

NAME: Key

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T  F  $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$

$$\frac{1}{a} + \frac{1}{b} = \frac{b}{b} \frac{1}{a} + \frac{a}{a} \frac{1}{b} = \frac{b+a}{ab}$$

T  F Given a function  $f$  with an inverse,  $f^{-1}(f(2)) = 2$

T  F  $\frac{-1}{3+i} = \frac{-3}{8} + \frac{1}{8}i$

$$\frac{-1}{3+i} \cdot \frac{(3-i)}{(3-i)} = \frac{-3+i}{9+3i-3i-i^2} = \frac{-3+i}{9+1}$$

T  F The number 5 is both a complex number and a polynomial.

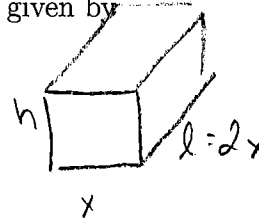
$$5 = 5 + 0i$$

$$5 = 5 \cdot 1 = 5 \cdot x^0$$

Show all your work. Reasonable supporting work must be shown to earn credit.

2. The height (in feet) of an open toy box, that is twice as long as it is wide, is given by  $\frac{27-4x^2}{12x}$ , where  $x$  is the width of the box.

- (a) [2] (Quiz1 #2) Find the length of the box when the width is 1.5 feet.



start (1.5)

(+) length = 2 · width  
 $= 2 \cdot 1.5$   
 $= 3$  (1.5)

plug in 1.5  
 some times 1.5

- (b) [3] (WordProblem #3) Write an expression in  $x$  whose value gives you the volume of the toy box.

start (1.5)

Volume = width · length · height (1)

$$= (x) (2x) \left( \frac{27-4x^2}{12x} \right)$$

(1.5) (1.5) (1.5)

3. Let  $f$  be the piece-wise defined function comprised a line and a parabola shown below.

(a) Estimate the following if possible:

i. [1] (WebHW §1.3 #6)  $f(-1)$

$\approx 1.5$

ii. [2] (WrittenHW §1.6 #36)

$(f + f)(0) = f(0) + f(0)$  (+5)

eval  $f(0)$  (+1) = 2 + 2 = 4 (+5)

iii. [2] (Quiz2 #1)  $(f(f(-4)))$

composition (+5)  $f(-1)$  (+1) = 1.5 (+5)

iv. [1] (WrittenHW §1.1 #104)

the maximum of  $f$ .

@  $x=0$   
with  $y$  value of 2

v. [2] (Transformations Activity #5) all possible  $x$  such that  $f(x) = 1$ .

$\approx -1.5$  and  $1.5$

looking for  $x$ -values (+5)  
got one (+1)  
got second (+5)

(b) [4] (PracticeExam #4) Find the formula for  $f$  in the indicated form:

$$f(x) = \begin{cases} \frac{1}{2}x + 1 & \text{if } -4 \leq x \leq -2 \\ -\frac{1}{2}x^2 + 2 & \text{if } -2 < x \leq 3 \end{cases}$$

( $y = mx + b$ ) line (+5)  
slope =  $\frac{\text{rise}}{\text{run}} = \frac{1}{2}$  (+5)

thru  $(-2, 0) \Rightarrow 0$  (+5)

