Exam 1 | TMath 120 | Practice

TMath 120

Key

NAME: This is a sample exam to be used for practice. This is *not* a template for the exam that will be given in class. Many of the questions on the exam will look quite different than those appearing here.

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

   T F $\frac{3x+y}{3z} = \frac{x+y}{z}$ $\qquad \frac{x+y}{z} = \frac{3(x+y)}{3z} = \frac{3x+3y}{3z}$

   T F $(x+y)^2 = x^2 + y^2$ $\qquad (x+y)^2 = (x+y)(x+y) = x^2 + xy + yx + y^2 = x^2 + 2xy + y^2$

   T F $|x| = x$ $\qquad$ let $x = -2$ then $|-2| = 2 \neq -2$

   T F $\dfrac{3+5i}{1-2i} = -\dfrac{7}{5} + \dfrac{11}{5}i$ $\qquad \frac{3+5i}{1-2i}\cdot\frac{1+2i}{1+2i} = \frac{3+6i+5i+10i^2}{1+2i-2i-4i^2} = \frac{3+11i-10}{1-4(-1)}$

   T F A cubic polynomial always has three complex roots.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. Find any real or imaginary x such that $3(7+x)^2 + 4 = 2$.

   $-4 \quad -4$

   $\frac{3(7+x)^2}{3} = \frac{-2}{3}$

   $(7+x)^2 = -2/3$

   $7 + x = \pm\sqrt{-2/3}$

   $x = -7 \pm \sqrt{-2/3}$

   $= -7 \pm i\sqrt{2/3}$

3. Find any real or imaginary x such that $\dfrac{1}{x+1} + \dfrac{1}{2} = \dfrac{1}{x+3}$.

   $\frac{1}{x+1} + \frac{1}{2} = \frac{1}{x+3}$

   $\frac{2}{2(x+1)} + \frac{x+1}{2(x+1)} = \frac{1}{x+3}$

   $\frac{2+x+1}{2(x+1)} = \frac{1}{x+3}$

   $(x+3)\,\frac{x+3}{2(x+1)} = \frac{1}{\cancel{x+3}}\,\cancel{(x+3)}$

   $\frac{\cancel{2(x+1)}(x+3)(x+3)}{2\cancel{(x+1)}} = 1 \quad 2(x+1)$

   1 $x^2 + 6x + 9 = 2x + 2$

   $x^2 + 4x + 7 = 0$

   $x^2 + 4x + 4 + 7 = 4$

   $(x+2)^2 + \cancel{7} = 4$

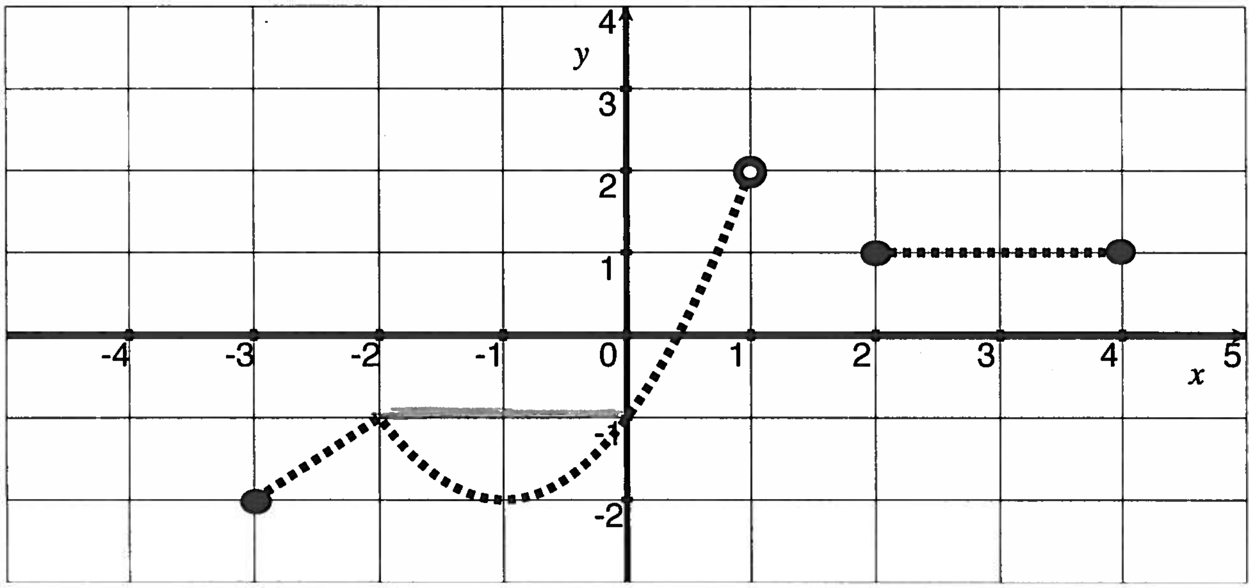
   $-\cancel{7} \quad -7$

   $(x+2)^2 = -3$

   $x + 2 = \pm\sqrt{-3}$

   $x = -2 \pm i\sqrt{3}$

4. [4] Let f be the function comprised of two lines and a parabola that has only been shifted (not vertically stretched) and whose graph is below:



