

NAME: Key

1. [10] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T F $\frac{1+2a}{6} = \frac{1+a}{3}$

$\frac{1+a}{3} = \frac{2(1+a)}{2 \cdot 3} = \frac{2+2a}{6}$

T F $(x^2)^3 = x^6$

$(x^2)^3 = x^2 x^2 x^2 = (xx)(xx)(xx)$

T F -3 is a root of $2x^3 - 9x + 5$

$2(-3)^3 - 9(-3) + 5 = 2 \cdot 27 + 27 + 5$

T F $\log_5 25 = 2$

$\log_5(5^2) = 2$

$= -54 + 32 \neq 0$

T F The domain of $\log_3(x+3)$ is $(-3, \infty)$.

Shift left 3 units $\rightarrow \log_3(x)$

T F $2 * 8^x = 16^x$

order of operations says perform exponents first

T F The pH scale is logarithmic.

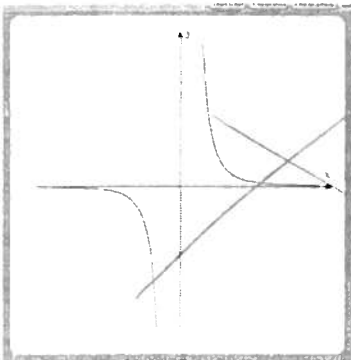
T F \$100 compounded continuously for one year with an annual interest rate of 5% is \$105.00.

$100e^{.05 \cdot 1}$ is a little more

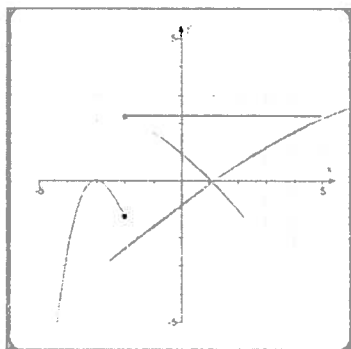
T F $\log(x+y) = (\log x)(\log y)$

T F $\log x + \log y = \log(xy)$

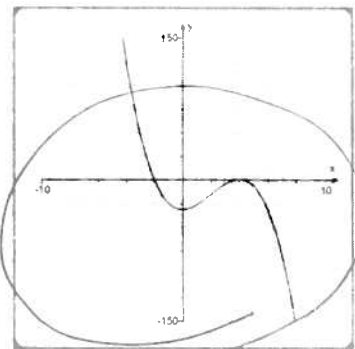
Show your work for the following problems. The correct answer with no supporting work will receive NO credit.



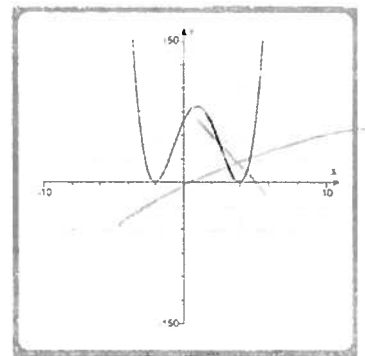
no b/c there are jumps



no b/c there are jumps



+1



no b/c end behavior

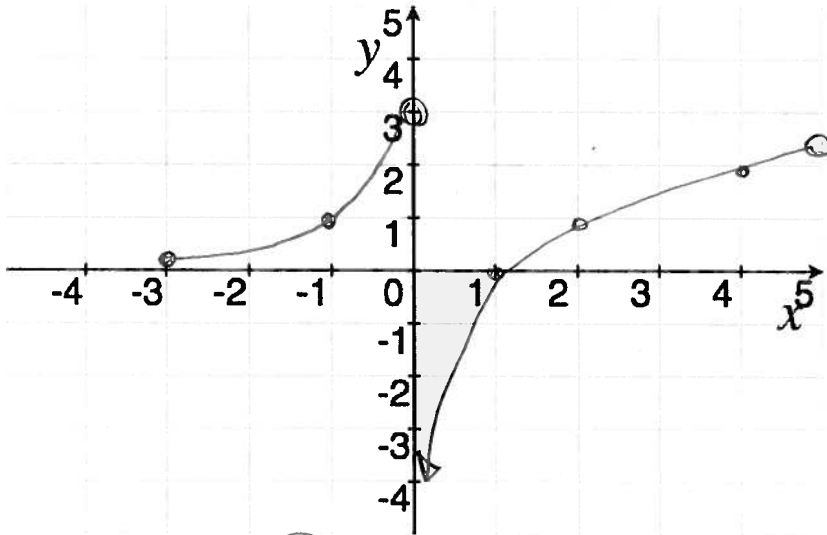
2. [2] (PolyWks #9) Identify all of the above graphs that could be the graph of an odd degree polynomial?

no others +1

3. [4] (Quiz3 #3) Carefully graph h on the axes provided where h is the piecewise defined function:

$$h(x) = \begin{cases} 3^{x+1} & \text{if } -3 \leq x < 0 \\ \log_2(x) & \text{if } 0 \leq x < 5 \end{cases}$$

graph shifted 1 unit left +.5



shape +.5
translated +.5

+5

shape +5

start/use def/etc +.5
end points +1

4. [4] (10/21 Lecture) Given that 2 is a root of $g(x) = 2x^3 - 20x^2 + 67x - 70$. Completely factor g .

rule $g(2) = 2(2)^3 - 20(2)^2 + 67(2) - 70 = 16 - 80 + 134 - 70 = 150 - 150 = 0$
 so 2 is a root. ☺

+1.5 Since 2 is a root $x-2$ is a factor of $g(x)$.

