

NAME: Key

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

(T) F $\frac{1}{\frac{a}{b}} = \frac{2}{ab}$

$\frac{2}{\frac{a}{b}} = \frac{2}{ab}$

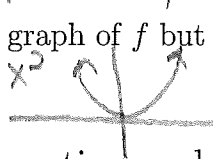
(T) F As $x \rightarrow \infty$, the output $f(x) = (x+4)^2(x-100)$ goes to ∞



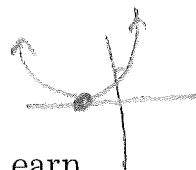
T (F) $(2+i)(3-i) = 5+i$

(T) F The graph $g(x) = f(x+1)$ is the graph of f but shifted left one unit.

consider $f(x) = x^2$



$g(x) = f(x+1) = (x+1)^2$



Show all your work. Reasonable supporting work must be shown to earn credit.

2. (WebHW2.2 #17) The length of a rectangle is $x^2 - 2x + 4$ centimeters. The area of a rectangle is $4x^4 - 8x^3 + 21x^2 - 10x + 20$ square centimeters.

- (a) [1] Find the area of the rectangle when $x = 1$.

$4(1)^4 - 8(1)^3 + 21(1)^2 - 10(1) + 20 = -4 + 11 + 20 = 27$

- (b) [3] Find the width of the rectangle as a function of x .

(1.5) (width)(length) = area

$\Rightarrow \text{width} = \frac{\text{area}}{\text{length}}$

$= \frac{4x^4 - 8x^3 + 21x^2 - 10x + 20}{x^2 - 2x + 4}$

(1.5) $x^2 - 2x + 4$
long division?

$x^2 - 2x + 4 \overline{) 4x^4 - 8x^3 + 21x^2 - 10x + 20}$
 $\underline{-(4x^4 - 8x^3 + 16x^2)}$

set up
(1.5)
alg/eq
(1.5)

$5x^2 - 10x + 20$
 $\underline{-(5x^2 - 10x + 20)}$

RJ

So
(width) = $4x^2 + 5$

3. Let f be the piece-wise defined graph comprised a line and two parabolas below.

(a) [2] (FunctionActivity #4)

Is f a function? Why or why not?

yes passes the vertical line test (1)

(b) [2] (Quiz1#1) Find the range of f ?

y values (1) $[-2, 2]$ values (1.5) endpoints (1.5)

(c) Estimate the following if possible:

i. [1] (Quiz1 #1) $f(-3)$

1 (1)

ii. [2] (WebHW§1.6 #1)

$(f \circ f)(-3)$

$f(-3) = 1$ then $f(1) = 2$

iii. [2] (WrittenHW§1.6 #68)

$(f(f(-4))) = f(-2) = 2$ (1) (1)

iv. [1] (WrittenHW§1.1 #104) The minimum of f .

lowest y-value @ $x = -4$ with $y = -2$ (1.5)

v. [2] (PracticeExam1 #6) All possible x such that $f(x) = 1$.

$x = -3$ or -1 or 1 (1.5) (1.5) (1.5) graph reading/y-value (1.5)

(d) [2] (InverseActivity#3) Does f have an inverse? Why or why not?

(1) No (1) fails the horizontal line test eg where would f^{-1} send 1? part or \Rightarrow could go to $-3, -1$ or 1

(e) [3] (WebHW1.5 #9) Graph $\frac{1}{2}f(x) + 3$.

(1.5) vert stretch first? (1.5)

vertical stretch by $\frac{1}{2}$ OR multiply y coord by $\frac{1}{2}$ (1.5)

