

NAME:

Key

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let  $f$  be a function, and  $x$ ,  $y$ , and  $z$  be real numbers with  $z \neq 0$ .

T  F  $\frac{2}{a} + \frac{1}{a+1} = \frac{2+1}{a+1} + \frac{1}{a+1} = \frac{4}{a+1}$  all  $\frac{2}{a} + \frac{1}{a+1} = \frac{2a+1}{a(a+1)} + \frac{a}{a(a+1)}$

T  F  $f(x+1) = f(x) + 1$  ex let  $f(x) = x^2$   $f(x+1) = (x+1)^2$  vs  $f(x)+1 = x^2+1$

T  F  $(x+3)^2 = x^2 + 9$   $(x+3)^2 = (x+3)(x+3) = x^2 + 3x + 3x + 9$

T  F  $\sqrt{-1} = i$  b/c  $i^2 = -1$

T  F  $(2+3i)(1-i) = 2*1 + 3*(-1)i = 2 - 3i$

$(2+3i)(1-i) = 2 - 2i + 3i - 3i^2 = 2 + i - 3(-1)$

- T  F A fifth degree polynomial always has five complex roots.

Complex Theorem about complex #'s

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] (Practice Exam #2) Find any real or imaginary  $x$  such that  $\frac{3}{x+1} - \frac{x}{x+1} = \frac{4}{x}$ .

alg   factors   $\frac{3}{x+1} - \frac{x}{x+1} = \frac{4}{x}$

~~$\frac{3-x}{x+1} = \frac{4}{x} \quad x+1$~~

$3-x = \frac{4}{x}(x+1)$

$3x - x^2 = 4x + 4$

$0 = x^2 + x + 4$

$+1/4 \quad +1/4$

$1/4 = (x^2 + x + 1/4) + 4$

$1/4 = (x+1/2)^2 + 4$

$-3.75 = (x+0.5)^2$   
 $\pm\sqrt{-3.75} = x+0.5$

$x = -0.5 \pm \sqrt{-3.75}$

or  $-1/2 \pm i\sqrt{15/4}$

or  $-1/2 \pm i\frac{\sqrt{15}}{2}$

or  $\frac{-1 \pm i\sqrt{15}}{2}$

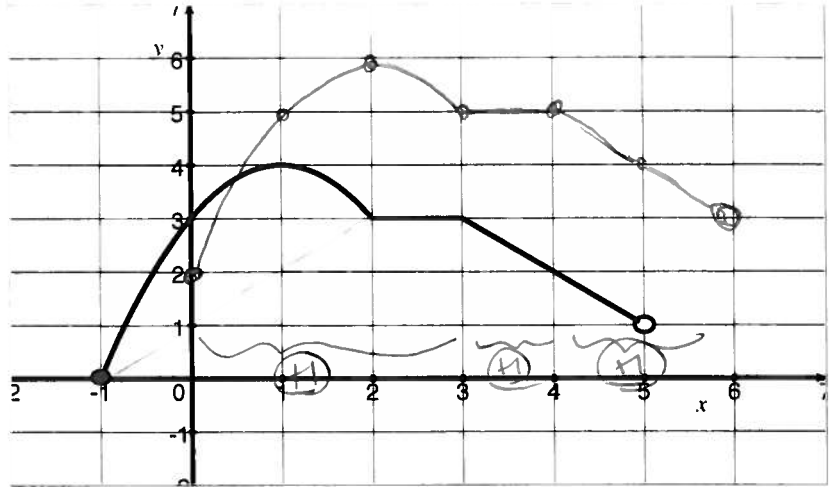
start  $+5$

solve the quadratic

$+1.5$

$+5$  know quadratic  
 $+5$  use quadratic formula  
 $-5$  both answers

3. Let  $f$  be the function comprised of two lines and a parabola that has only been shifted (not vertically stretched) and whose graph is below:



- (a) [4] (PracticeExam1 #3) Estimate the following if possible:

i.  $f(4) = 2$  (x1)

ii.  $\frac{6}{f(0)} + 3 = \frac{6}{3} + 3 = 2 + 3 = 5$  (x1)

iii.  $(f \circ f)(-1)$

$f(f(-1)) = f(0) = 3$  (x1)

- (b) [3] (§1.5 #18) Draw the graph of  $g$  if  $g(x) = f(x-1) + 2$ .

draw each

vert shift up by 2 + .5, horiz shift right by 1

- (c) [1] (§1.1 #48) Identify the  $y$  intercept.