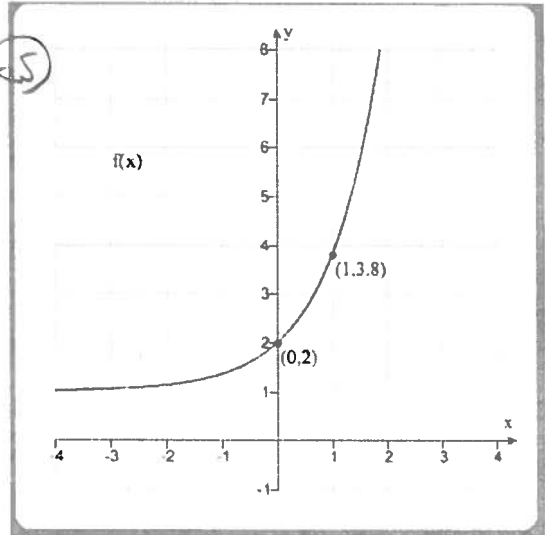
+1.5 So $(x-2) \cdot ? = g(x)$
 $\Rightarrow ? = \frac{g(x)}{x-2}$ long div

algorithm +.5

$$\begin{array}{r} x-2 \overline{) 2x^3 - 20x^2 + 67x - 70} \\ \underline{-(2x^3 - 4x^2)} \\ -16x^2 + 67x - 70 \\ \underline{-(-16x^2 + 32x)} \\ 35x - 70 \\ \underline{-(35x - 70)} \\ 0 \end{array}$$

+1.5 So $g(x) = (x-2)(2x^2 - 16x + 35)$
 (we need to see if $2x^2 - 16x + 35$ factors
 or see if it has any roots
 $0 = 2x^2 - 16x + 35$
 $0 = x^2 - 8x + 35/2$
 $+16 \quad +16$
 $16 = (x-4)^2 + 35/2$
 $-35/2 = (x-4)^2$
 2 \Rightarrow no real roots for this

5. Let f have the graph provided to the right.



(a) [2] What is the range of f ? *y-values* (+.5)

$(1, 2)$

(b) [2] Does f have an inverse?

Why or why not?

yes it passes the horizontal line test (+.5)

(c) [1] Find $f(1)$

3.8

(d) [3] (WebHW7 #15) Write the equation of the form $f(x) = a^x + b$ from the given graph.

(+.5) { passes thru (0, 2) so $2 = a^0 + b \Rightarrow 2 = 1 + b \Rightarrow b = 1$

(+.5) { passes thru (1, 3.8) so $3.8 = a^1 + 1 \Rightarrow 2.8 = a$

so $f(x) = 2.8^x + 1$

plug in points (+.5)

algebra (+.5)

(e) [2] (WebHW7 #15) Find $f(3)$

$$f(3) = 2.8^3 + 1 = 22.952$$

start (+.5)
plug in 3 (+.5)

6. Find all x that satisfy the following:

(a) [3] (WebHW8 #13)

$$4 \cdot 3^{2x-3} + 4 = 15$$

$$4 \cdot 3^{2x-3} = 11$$

$$3^{2x-3} = \frac{11}{4}$$

$$\log_3 3^{2x-3} = \log_3 \frac{11}{4}$$

$$4 \cdot 3^{2x-3} + 4 = 15$$

$$4 \cdot 3^{2x-3} = 11$$

$$3^{2x-3} = \frac{11}{4}$$

$$2x - 3 = \log_3 \frac{11}{4}$$

$$2x = \log_3 \frac{11}{4} + 3$$

$$x = \frac{\log_3 \frac{11}{4} + 3}{2}$$

$$\approx 1.96$$

(b) [4] (§3.4 #67)

$$\log_3(2x - 7) = 2 + \log_3(4x - 1)$$

$$\log_3(2x - 7) - \log_3(4x - 1) = 2$$

$$3 \log_3 \frac{2x-7}{4x-1} = 3^2 \Rightarrow \frac{2x-7}{4x-1} = 9$$

$$\frac{2x-7}{4x-1} = 9$$

$$2x-7 = 9(4x-1)$$

~~$$x = \frac{1}{7}$$~~

check
no solutions

$$\text{or } \log_3(2x-7) = \log_3 3^2 + \log_3(4x-1)$$

$$\log_3(2x-7) = \log_3 9(4x-1)$$

$$2x-7 = 9(4x-1)$$

$$2x-7 = 36x-9$$

$$2 = 34x$$

~~$$x = \frac{1}{17}$$~~

use correct (+.5)
order of ops (+.5)
inverse (+.5)

log properties (+.5)
use exp (+.5)
use sym (+.5)
order of ops (+.5)
check (+.5)

