#3a-7 were unclear
Poden reducing fractions $\frac{3x}{3x+2} \neq \frac{x}{x+2}$

Exam 1

TMath 120

Fall 2013

NAME: Key

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

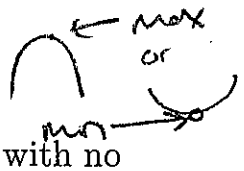
T (F) $\frac{1}{x} + \frac{2}{x+1} = \frac{4}{x+1}$ $x(\frac{1}{x} + \frac{2}{x+1}) = x(\frac{x+1+2x}{x(x+1)}) = \frac{3x+1}{x(x+1)}$

T (F) $(f \circ f)(x) = f(x) \cdot f(x)$ $(f \circ f)(x) = f(f(x))$ composition
↳ mult

T (F) The lines defined by $3x - 7 = y$ and $2y + 6x = 5$ are parallel. $2y = -6x + 5 \Rightarrow y = -3x + \frac{5}{2}$

(T) F $(2+i) - (5-i) = -3+2i$ $2+i-5+i = -3+2i$

(T) F The vertex is the extrema (min or max) of a quadratic function.



Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] (Practice Exam #2) Find any real or imaginary x such that $\frac{1}{3} - \frac{1}{x-2} = \frac{1}{x}$.

$x-2\ \frac{1}{3} - \frac{1}{x-2}3 = \frac{1}{x}$

Start (1.5)

$\frac{x-2}{3(x-2)} - \frac{3}{3(x-2)} = \frac{1}{x}$

Factor addition (+1)

Legal Denoms (+1)

$\frac{x-2-3}{3(x-2)} = \frac{1}{x}$

$x\ \frac{x-5}{3x-6} = \frac{1}{x}$

$3x-6\ \frac{x^2-5x}{3x-6} = 1\ 3x-6$

$x^2 - 5x = 3x - 6$
$-3x \quad -3x$

$x^2 - 8x = -6$

$+(\frac{8}{2})^2 \quad +(\frac{8}{2})^2$

$x^2 - 8x + 16 = -6 + 16$

$(x-4)^2 = 10$

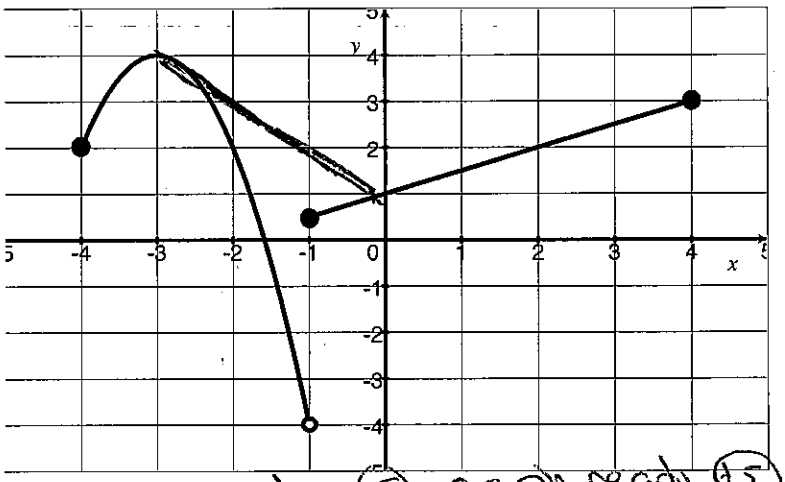
$x - 4 = \pm\sqrt{10}$

$x = 4 \pm \sqrt{10}$

solving (+1)

both answers (+.5)

3. Let f be the piece-wise defined function comprised a line and a parabola whose graph is below:



(a) Estimate the following *if* possible:

i. [1] (§1.3 #56) $f(4) = 3$

ii. [1] (WebHW1 #16) the y-intercept 1

iii. [2] (PracticeExam #4) $(f \circ f)(-3) = f(f(-3))$
$= f(4) = 3$ (+.5) (+.5)

composition (5) graph ready (+.5)

(b) [1] (transform Wks #5) Find the range of f.
i.e. what the y-coord range from & to
$(-4, 4]$

(c) [2] (PracticeExam #6) Find all possible input(s) so that $f(x) = 2$.
if $x = -4, -2$ or 2 (+.5) (+.5) (+.5) then $f(x) = 2$
graph ready (+.5)

(d) [2] (Inverse Wks #3) Does f have an inverse? Why or why not?
no? (+1) since $f(-4) = 2 = f(-2)$ (+1)
so the inverse would have to send 2 to both places?