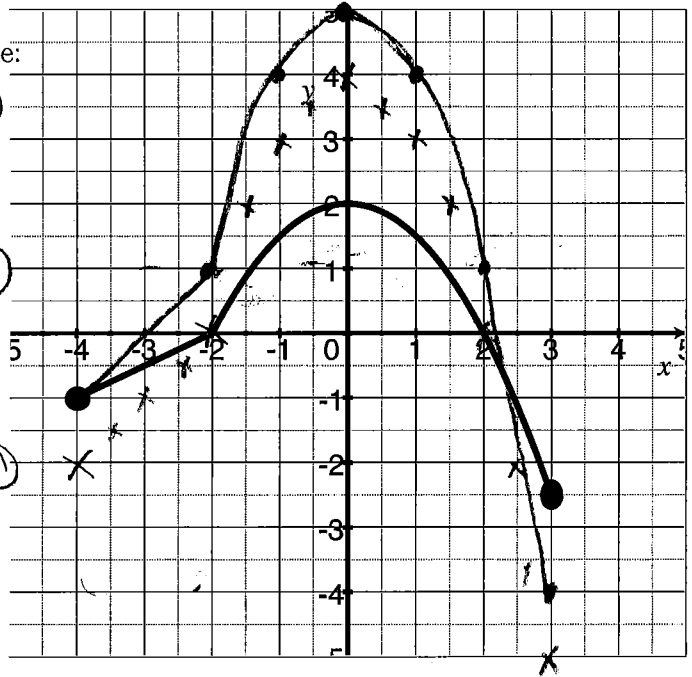
$0 = \frac{1}{2}(-2) + b$

$0 = -1 + b$  (+5)

(c) [3] (WebHW3 #16) Graph  $2f(x) + 1$ .

vertical stretch by 2 or multiply coord by 2 (+5)  
shift graph up 1 (+5)  
B same  
order (+5)

graphed in order (+5)  
graphed vert stretch (+5)  
graphed up 1 (+5)



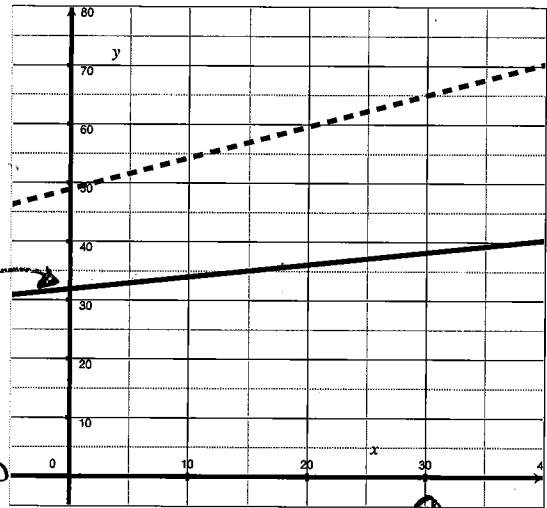
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4. (LineActivity #7) The functions below (approximately) return the median annual income (reported in thousands of dollars) of Americans since 1960 for Asian Americans (A) and Latinx Americans (L).

- (a) [2] Let  $x$  be the years since 1960, then  $A(x) = .537x + 48.900$ , and  $L(x) = .211x + 31.886$ .

Identify which line is  $L$  and justify your choice. (+1)

$L$  is the solid line with a  $y$ -intercept of  $\approx 32$  justify (+1)



- (b) [2] Which population has a bigger median annual income in 1990? Justify your answer.

1990 corresponds w/  $x=30$

Notice the dotted line or the Asian American's have higher values than the solid line  $\Rightarrow$  Asian American's have a bigger median annual income

- (c) [3] Will  $A$  and  $L$  intersect? Justify your answer and interpret the practical significance.

(+1) Not when  $x$  is positive.

(+1) Notice  $A$  has a larger slope & has a higher  $y$ -intercept. This means  $A$  will continue to grow faster than  $L$  & remain above  $L$

(+1) Practically this means Asian American's will continue to have a higher median annual income than Latinx American's

- (d) [3] (Written HW §1.7 #78) Find the algebraic rule/expression for  $A^{-1}(x)$ .

$$A(x) = .537x + 48.9$$

$$\text{OR } x = .537y + 48.9$$

$$A(x) - 48.9 = .537x$$

$$\frac{A(x) - 48.9}{.537} = x$$

try solve for  $y$   
hard b/c  $x+y$  mean something here?  
really - just solve for  $x$

$$\text{OR } 1.862A(x) - 91.061 = A^{-1}(x)$$

5. [3] (Practice Exam, #7) Find the real or complex solutions to  $3(7+x)^2 + 4 = 2$ .

