Exam 2 # TMath 120 Autumn 2022

NAME: Key

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T F $\frac{1}{a}+\frac{1}{ab}=\frac{b+1}{ab}$ $\frac{1}{a}\cdot\frac{b}{b}+\frac{1}{ab}=\frac{b+1}{ab}$

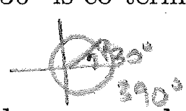
T F $x^{\frac{1}{2}}=x^{-2}$ $x^{1/2}=\sqrt{x}$ $x^{-2}=\frac{1}{x^2}$

T F $(x2)^3=x^38$ $(x2)^3=(x2)(x2)(x2)=x^3\cdot 8$

T F $\frac{\log(x)}{\log(y)}=\log(x)-\log(y)$ $\log(x)-\log(y)=\log\left(\frac{x}{y}\right)$

T F $\log_3(\log_3(3))=0$ $\log_3(\log_3(3))=\log_3(1)=0$

T F 30° is co-terminal with 390°. $30°+360°=390°$



Show all your work. Reasonable supporting work must be shown to earn credit.

2. [2] (WebHW4.1 #12) Let $\theta=\frac{-\pi}{3}$ radians. Convert θ into degrees.

$\frac{-\pi}{3}\,\cancel{rad}\cdot\frac{180°}{\pi\,\cancel{rad}}=\frac{-180\cancel{\pi}}{3\cancel{\pi}}°=-60°$

start (.5) factor (.5) orientation right (.5) neg sign (.5)

3. [3] (WebHW41.#7) Solve for x in $\log_4(x^2-9x+22)=1$

$\cancel{4^{\log_4}}(x^2-9x+22)=4^1$ use exp (.5) use right (.5)

$x^2-9x+22=4$
$-4 \quad -4$

$x^2-9x+18=0$

quad formula ← factoring → technology } solve quadratic (+2)

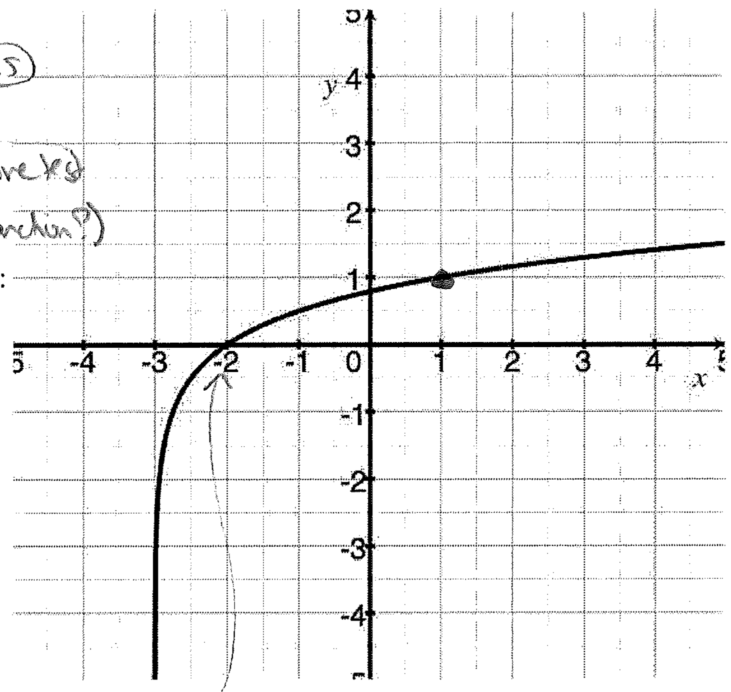
$(x-3)(x-6)=0$

$\Rightarrow x=3$ or 6

note
$\log_4(6^2-9\cdot6+22)=1$ ✓
$\log_4(3^2-9\cdot3+22)=1$ ✓

11

4. Let f be a logarithmic function that has been horizontally shifted.



(a) [2] (PracticeExam2#9) line test (+.5)
Does f have an inverse?
Why or why not? (+1)
yes? passes the horizontal line test
(+.5) (which will be an exp function?)

