Exam 2 **TMath 120** Autumn 2015

NAME: Key

1. [8] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

T (F) $\frac{3}{a} + \frac{4}{a+1} = \frac{3+1}{a+1} + \frac{8}{a+1}$ $\frac{a+1}{a+1}\frac{3}{a} + \frac{4}{a+1}\frac{a}{a} = \frac{3(a+1)+4a}{a(a+1)} = \frac{7a+3}{a(a+1)}$

(T) F $4x$ is a polynomial. of degree one

T (F) $\frac{1}{2} * 6^x = 3^x$ let $x=2$, $3^2=9$ but $\frac{1}{2}\cdot 6^2 = \frac{1}{2}\cdot 36 = 18$

T (F) $(x^2)^3 = x^5$ $(x^2)^3 = x^2x^2x^2 = xxxxxx = x^6$

T (F) $\log(x) + \log(x) = \log(x+x)$ $\log x + \log x = \log(x\cdot x)$

(T) F $\log_4(16) = 2$ $\log_4 16 = \log_4 4^2 = 2$

(T) F $\log(x^{3x+2}) = (3x+2)\log(x)$

(T) F $x^2 - 2x + 1$ divides $x^4 - 2x^2 + 1$

$$x^2-2x+1 \overline{)\,x^4+0x^3-2x^2+0x+1}$$
$$x^2+2x+1$$
$$-(x^4-2x^3+x^2)$$
$$2x^3-3x^2+0x+1$$
$$-(2x^3-4x^2+2x)$$
$$x^2-2x+1$$

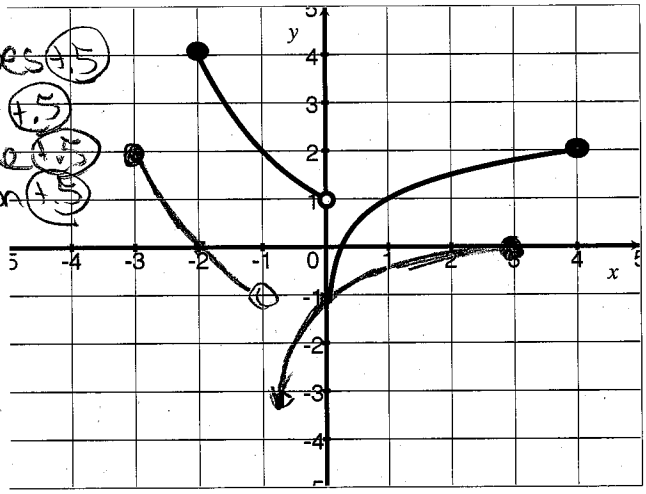
Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [3] (Quiz3 #2) Simplify $\left(\frac{a^3b^{-3}}{a\sqrt{b}}\right)^2$

$\frac{(a^3)^2(b^{-3})^2}{a^2(\sqrt{b})^2} = \frac{a^6b^{-6}}{a^2b} = \frac{a^4}{b^7}$

+1 +1 combine a's +.5 combine b's +.5

3. Let f be the piece-wise defined function comprised of an exponential ($b^x = y$) and logarithmic ($\log_b(x) = y$) function graphed below. Each piece may have undergone vertical shifts (but no other transformations).



(a) [2] (WebHW7 #22) Find the range of f.

y-values (+.5) endpt (+.5) include (+.5) notation (+.5)

$(-\infty, 4]$

or

$y \leq 4$

(b) [2] Estimate the following *if* possible:

i. $f(1)$ 1 (+1)

ii. $f(-1)$ 2 (+1)

(c) [4] (§3.1 #54 & §3.2 #62) Find the equation/rule for h in the indicated form:

$$h(x) = \begin{cases} \left(\frac{1}{2}\right)^x & -2 \leq x < 0 \\ \log_4(x) + 1 & 0 < x \leq 4 \end{cases}$$

(+.5) exponent $b^x + c = y$ (may have vertical shift)

would pass thru $(0,1)$: $b^0 + c = 1$, $1 + c = 1$, $\Rightarrow c = 0$

passes thru $(-2,4)$ (+.5): $b^{-2} = 4$, $\frac{1}{b^2} = 4$, $1 = 4b^2$, $\frac{1}{4} = b^2 \Rightarrow b = \frac{1}{2}$ (+1)