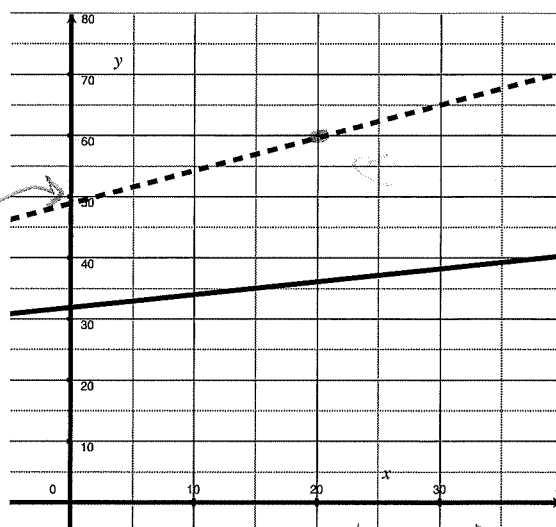
shift up by 3 units (1.5)

shape (1)

2

4. (LineActivity #7) The functions below (approximately) return the median annual income (reported in thousands of dollars) of Americans since 1960 for Asian Americans (A) and Latinx Americans (L).

- (a) [2] Let x be the years since 1960, then $A(x) = .537x + 48.900$, and $L(x) = .211x + 31.886$. Identify which line is A and justify your choice.



① the dotted line
① note the y-intercept is ≈ 50

- (b) [2] Which population's median annual income is increasing faster? Justify your answer.

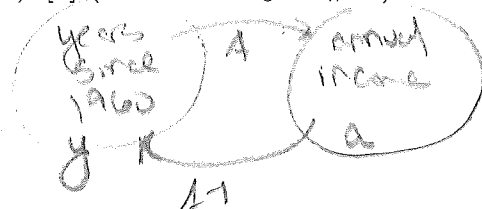
① the slope of line in median wage per year is higher (.537) vs (.211)
① the graph shows this b/c the dotted line is steeper.

- (c) [2] (WebHW7 #5) Given that A and L both have inverses, estimate $A^{-1}(60)$. Interpret your answer in terms of median annual income and year.

①.5 $A^{-1}(60) = x$ exactly when $60 = A(x)$
so what x value give output of 60? $\approx x = 20$

so 1960 + 20 or 1980 + The median annual income for Asian Americans was 60 thousand in 1980.

- (d) [3] (WrittenHW§1.7 #78) Find the algebraic rule/expression for $A^{-1}(x)$.



$$a = .537y + 48.90$$

$$a - 48.90 = .537y$$

$$\frac{a - 48.90}{.537} = y$$

swap x & y's
algebra
get it

So $1.862a - 91.06 = y$

5. [3] (PracticeExam #3) Find any real or imaginary solutions to

Sketch

$$\frac{2}{x+2} = \frac{1}{x-2} + \frac{3}{5}$$

$$5(x+2)(x-2) \left(\frac{2}{x+2} = \frac{1}{x-2} + \frac{3}{5} \right) 5(x+2)(x-2)$$

$$10(x-2) = 5(x+2) + 3(x+2)(x-2)$$

$$10x - 20 = 5x + 10 + 3(x^2 - 4)$$

$$-10x + 20 = 5x + 10 + 3x^2 - 12$$

$$0 = -5x + 30 + 3x^2 - 12$$

$$0 = 3x^2 - 5x + 18$$

$$x = \frac{5 \pm \sqrt{25 - 4(3)(18)}}{2(3)}$$

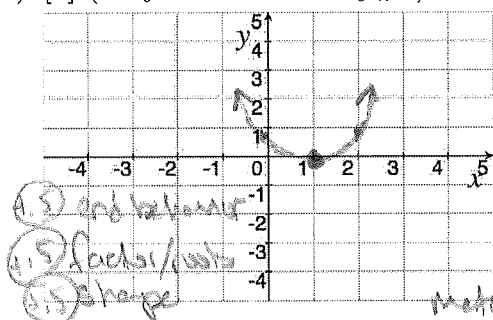
$$= \frac{5 \pm \sqrt{-191}}{6} = \frac{5 \pm \sqrt{191}i}{6}$$

complete square
factoring
technology

side quad

6. Provide a graph AND an algebraic rule/expression for each of the functions described below:

- (a) [4] (Polynomial Activity#4) A degree 4 polynomial that has $x - 1$ as a factor.