(b) Estimate the following *if* possible:

i. [1] (WebHW3.1#9) $f(1)$

1

ii. [2] (§3.2#56)
The domain of f. x values (+.5)
(+.5) (+.5)
$(-3, \infty)$
open paren (+.5)

iii. [2] All x such that $f(x) = 0$.

-2

(c) [3] (LogFunctionActivity#3) Find the algebraic rule/formula/equation for f.

told looks like $\log_b(x)$ but horizontally shifted ie $\log_b(x+h) = f(x)$
(+.5) (+.5)

graphically $\log_b(x) = y$
passes thru $(1,0)$ now its $(-2,0)$
$\Rightarrow$ shift LEFT 3 units
$\Rightarrow h = 3$ so $\log_b(x+3) = f(x)$ (+1)

passes thru $(1,1)$ so (+1)
$\log_b(1+3) = 1$
$\log_b(4) = 1$
$4 = b^1$
So $\log_4(x+3) = f(x)$

5. Given $\log_5(x) = 2$ and $\log_5(y) = 8$.

(a) [2] (LogFunctionsActivity#2) Find x.

$5^{\log_5(x)} = 5^2 \Rightarrow x = 5^2$ or 25
got it (+.5)

use exp (+.5)
use correctly (+.5)
looking for x (+.5)

(b) [3] (Quiz3 #2) Find $\log_5(\frac{5x}{y})$.

$\log_5\left(\frac{5x}{y}\right)$ try prop (+.5)
use correctly (+1)
plug in $\log_5(x)$ (+.5)
and $\log_5(y)$ (+.5)

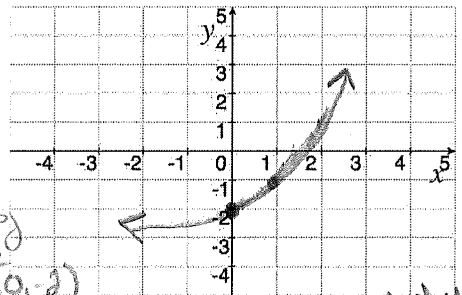
$= \log_5(5x) - \log(y)$
$= \log_5(5) + \log_5(x) - \log(y)$
(+.5)
$= 1 + 2 - 8 = -5$

got it (+.5)

5

6. Provide a graph AND an algebraic rule/expression for each of the graphs described below:

   (a) [4] (WebHW3.1#5) An exponential function vertically shifted so that it passes through $(0, -2)$.



(.5) horiz asy
(.5) shape
(.5) thru (0,-2)

match (1)

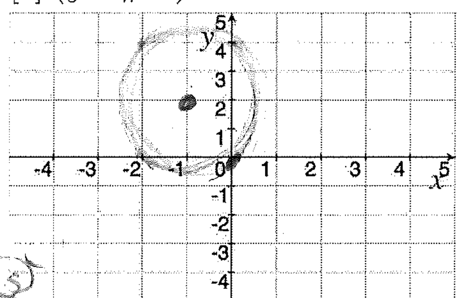
There are lots of correct answers!

$y = b^x + v$ (.5)

shifted down 3 units so (.5)

$y = 2^x - 3$ (.5)

   (b) [4] (§1.1#98) A circle centered at $(-1, 2)$ that passes through $(0, 0)$.



shape (.5)
center (.5)

match (1)

(.5) $(x-h)^2 + (y-k)^2 = r^2$ (-> was for scale)

(.5) note radius = dist between $(-1,2)$ & $(0,0)$
$= \sqrt{1^2 + 2^2} = \sqrt{5}$

so

$(x+1)^2 + (y-2)^2 = (\sqrt{5})^2$

plug in $(-1,2)$ (.5) signs (.5)

7. (WebHW3.4 #14) The number of people in a community who became infected during an epidemic t weeks after its outbreak is well approximated by $f(t) = \dfrac{35,000}{1 + ae^{-kt}}$ where 35,000 people of the community are susceptible to the disease. Assume that 3,000 people were infected initially, and 8,525 were infected by the end of the fourth week.