$$3(7+x)^2 + 4 = 2$$

$$-4 - 4$$

$$\text{OR } 3(7+x)^2 + 4 = 2$$

$$3(7+x)^2 = -2$$

$$\frac{3(7+x)^2}{3} = \frac{-2}{3}$$

$$(7+x)^2 = -\frac{2}{3}$$

$$7+x = \pm \sqrt{-\frac{2}{3}}$$

$$x = -7 \pm i\sqrt{\frac{2}{3}}$$

$$3(49 + 14x + x^2) + 4 = 2$$

$$147 + 42x + 3x^2 + 4 = 2$$

$$3x^2 + 42x + 149 = 0$$

$$x = \frac{-42 \pm \sqrt{42^2 - 4(3)(149)}}{2(3)}$$

$$x = \frac{-42 \pm \sqrt{1764 - 1788}}{6} = -7 \pm \frac{i\sqrt{24}}{6}$$

$$x = -7 \pm i\sqrt{\frac{16}{9}} = -7 \pm i\sqrt{\frac{2}{3}}$$

Start (+1)  
Answer (+3)  
Justify (+1)

start (+5)  
algebra (+1.5)  
solved for correct thing (+1)

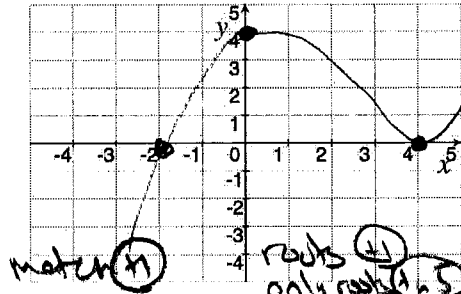
(+5) start  
(+1.5) algebra  
(+1) complex

(+5) quad form use right  
(+1) i showing up

24  
26  
50

6. Provide a graph AND an algebraic rule/expression for each of the functions described below:

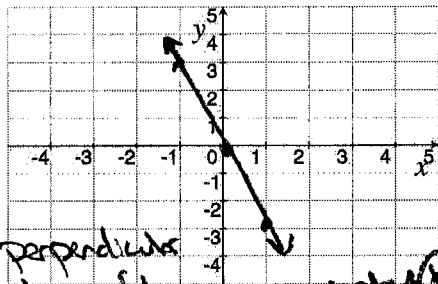
(a) [4] (Polynomial Activity #4) A degree 3 polynomial whose only roots are  $-2$  &  $4$ .



$$\frac{1}{8}(x+2)(x-4)^2 = y$$

note: there are many answers.

(b) [4] (Line Activity #13) A line perpendicular to  $y = \frac{1}{3}x + 2$ .



slope  $\perp$  to  $\frac{1}{3}$  is  $-3$  So  $y = -3x$

note: there are many answers

perpendicular slope (1)  
eg of line (1.5)

7. Choose ONE of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

No, doing both questions will not earn you extra credit.

(a) (Workshop) A salesperson find that her sales average 41 cases per store when she visits 20 stores a week. Each time she visits three additional stores per week, the average sales per store decrease by 2 cases.

- i. [3] Write a rule/expression that returns the salesperson's total sales as a function of stores she visits each week.
- ii. [2] How many stores should she visit if she wants to maximize her sales?

(b) (WordProblems #3)) A full radiator contains 8 quarts of fluid, 40% of which is antifreeze.

- i. [3] Write a rule/expression that returns the percentage (as a decimal) of antifreeze as a function of antifreeze that you add.
- ii. [2] How much fluid should be drained and replaced with pure antifreeze so that the new mixture is 60% antifreeze?