@ 3 for  $f$  2 for  $g$  (take either)

- (d) [2] (Quiz2 #3) Find the average rate of change of  $f$  from  $x = -1$  to  $x = 2$

$$\frac{f(b)-f(a)}{b-a} = \frac{f(-1)-f(2)}{-1-2} = \frac{0-(3)}{-3} = 1$$

or slope of line connecting  $(-1, 0)$  to  $(2, 3)$   $\Rightarrow \frac{3}{3}$  or 1

- (e) [4] (PracticeExam#4) Find the piece-wise defined rule of  $f$  in the indicated form.

$$f(x) = \begin{cases} (x-1)^2 + 4 & \text{if } -1 \leq x < 2 \\ 3 & \text{if } 2 \leq x < 3 \\ -x + 6 & \text{if } 3 \leq x < 5 \end{cases}$$

form (x1)

(x1) parabola  $\Rightarrow$  quadratic vertex @  $(1, 4)$

horiz line @  $y = 3$

downward sloping line  $y = mx + b$

$m = \frac{-1}{1} = -1$  passes thru  $(4, 2)$  so

$2 = (-1)(4) + b \Rightarrow b = 6$  (x1)

so  $y = -x + 6$

so (x1)  $-(x-1)^2 + 4$  note: the directions said there was no vertical stretches

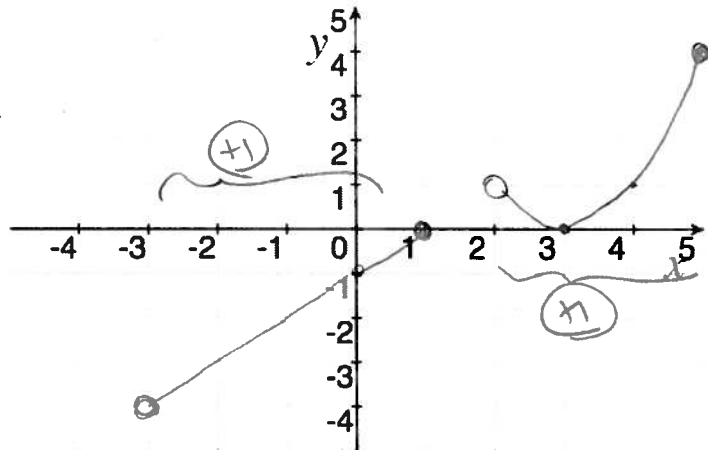
4. Let  $h$  be piece-wise defined by:

$$h(x) = \begin{cases} x - 1 & -3 < x \leq 1 \\ (x - 3)^2 & 2 < x \leq 5 \end{cases}$$

(a) [3] (WebHW1 #19)

Graph  $h$ .

endpoints (+)



(b) [2] (WebHW2 #11) Is  $h$  a function? Why or why not?

yes b/c the graph passes the vertical line test.

(c) [2] (TransformationWks #5) Find the domain of  $h$ .

$-3 < x \leq 1$  and  $2 < x \leq 5$  or  $(-3, 1] \cup (2, 5]$

(d) [2] (§1.1 #48) Identify the  $x$ -intercept(s).

$x = 1$  and  $x = 3$

(e) [1] (Quiz1 #3) Find all possible input(s) so that  $h(x) = -2$ .

i.e. find all inputs so that  $y = -2$   $x = -1$

5. (Exam1 Aut2013 #6) Let  $\alpha(x) = \frac{x}{x+2}$  and  $\beta(x) = 2 - \sqrt{x+1}$ . Both  $\alpha$  and  $\beta$  have inverses that exist.

(a) [4] (§1.7 #56) Find  $(\alpha^{-1})(x)$

$$\frac{y}{y+2} = x$$

$$\Rightarrow y = x(y+2)$$

$$y = xy + 2x$$

$$y - xy = 2x$$

$$y(1-x) = 2x$$

$$y = \frac{2x}{1-x}$$

order of op (+)  
algebra (+)  
+5 clear den  
+5 distrib

(b) [2] (§1.6 #38)  $(\alpha \circ \beta)(x)$ .

$$(\alpha \circ \beta)(x) = \alpha(\beta(x))$$

$$= \alpha(2 - \sqrt{x+1}) = \frac{2 - \sqrt{x+1}}{2 - \sqrt{x+1} + 2} \text{ or } \frac{2 - \sqrt{x+1}}{4 - \sqrt{x+1}}$$

(c) [1] (§1.7 #26)  $(\alpha \circ \alpha^{-1})(\pi)$ .