   (a) [4] Find the values for a and k so f models this epidemic.

   (b) [1] Use f to predict the number of infected people at the end of the 8th week.

a) have

t	$f(t)$
0	3000
4	8525

set up values (.5)

so

$3000 = \dfrac{35,000}{1 + ae^{-k \cdot 0}}$ plug in (0,3000) (.5)

$3000 = \dfrac{35000}{1 + a \cdot 1}$

alg (1)

$\dfrac{(1+a)(\cancel{3000})}{\cancel{3000}} = \dfrac{35000}{3000}$

$1 + a = 11.\overline{66}$

$\Rightarrow a = 10.\overline{66} = 32/3$

$8525 = \dfrac{35000}{1 + 10.\overline{66}e^{-k \cdot 4}}$ plug in (4,8525) (.5)

$\Downarrow$

$\dfrac{(1 + 10.\overline{66}e^{-k4})\cancel{8525}}{\cancel{8525}} = \dfrac{35000}{8525}$

alg (.5)

$1 + 10.\overline{66}e^{-k4} = 4.10557$

$10.\overline{66}e^{-k4} = 3.10557$

$e^{-k4} = .29115..$

$\Rightarrow \cancel{\ln e}^{-k4} = \ln(.29115)$ use ln (.5) use right (.5)

$-k4 \overset{3}{=} -1.23393$

$\Rightarrow k = .30848$

$f(t) = \dfrac{35000}{1 + 10.\overline{66}e^{-.30848t}}$

b) plug 8 into t (.5)

order of op (.5)

$\approx 18,382$

13

36
19
45

8. (LogsPractice2#5&6)

Decibels: The loudness of a sound (measured in decibels) is related to intensity I by

$$10 \log \left(\frac{I}{S} \right)$$

where $S = 10^{-12}$ W/m^2.

(a) [3] France passed a law limiting iPods and other MP3 players to a maximum possible volume of 100 decibels. Find the maximum intensity (in W/m^2) an iPod is legally allowed to output in France.

(b) [3] Normal conversation has a sound level of about 65 decibels. How many more times intense than normal conversation is the sound an iPod operating at the French maximum of 100 decibels?

start (.5)

a) find I_{max} given

$\frac{100}{10} = \frac{10 \log\left(\frac{I_{max}}{10^{-12}}\right)}{10}$ (+1)

$10^{10} = 10^{\log\left(\frac{I_{max}}{10^{-12}}\right)}$ use exp (.5) use correctly (.5)

$10^{10} = \frac{I_{max}}{10^{-12}}$

$10^{-12} \cdot 10^{10} = I_{max}$ alg (.5)

$10^{-12+10} = I_{max}$

$10^{-2} = I_{max}$

$.01 = I_{max}$ (+.5)

b) Looking for ? where $I_{max} = ?\, I_{conversation}$ (+1)

or $\frac{I_{max}}{I_{conversation}} = ?$

Given $65 = 10 \log\left(\frac{I_{conversation}}{S}\right)$ (.5)

$\Downarrow$

$6.5 = \log\left(\frac{I_{conversation}}{S}\right)$ alg (.5)

$\Downarrow$

$10^{6.5} = \frac{I_{conversation}}{S}$

$\Downarrow$

exp (.5) $S\, 10^{6.5} = 10^{-12} \cdot 10^{6.5} = I_{conversation}$

So

return/plug in (.5) $\frac{I_{max}}{I_{conversation}} = \frac{10^{-2}}{10^{-12} \cdot 10^{6.5}} = 10^{-2+12-6.5}$

$= 10^{3.5} \approx 3162$ times

6