7) a) i) Total Sales =  $\left( \begin{matrix} \text{Total stores} \\ \text{visited each} \\ \text{week} \end{matrix} \right) \cdot \left( \begin{matrix} \text{\# of cases} \\ \text{sold at} \\ \text{each store} \end{matrix} \right)$

(+) let  $v = \#$  of stores visited/week  
 $s = \#$  of cases sold/store

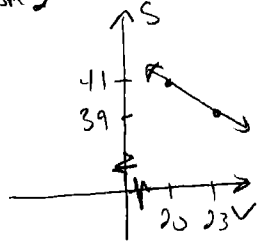
$$= v \cdot s$$

$$= v \left( -\frac{2}{3}v + \frac{163}{3} \right)$$

$$= -\frac{2}{3}v^2 + \frac{163}{3}v$$

Note  $s$  is a function of  $v$  so

| $v$ | $s$ |
|-----|-----|
| 20  | 41  |
| 23  | 39  |
| 26  | 37  |



a line connecting  
 $s = mv + b$

$$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{3}$$

thru  $(20, 41)$  so

$$41 = -\frac{2}{3}(20) + b$$

$$\Rightarrow 41 + \frac{40}{3} = b$$

$$\frac{163}{3} = b$$

$$\text{so } s = \left(-\frac{2}{3}\right)v + \frac{163}{3}$$

Step (+.5)

ii) how many stores to max. sales?

(+.5) we need to find the vertex

(+) technology  $\rightarrow$  complete  $\square$   
 $-\frac{b}{2a}$

$\Rightarrow$  when  $v =$

$\Rightarrow$  round up to

stores (+.5)

b) i) antifreeze percentage as a decimal

$$= \frac{\text{antifreeze}}{\text{total mixture}}$$

$$= \frac{\text{original antifreeze} + \text{added}}{\text{B quarts}}$$

note the radiator will stay full

$$= \frac{.4 \cdot \text{Mix Keep} + x}{\text{B}}$$

$$= \frac{.4(\text{B} - x) + x}{\text{B}} = \frac{3.2 - .4x + x}{\text{B}} = \frac{3.2 + .6x}{\text{B}}$$

40% of B quarts is 3.2

let  $x$  be the antifreeze added  
 $=$  amount of mixture removed.

$$\Rightarrow \text{Keep } \text{B} - x$$

Step (+.5)

ii) Find  $x$  so that 65% antifreeze

$$(.5) .6 = \frac{3.2 + .6x}{\text{B}} \Rightarrow 4 \cdot \text{B} = 3.2 + .6x$$

$$\Rightarrow 1.6 = .6x$$

$$\Rightarrow x = 2.7 \text{ quarts}$$

round (+.5)

# Transforming Functions continued

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

Let  $f(x) = x^2$  for the *entirety* of this worksheet.

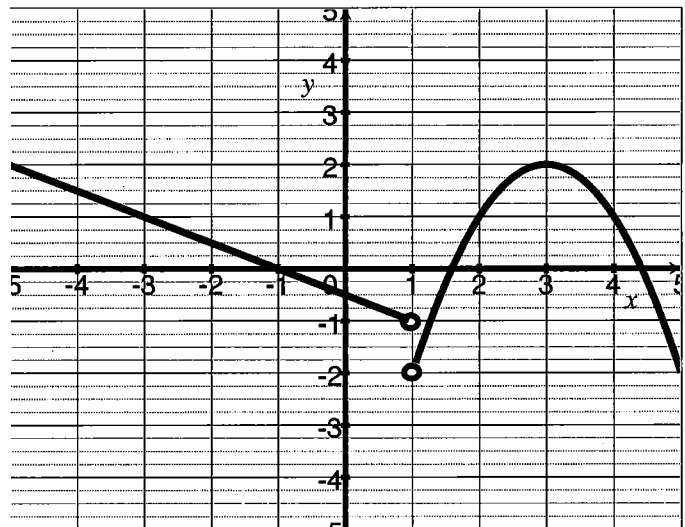
1. The graph of a piece-wise defined function labeled  $g$  is below. To be explicit, all the pieces below make up the graph of  $g$ .

(a) Is  $g$  a function?  
Why or why not?

(b) Find the domain of  $g$ .

(c) For what value(s)  
of  $x$  does  $g(x) = 1$ ?

(d) Given that  $g$  is comprised  
of a line and a parabola,  
find the piece-wise defined  
algebraic rule of  $g$ .



(e) Draw the graphs of  $m(x) = -2g(x) + 1$  on the set of axes.