Estimate the following *if* possible:

$f(-3) = -2$

$$\frac{f(-3)-1}{f(-1)} = \frac{-2-1}{-2} = \frac{-3}{-2} = 3/2$$

$f(1)$ is not defined

$(f \circ f)(0) = f(f(0))$
$= f(-1)$
$= -2$

$f(-1)f(2)$

$-2 \cdot 1 = -2$

$f(-1-2) = f(-3) = -2$

The average rate of change of f from $x = -2$ to $x = 0$

$$\frac{f(-2)-f(0)}{-2-0} = \frac{(-1)-(-1)}{-2} = \frac{-1+1}{-2} = 0$$

i.e. slope of line drawn above

The piece-wise defined rule of f:

$$f(x) = \begin{cases} x+1 & \text{if } -3 \le x < -2 \\ (x+1)^2 - 2 & \text{if } -2 \le x < 1 \\ 1 & \text{if } 2 \le x \le 4 \end{cases}$$

5. Let $\alpha(x) = \frac{1}{x-2}$ and $\beta(x) = \frac{\sqrt{x+4}}{x}$.

(a) Find the domain of β.

all x but when den = 0 or when stuff under sqrt < 0

so all ~~all~~ x but x = 0 or when $x + 4 < 0$

$\Rightarrow x < -4$

So $x \neq 0$ and $x \geq -4$ or $(-4,0) \cup (0,\infty)$

(b) Find the rule of $\beta \circ \alpha$. Simplify.

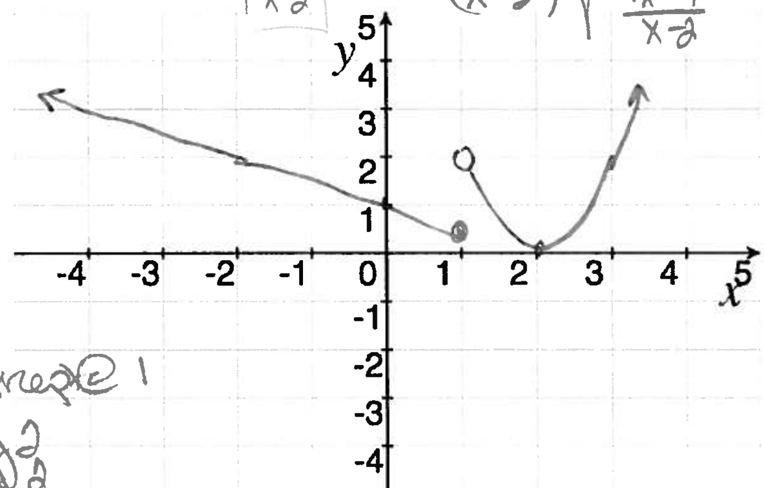
$(\beta\circ\alpha)(x) = \beta(\alpha(x))$

$= \beta\left(\frac{1}{x-2}\right) = \dfrac{\sqrt{\frac{1}{x-2}+4}}{\frac{1}{x-2}} = \sqrt{\frac{1}{x-2} + \frac{4}{1}\cdot\frac{x-2}{x-2}} \div \frac{1}{x-2}$

$= (x-2)\sqrt{\frac{4x-7}{x-2}}$

6. Let h be the *function* defined by:

$$h(x) = \begin{cases} -\frac{1}{2}x + 1 & x \leq 1 \\ 2(x-2)^2 & 1 < x \end{cases}$$



(a) Graph h. (Explaining graph transformations is worth partial credit.)

line w/ slope $-\frac{1}{2}$ + y intercept @ 1

parabola stretch vert by 2 + shifted right 2

(b) What are the coordinates of the vertex on the piece of the graph above that is a parabola?

(2, 0)

(c) Identify the x-intercept(s).

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(d) Find all possible input(s) so that $h(x) = 1$.

when $x = 0$ and when $1 = 2(x-2)^2 \Rightarrow \frac{1}{2} = (x-2)^2 \Rightarrow \pm\sqrt{\frac{1}{2}} = x - 2$, $x = 2 \pm \frac{1}{\sqrt{2}}$

(e) What is the range h?

$[0, \infty)$

(f) On what interval(s) is h increasing?

$(2, \infty)$