(+.5) logarithmic $\log_b(x) + d = y$ (may have vertical shift)

passes thru $(1,1)$: $\log_b(1) + d = 1$, $0 + d = 1$, $d = 1$ (+1)

passes thru $(4,$: $\log_b(4) + 1 = 2$, $-1 \quad -1$, $\log_b 4 = 1$, $b^1 = 4$

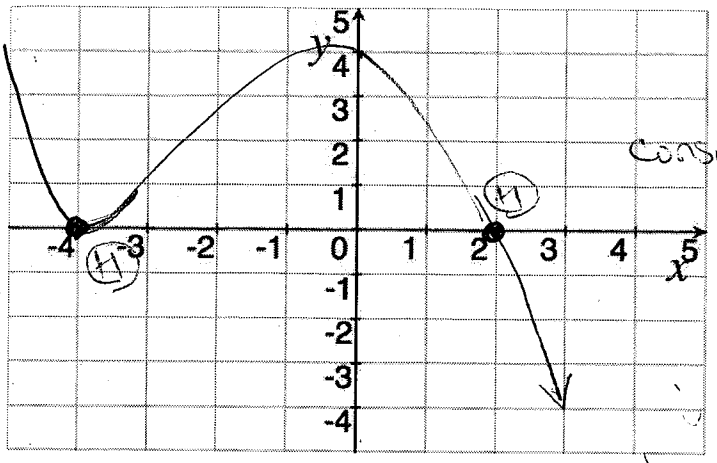
(d) [3] (WebHW7 #13) Draw the graph of g if $g(x) = f(x+1)$-2.

get it (+1.5)

(+1) horz shift left 1

(+.5) vertical shift down 2

4. Let $h(x) = -\frac{1}{32}(x-2)^3(x+4)^2$



(a) [1] (WebHW6 #1)
What is the degree of h?

5

Consistent w/ a-c

(b) [2] (Polynomial Wks #8)
Describe the end behavior of h.

(+1) as $x \to \infty$, $y \to -\infty$
(+1) as $x \to -\infty$, $y \to \infty$

— or —

as x gets very large y gets very negative and as x gets very negative, y gets very large

+1 partial for opposite behavior

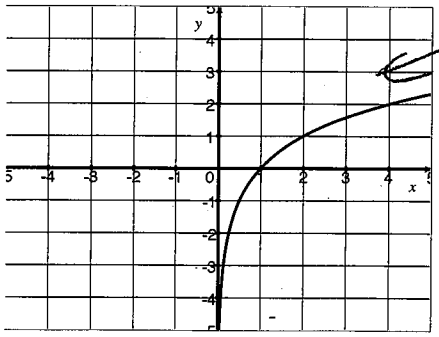
(c) [2] (§2.2 #52) Identify the zeros.

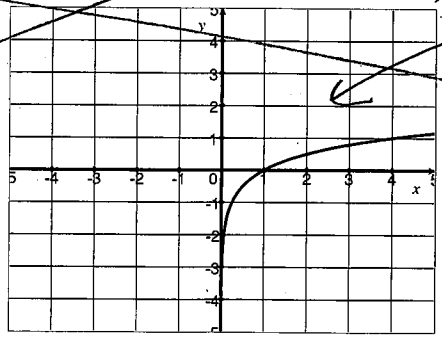
$(x-2)$ is a factor $\Rightarrow$ (+1) 2 is a root similarly (+1) -4

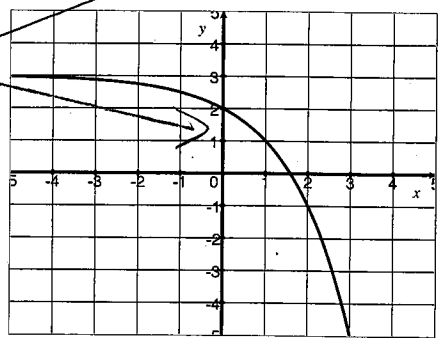
(d) [3] (§2.2 #52) *Sketch* a graph of h, clearly indicate end behavior and shape around any x-intercepts. $h(0) = \frac{-1}{32}(0-2)^3(0+4)^2 = \frac{+1}{2^5}\cdot 8 \cdot 16 = +4$

5. [3] (§3.2 #62) Match the algebraic equations to the graphs:

$a(x) = -2^x + 3$ $b(x) = \log_2(x)$ passes thru (2,1) $c(x) = \log_4(x)$ passes thru (4,1)







b c a

2x vertical flip up by 2

6. Solve for x

(a) [3] (WebHW8 #14) $2^{3-x} = 5^x$

$\ln 2^{3-x} = \ln 5^x$

$(3-x)\ln 2 = x \ln 5$

$3\ln 2 - x\ln 2 = x\ln 5$

$3\ln 2 = x\ln 5 + x\ln 2$

$3\ln 2 = x(\ln 5 + \ln 2)$

$x = \frac{3\ln 2}{\ln 5 + \ln 2} \approx .903$

or $\log_2(5^x) = 3 - x$

$x\log_2(5) = 3 - x$

$x + x\log_2(5) = 3$

$x(1 + \log_2 5) = 3$

$x = \frac{3}{1+\log_2 5}$

(b) [4] (PracticeExam2 #8) $\log(x-16) = 2 - \log(x-1)$

$\log(x-16) + \log(x-1) = 2$

$\log[(x-16)(x-1)] = 2$

$10^2 = (x-16)(x-1)$

$100 = x^2 - 17x + 16$

$x^2 - 17x - 84 = 0$

$(x-21)(x+4) = 0$

$\Rightarrow x = 21$ or $\cancel{-4}$

+log(x-1) +log(x-1)

or

7. [4] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (WebHW9 #7) Earthquakes are measured with the Richter scale calculated by $\log\left(\frac{I}{I_0}\right)$ where I is the intensity of an earthquake and I_0 is a "standard earthquake" (about .0001). Suppose earthquake A registers 4 more points on the Richter scale than earthquake B. How are their corresponding intensities related?