(e) [2] (Quiz2 #3) Find the average rate of change of f as x changes from -3 to 0.
i.e. the slope of the above (+1)
i.e. $-3/3 = -1$ (+1)
or $\frac{f(-3) - f(0)}{-3 - 0} = \frac{4 - 1}{-3} = \frac{3}{-3} = -1$ (+1)

(f) [5] (PracticeExam #4) Find the formula for f in the indicated form:

$$f(x) = \begin{cases} -2(x+3)^2 + 4 & \text{if } -4 \le x < -1 \\ \frac{1}{2}x + 1 & \text{if } -1 \le x \le 4 \end{cases}$$

quad (+.5)

if $-4 \le x < -1$

parabola w/ vertex $(-3, 4)$ (+.5)
so $y = a(x+3)^2 + 4$ (+1)
passes thru $(-4, 2)$ so
$2 = a(-4+3)^2 + 4$
$\Rightarrow -2 = a(1)$
so $y = -2(x+3)^2 + 4$
sign of a (+.5) (+.5) 2

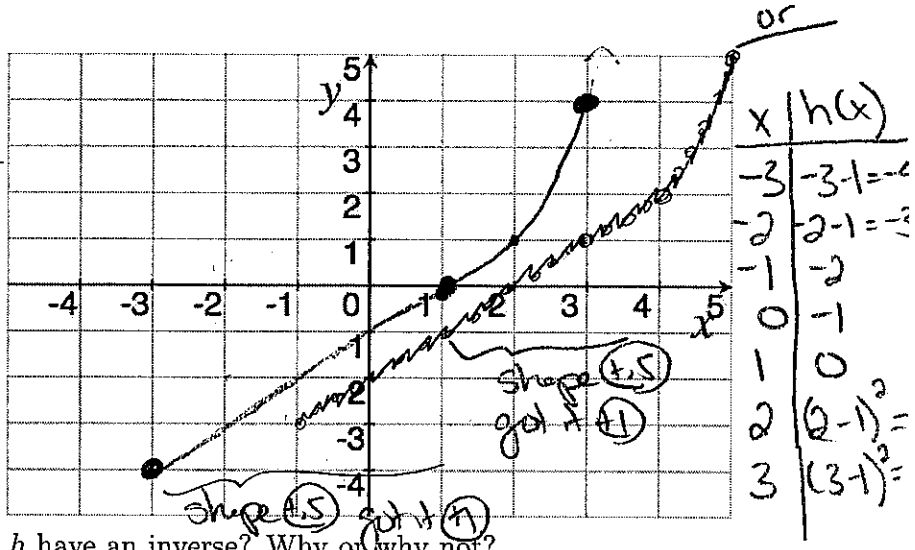
line (+.5)

if $-1 \le x \le 4$

slope $= \frac{\text{rise}}{\text{run}} = \frac{1}{2}$ (+.5)
y-intercept $= 1$
so $y = \frac{1}{2}x + 1$ (+.5) (+.5)

4. Let h be the *function* defined by:

$$h(x) = \begin{cases} x-1 & -3 \le x \le 1 \\ (x-1)^2 & 1 < x \le 3 \end{cases}$$



(a) [3] (WebHW1 #19) Graph h.

note $(x-1)^2$ is x^2 shifted right one unit. partial +.5

(b) [2] (InverseWks #2) Does h have an inverse? Why or why not?

yes? +1 no output is mapped to by more than 1 input +1

or

it passes the horizontal line test

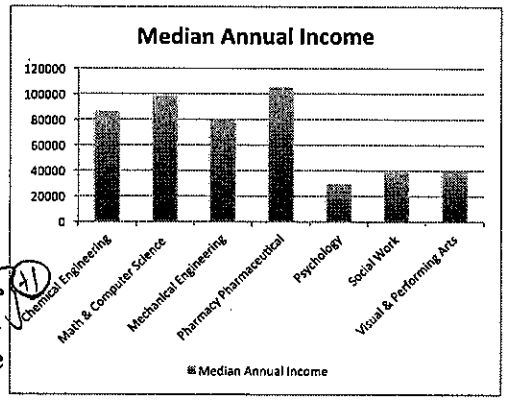
(c) [3] (WebHW3 #6) The function m is defined by $m(x) = h(x-2)+1$. Graph m on the above set of axes.

note: it does not matter if you do the horiz or vert shifts first.

+1 shift right 2 units +1.5 vertical shift up one unit +1.5

did it +1

5. (PracitceExam #9) Let the domain of f be undergraduate majors and $f(x)$ be the median annual earnings of people with the the undergraduate major x.



(a) [2] Is f a function? Why are why not?

yes - each undergrad major corresponds +1 to only one median annual income +1

(b) [4] Some data of f is shown in the graph on the right, what is f(Psychology) and what does it mean?

f(Psychology) = 30,000 +1

This means the median annual income for people with an undergrad major +1 in Psychology is about +1 +3 $30,000. +1

6. Let $\alpha(x) = \dfrac{x}{x+2}$ and $\beta(x) = 2 - \sqrt{x+1}$. Both α and β have inverses that exist.

(a) [2] (§1.6 #20) Find $(\alpha - \beta)(x)$

distribute neg (+1)

$\alpha(x) - \beta(x)$

$\dfrac{x}{x+2} - (2 - \sqrt{x+1}) = \dfrac{x}{x+2} - 2 + \sqrt{x+1}$

(+.5) (+.5)

(b) [2] (§1.6 #38) $(\alpha \circ \beta)(x)$.