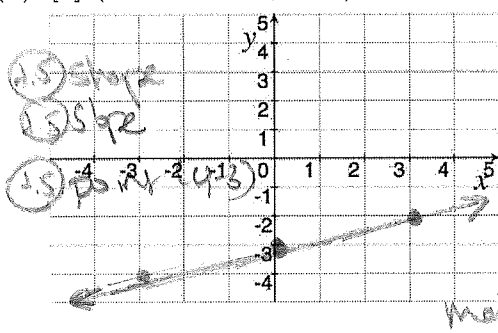


there are lots of great answers?

$(x-1)^4$ works

poly (1.5)
degree (1.5)
factor (1.5)

- (b) [4] (Line Activity#12) A line parallel to $y = \frac{1}{3}x + 2$ and through $(0, -3)$.



$y = \frac{1}{3}x - 3$

line (1.5)
slope (1.5)
y-intercept (1.5)

7. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

No, doing both questions will not earn you extra credit.

- (a) (WordProblems #4) "The Interview" generated roughly \$15 million in online sales and rentals during the first 4 days of availability, Sony Pictures said on Sunday. Sony did not say how much of that total represented \$6 digital rentals versus \$15 sales" -New York Times Dec 28th 2014.

i. [3] Write a rule/expression that returns the total sales as a function of digital rentals and sales.

ii. [2] Given that "there were a total of 2 million transactions [digital rentals and sales] over all." (also from New York Times), find how much of the total \$15 million was the result of the digital rentals and how much was from the \$15 sales.

- (b) (WebHW2.2) A farmer would like to fence land near a river so that the river will act as one of the sides of the pasture. The farmer has 200 meters of fencing that she can use.

i. [3] Write a rule/expression that returns the area fenced in a rectangular pasture as a function of the width of the pasture.

ii. [2] How wide and long should the pasture be to maximize the area enclosed?

(a) Start (1.5)

i) let $r = \#$ of digital rentals | defined variables
 $s = \#$ of sales (1)

(1.5) Total sales = \$ from rentals + \$ from sales

$$= 6r + 15s \quad (1)$$

So

$$15 \text{ mil} = 6r + 15s$$

ii) given $r + s = 2 \text{ mil}$ (1.5)

we can sub into the equation

$$r = 2 - s$$

let r and s be in millions

$$\Rightarrow 15 = 6(2 - s) + 15s \quad \text{sub (1)}$$

$$\Rightarrow 15 = 6(2 - s) + 15s$$

$$15 = 12 - 6s + 15s$$

$$-12 = -12$$

$$\frac{4}{9} = \frac{9s}{9}$$

$$.44 = s$$

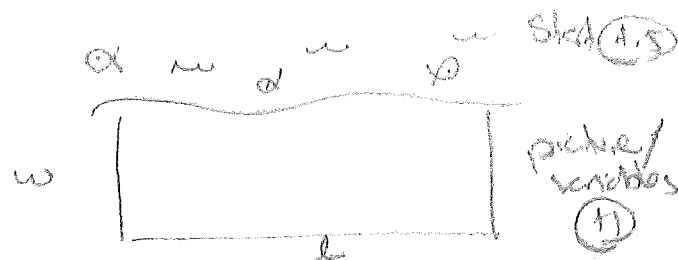
got it (1.5)

so .44 mil sales
and

$$r = (2 - .44)$$

1.56 mil rentals

(b)



$$i) (1.5) \text{ Area} = w \cdot l \quad (*)$$

$$\text{note } w + w + l = 200 \quad (1.5)$$

$$\Rightarrow 2w + l = 200$$

$$\Rightarrow l = 200 - 2w$$

sub in for * equation

$$\text{Area} = w(200 - 2w) \quad (1.5)$$

ii) to find maximum of Area (1.5)
looking for vertex

$$\text{Area} = 200w - 2w^2$$

$$= 2w(100 - w)$$

complete square \leftarrow roots @ 0 and 100 \rightarrow Technology

$$\Rightarrow \text{max in middle is } 50$$

find max (1)

$$\text{So } w = 50 \text{ and } l = 200 - 2(50)$$

$$l = 200 - 100$$

got word l (1.5)

$$(l = 100)$$