{6-3}\cancel{a^3}$ combine bases (+1)

$\tfrac{1}{3}\cancel{1}9\, a^7c^{10}b^3$ scalars (+1)

10. [5] (Story Problem Worksheet) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

    (a) A salesperson finds that his sales average \$400 per store when he visits 25 stores a week. Each time he visits an additional store per week, the average sales per store decrease by \$30. How many stores should he visit if he wants to maximize his sales?

    (b) Potassium ferrate has been considered for use in batteries but costs \$100 per gram. You have a battery case that is currently *full* with 50 grams of a mixture that is 10% potassium ferrate. You would like to build the battery but you need a higher concentration of the potassium ferrate (40% should do it). What is the minimum amount of potassium ferrate you will have to buy and add to the battery case (after you dumped out some of the original mixture to make room) to get the cathode to work?

(a)

+1 stated

let x be the # of additional stores visited then The total sales is (# of stores)·(ave \$ per store)

+1 defined variable

or $(25+x)(400-30x)$

+1 relation between variables

$10{,}000 + 400x - 750x - 30x^2$ +.5 # brought in is # stores · \$/store

or $-30x^2 - 350x + 10{,}000$

+1 alg

+.5 got it

max happens at the vertex b/c the graph is shaped like ∩

when $x = \frac{350}{2(-30)} = \frac{35\cancel{0}}{-6\cancel{0}} \approx -5.8$

So he should visit about $25 - 5.8 \approx 19$ stores a week.

(he'd make about $(19.2)(574) \approx$ ~~[illegible]~~ \$11,020.80)

(b) 50 grams with 10% potassium ferrate.

want 50 .4 grams of potassium ferrate.

let x be the # of grams of potassium ferrate you add

let y be the # of grams of mixture you keep

Total mixture : $x + y = 50$

total potassium ferrate: $x + .1 \cdot y = 50 .4$

$\Rightarrow$ $y = 50 - x$ so

$$x + .1(50-x) = 20$$

$$x + 5 - .1x = 20$$

$$.9x + 5 = 20$$

$$.9x = 15$$

$$x = 15/.9 = \frac{150}{9}$$

(+1) started
(+1) defined variables
(+1) relation between variables
(+.5) concentration calc.
(+1) alg
(+.5) got it.

The min amount of potassium ferrate we'd have to buy is

$\frac{150}{9}$ grams $\approx 16.\overline{6}g \approx 17$ grams

so it would cost \$1,700 ?

[vs a cost of 2000 so we save \$300 with this mixture problem]

$$\begin{array}{r} 16.6 \\ 9\overline{)150} \\ -9 \\ \hline 60 \\ 54 \\ \hline 60 \end{array}$$