$(\alpha \circ \beta)(x) = \alpha(\beta(x)) = \alpha(2 - \sqrt{x+1})$

(+.5) (+.5)

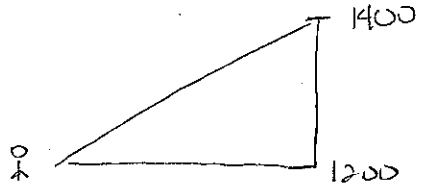
$= \dfrac{2 - \sqrt{x+1}}{2 - \sqrt{x+1} + 2} = \dfrac{2 - \sqrt{x+1}}{4 - \sqrt{x+1}}$

(+.5)

(c) [1] (§1.7 #26) $(\alpha \circ \alpha^{-1})(\pi)$.

π. because α^{-1} 'undoes' α?

7. [1] (Word Problem Wks) A path that ~~is~~ would be 3 miles long if on flat ground starts at 1200 ft and ends at 1400 ft above sea level. How steep the assent (slope/average rate of elevation gain) of the path?.



$\dfrac{\text{rise}}{\text{run}} = \dfrac{1400 - 1200}{3\text{mi}} = \dfrac{200}{3}$ ft/mi

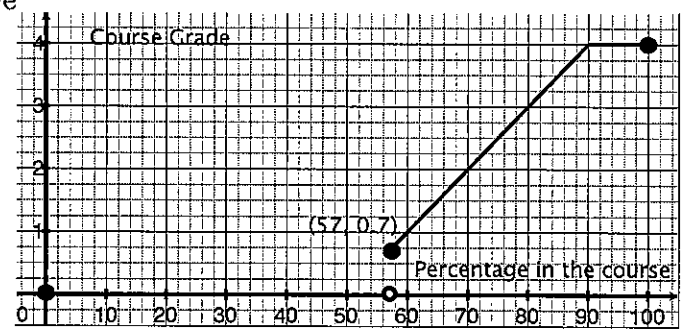
$= 66.\overline{6}$ ft/mi

8. [5] (Word Problem Wks) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) Seismic waves travel at about 4km/s but Megan has (a really fast!) carrier pigeon that travels 6km/s. Assume that Megan's first instinct when feeling a quake is to "tweet" the experience and that process (recognizing it's an earthquake, finding her carrier pigeon, attaching a message to the bird's leg, and the message being broadcast) takes 8 minutes. How far does a Megan follower have to be from Megan to know there is an earthquake before feeling it? (inspired by http://xkcd.com/723)

(b) James T. Kirk is in this course and would like to know if it is still possible to earn a 2.0 now that he's taken two exams. He has looked at the gradebook on ~~WebAssign~~ MyMathLab and has computed the averages listed below.
Assuming James' work does not drastically change in the remaining 3 weeks and his averages remain about the same, find what grade he needs to get on the final to receive a 2.0 in the course. In case you don't remember, the weights specified in the syllabus and the graph of the function f that takes your class percentage x and returns your score on a 4. scale are also provided.

	weight	James' ave
Mini-Quizzes	5%	95%
WebAssign	10%	10%
WrittenHW	15%	0%
Quizzes	15%	70%
2 Exams	30%	100%
Final	25%	



start (+.5)

a) let t_p be the time the pigeon is in flight
& let t_w be the time the wave has been traveling. } def. variables (+1)

We want to know when does the pigeon catch up to the wave?
(+.5) we want the time so distance traveled by wave = distance traveled by pige

$d = r \cdot t$ (+5)

$4\ km/s \cdot t_w\ s = 6\ km/s \cdot t_p\ s$

(+5) { Note $t_w = t_p + 8\ min = t_p + 8\ min \cdot \frac{60s}{1\ min} = t_p + 480$

So $4(t_p + 480) = 6t_p$

alg (+1) $\Rightarrow 4t_p + 1920 = 6t_p$

$\Rightarrow 1920 = 2t_p$

$\Rightarrow t_p = 960s$

so 960 seconds after the pigeon has been released - it catches up to the wave so.

interpret (+1) Followers need to be greater than $6 \cdot 960\ km = 5760\ km$ away.

start +.5 def variables +1

(b) To earn a 2.0 in the course Kirk needs a 70% according to the graph given +.5

So far Kirk has earned:

+.5

95% of the 5 pts for mini quizzes	$0.95 \cdot 5$
10% of the 10 pts for webwk	$0.10 \cdot 10$
0% of the 15 for written hw	$0.0 \cdot 15$
70% of the 15 for quizzes and	$0.70 \cdot 15$
100% of the 30 for exams	$1.00 \cdot 30$
+ an unknown x% of the 25 for the final	$\frac{x}{100} \cdot 25$ +

So Kirk needs to find the x% so that

+.5 $.95 \cdot 5 + .1 \cdot 10 + 0 \cdot 15 + .7 \cdot 15 + 1 \cdot 30 + \frac{x}{100} \cdot 25 = 70$

$4.75 + 1 + 0 + 10.5 + 30 + \frac{x}{4} = 70$

$46.25 + \frac{x}{4} = 70$

$-46.25 \quad -46.25$

$\frac{x}{4} = 23.75$

$\Rightarrow x = 4 \cdot 23.75$

$= 95\%$

+1 +1

interpret correctly +1

yes it is still possible.

note: he really should have done his homework!