7. [3] (exp wks #2) Simplify: $\frac{(6st^3)^2}{2t^{-1}\sqrt{s}}$

$$\frac{6^2 s^2 (t^3)^2}{2 t^{-1} s^{\frac{1}{2}}}$$

$$\frac{36 s^2 t^6}{2 s^{\frac{1}{2}}}$$

$$\frac{18 s^{\frac{3}{2}} t^7}{1}$$

8. [4] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

(a) (§3.5 example 11) A star can be classified by its brightness relative to others. If two stars of magnitudes m_1 and m_2 have apparent brightness b_1 and b_2 , respectively, then $m_1 - m_2 = 2.5 \log \left(\frac{b_1}{b_2} \right)$

- [1] Find the relative ~~brightness~~ ^{magnitudes} of two stars that have the same level of brightness.
- [3] Find the magnitude m of a star that is 650 times as bright as one of magnitude 7.25.

(b) (§3.1 #92) Fidelity Federal offers three type of investments: (i) 9.7% compounded annually, (ii) 9.6% compounded monthly, and (iii) 9.5% compounded continuously. Which investments is the best deal?

a) i) if $b_1 = b_2$ then $m_1 - m_2 = 2.5 \log \frac{b_1}{b_2} = 2.5 \log(1) = 0$
 i.e. no relative brightness

ii) Given the brightness is $650 \cdot b_2$ we want to find m so that

$$m - 7.25 = 2.5 \log \frac{650 b_2}{b_2}$$

$$m = 7.25 + 2.5 \log 650$$

$$\approx 14.28$$

(b) Let's see which one returns more after investing \$100 for 1 year

- $100 (1 + \frac{.097}{1})^{11} = 109.70$
- $100 (1 + \frac{.096}{12})^{12 \cdot 1} = 110.03$
- $100 e^{.095 \cdot 1} = 109.97$

identify correct one

~~the option @ 9.7%~~
~~the option @ 9.5%~~
 Growth the second option @ 9.6%

9. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

(a) (Fall12 Exam2#9) Legislation passed last in 2010 to let the University of Washington (UW) raise its own tuition, but previous to that the state set the maximum percentage increase that tuition could be raised, call it r . Between '07 and '10, the University raised tuition by as much as the state would allow for each year. The '07 to '08 school year had a tuition of approximately \$6000. In the '10 school year the tuition was approximately \$8,700.

- [4] Find percentage rate that UW is raising its tuition every year?
- [2] If the University of Washington continues increasing tuition rates at the above fixed percentage, when will the cost of tuition exceed \$100,000?

(b) (Word Problem Wks #4) Chad just graduated but does not have a job lined up yet. In his last year of graduate school he made ends meet by using his credit card and now has a balance of \$8,000. His credit card compounds monthly with an annual interest rate of 21.9%. Assume the worst and that Chad won't find a job for the next two years and will have to move in with his parents. He won't be able to make any payments on his credit card bill.

Just before graduation, Chad received an ad for a State Farm Good Neighbor Visa Credit Card. The card will transfer his balance (with a 3.90% balance transfer fee) and then give him a lower annual interest rate of 15.2% that is still compounded monthly. Should he take this option instead of staying with his first credit card?

a) let t be years since the '07-'08 } #1

t	tuition
0	6000
1	
2	
3	8700

percentage increase
 \Rightarrow exponential growth
 $\Rightarrow P(1 + \frac{r}{n})^{nt}$ #1.5
 sta #1.5

b/c (0,6000) $\Rightarrow 6000 = P(1+r)^0 \Rightarrow P = 6000$ #1.5
 b/c (3,8700) $\Rightarrow 8700 = 6000(1+r)^3$ #1.5
 $\Rightarrow (1+r)^3 = \frac{87}{60} \Rightarrow 1+r = \sqrt[3]{\frac{87}{60}}$ #1
 $\Rightarrow r = \sqrt[3]{\frac{87}{60}} - 1$
 $\approx 13.385\%$
 so 13%

ii) find t so that $100000 = 6000(1.13)^t$ #1
 $\frac{50}{3} = 1.13^t$
 $\ln \frac{50}{3} = t \ln(1.13)$
 $t = \frac{\ln \frac{50}{3}}{\ln(1.13)} \approx 23.1$ years #1
 order of op #1
 0.2134
 1.220

(b) current credit card balance after 2 years.
 $8000(1 + \frac{.219}{12})^{12 \cdot 2}$
 $= \$12,347.93$
 using $P(1 + \frac{r}{n})^{nt}$ #1.5
 calculate #1.5

new credit card balance after 2 years.
 $(8000 + .039 \cdot 8000)(1 + \frac{.152}{12})^{12 \cdot 2}$
 $= 8312 \cdot 1.35 = \$11,243.51$
 using $P(1 + \frac{r}{n})^{nt}$ #1.5
 calculate #1.5
 Take the new card