$\pi$  b/c  $\alpha$  and  $\alpha^{-1}$  undo each other.  
+5 if use formula for  $\alpha^{-1}$

6. [4] (PracticeExam #11) A rectangular box with a volume of  $60 \text{ ft}^3$  has a square base. Find a function that models its surface area  $S$  in terms of the length  $x$  of one side of its base.

start +.5

volume variable



Surface area = Area of bottom + Area of top + Area of front + Area of back + Area of right side + Area of left side

$$= x \cdot x + x \cdot x + xh + xh + xh + xh$$

$$= 2x^2 + 4xh$$

Volume that we know

$$x \cdot x \cdot h = 60 \text{ ft}^3$$

$$\Rightarrow h = \frac{60}{x \cdot x} = \frac{60}{x^2}$$

So Surface Area =  $2x^2 + 4xh$

$$= 2x^2 + 4x \left( \frac{60}{x^2} \right)$$

$$= 2x^2 + \frac{240}{x}$$

stop +.5

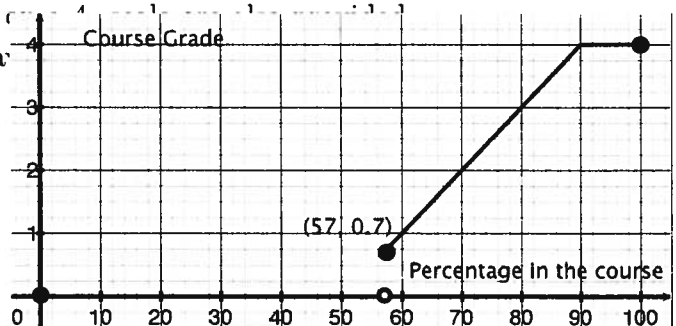
7. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit. No, doing both questions will not earn you extra credit.

(a) Zoe is going to check the accuracy of the speedometer in her car. Zoe turns on cruise control to 65 miles per hour. Zoe then uses a stop watch and determines it takes her 2 minutes to get from mile post 166 to 169. Is Zoe's speedometer correct? If not, is the speedometer over reporting or under reporting her actual speed?

(b) Leonard McCoy is in this course and during the seventh week he logged into MyMathLab and computed the average of his WebAssign, WrittenHW's, Quizzes, and Exams. The numbers are reported below.

McCoy's work did not change drastically in the remaining 3 weeks of the course and ultimately he earned a 3.0 in the course. What grade did he earn on the final exam to earn a 3.0 for the course? In case you don't remember, the weights specified in the syllabus and the graph of the function  $f$  that takes your class percentage  $x$  and returns your score

	weight	McCoy's a
Mini-Quizzes	5%	95%
WebAssign	10%	100%
WrittenHW	15%	95%
Quizzes	15%	65%
2 Exams	30%	70%
Final	25%	?



a) actual speed =  $\frac{\text{change in distance}}{\text{time}} = \frac{169 - 166 \text{ mi}}{2 \text{ min}} = \frac{3 \text{ mi}}{2 \text{ min}}$

$= \frac{3 \text{ mi}}{2 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 90 \frac{\text{mi}}{\text{hr}}$

dim analysis (1)  
pathway/plan (+.5)

speedometer is reporting 65 mi/hr

interpretation (+1)  
Dean - her speedometer is way off + in the direction that is going to get her in trouble!  
The speedometer is under reporting her speed. (+.5)

7b) Let  $x = \text{McCoy's \% on the final exam.}$  (+.5)

note earning a 30 in the course means McCoy's course % was 30 by looking @ the graph (+.5)

So (+.5)

$$5 \cdot .95 + 10 \cdot 1.00 + 15 \cdot .95 + 15 \cdot .65 + 30 \cdot .70 + 25 \cdot x = 80$$

$$\Rightarrow 4.75 + 10 + 14.25 + 9.75 + 21 + 25x = 80$$

$$\Rightarrow 59.75 + 25x = 80$$

$$\Rightarrow 25x = 20.25$$

$$\Rightarrow x = .81$$

so McCoy earned 81% on the final exam.

arithmetic (+1)  
pathway/plan (+.5)