(b) (WordProblem #10) Assume that the vinegar in this problem has a pH level of 3.1. Note that the pH level of an object is computed by $-\log[H^+]$ where $[H^+]$ is the concentration of hydrogen ions (in Moles) per liter. If you found a substance X whose $[H^+]$ was three times as intense as vinegar, find its pH measure?

(a) Let I_A be the intensity of earthquake A and similarly for I_B.

Given Richter Scale of $A = 4 +$ Richter Scale of B

looking for I_A/I_B

$\log \frac{I_A}{I_0} = 4 + \log\left(\frac{I_B}{I_0}\right)$

$\log \frac{I_A}{I_0} - \log \frac{I_B}{I_0} = 4$

$\log\left(\frac{I_A/I_0}{I_B/I_0}\right) = 4$

$\log \frac{I_A}{I_B} = 4$

$10^4 = \frac{I_A}{I_B}$

So earthquake A is 10,000 times more intense than B

(b) let $[H^+]_V$ be the concentration of hydrogen ions of vinegar and similarly for substance X

Given $[H^+]_X = 3[H^+]_V$

looking for pH of X

$-\log[H^+]_X = -\log 3[H^+]_V$

$= -(\log 3 + \log[H^+]_V)$

$= -\log(3) +$ pH of vinegar

So pH level of $-\log(3) + 3.1 \approx$

8. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

   (a) (§3.1 #88) The population in Sometown, USA was 12,000 in 2000 and grew to 15,000 in 2010. Assume that the population will continue to grow exponentially and determine when the population will double from the 12,000 in 2000.

   (b) (§3.1 #87) Ms. Ann Scheiber retired from government service in 1941 with a monthly pension of \$83 and \$5000 in savings. At the time of her death in January 1995 at the age of 101, Ms. Scheiber had turned the \$5000 into \$22 million through shrewd investments in the stock market. If she had lived longer and kept the same rate of annual return, when would she have \$25 million dollars?

(a) start (+1.5) notation/def (+1.5)

Pe^{rt} } = pop @ time t (+1.5)

(+.5) 12,000 years from 2000 (+.5)

$12{,}000e^{rt}$ = pop @ time t

(+.5) (we want to know when otpt is 24,000

So we need to find r first) (+1.5)

$12{,}000e^{r\cdot 10} = 15{,}000$ (+1)

$e^{r\cdot 10} = 5/4$

$r \cdot 10 = \ln 5/4$

$r = \frac{1}{10}\ln 5/4 \approx$

(+.5) So find t given $12000e^{t(\frac{1}{10}\ln 5/4)} = 24{,}000$

$\Rightarrow e^{t(\frac{1}{10}\ln 5/4)} = 2$

$\Rightarrow \left(e^{\ln(5/4)^{1/10}}\right)^t = 2$ (alg +1.5, use prop +1.5)

$\left[\left(\frac{5}{4}\right)^{1/10}\right]^t = 2$

$t \ln\left(5/4^{1/10}\right) = \ln 2$

$t = \ln 2 / \ln 5/1^{1/10} \approx$

(b) start (+1.5) notation/def (+1.5)

$P(1+r)^t$ } = \$ at time t (+1.5)

(+.5) 5,000 years from 1941 (+.5)

$5000(1+r)^t$ = \$ at time t

(+.5) (we want to know when otpt is \$25m

So we need to find r first) (+1.5)

$5000(1+r)^{54} = 22{,}000{,}000$ (+1)

$(1+r)^{54} = 22{,}000$

$1+r = (22000)^{1/54}$

$r = (22000)^{1/54} - 1 \approx$

(+.5) So to find t given

$5000\left(1+(22000)^{1/54}-1\right)^t = 25{,}000{,}000$ (+.5)

$\left(22000^{1/54}\right)^t = 5{,}000$

$\ln\left(22000^{1/54}\right)^t = 5{,}000$ (alg +.5, prop +.5)

$t \ln(22000^{1/54}) = 5{,}000$

$t = \frac{5000}{\ln 22000^{1/54}} \approx$

$$\begin{array}{r} 22 \\ 22 \\ \quad 6 \\ \hline 50 \end{array}$$