EXAM 2

TMath 120

Fall 2013

NAME: Key

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

   (T) F $\pi x^3 - (\sqrt{5})x + 3$ is a polynomial.

   T (F) $\frac{1}{x} + \frac{2}{x+1} = \frac{4}{x+1}$ $\quad \frac{1}{x} + \frac{2}{x+1} = \frac{(x+1)(1)}{x(x+1)} + \frac{2(x)}{(x+1)(x)} = \frac{x+1+2x}{x(x+1)}$

   T (F) $\frac{-2^{-2}}{6^{-1}} = \frac{3}{2}$ $\quad \frac{-2^{-2}}{6^{-1}} = \frac{-6}{2^2} = \frac{-6}{4} = \frac{-3}{2}$ missing negative sign

   T (F) $\log(u + v) = \log(u) + \log(v)$ $\quad \log(u)+\log(v) = \log(u \cdot v)$

   (T) F $\log_6 36 = 2$ $\quad \log_6 36 = \log_6(6^2) = 2$ or $\log_6 36 = \frac{\log 36}{\log 6}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [5] (PracticeExam #6) The function below "passes the horizontal line test" so it has an inverse, find the formula for the inverse.

$$y = \log_3\left(\frac{2x}{x-5}\right)$$ (+.5) start

$x = \log_3\left(\frac{2y}{y-5}\right)$ swap x's & y's (+1)

$3^x = \left(\frac{2y}{y-5}\right)$ def. of log/cancel prop (+1)

$y-5 \quad 3^x = \frac{2y}{y-5}$ ~~$y-5$~~ order of operation (+1)

notation (+.5)

$3^x y - 5 \cdot 3^x = 2y$ algebra (+1)

$3^x y - 2y = 5 \cdot 3^x$

$y(3^x - 2) = 5 \cdot 3^x$

So $y = \frac{5 \cdot 3^x}{3^x - 2}$

$((x^3)^2 = x^3 x^3 = xxx\ xxx = x^6$

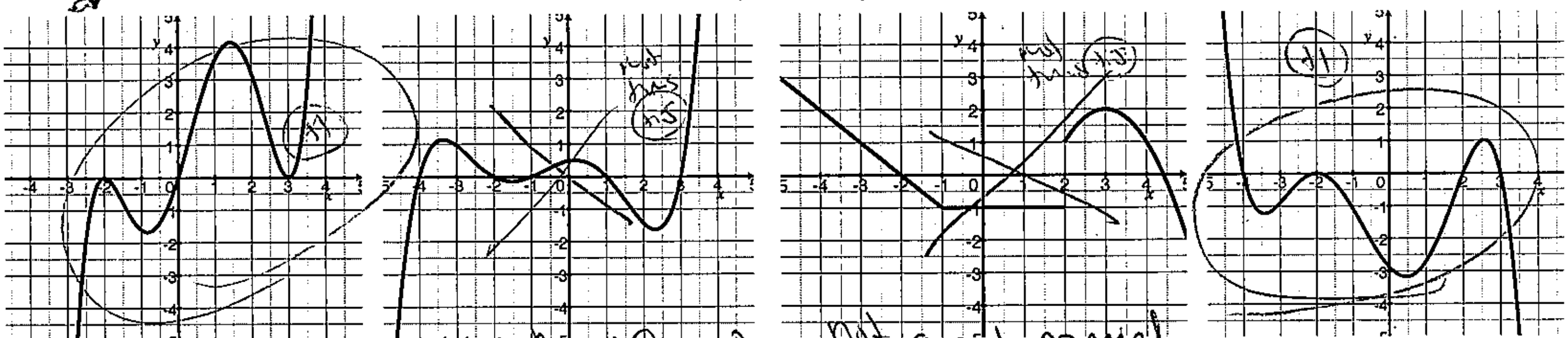
3. [4] (ExponentWks #2) Simplify: $\dfrac{(2zx^3)^2}{10x^{-1}\sqrt{z}}$ $= \dfrac{2^2 z^2 (x^3)^2}{10x^{-1} z^{1/2}}$ dist (+1) (+.5)

$= \dfrac{4z^2x^6}{10x^{-1}z^{1/2}}$ (+.5) $= \dfrac{2}{5} z^{2-1/2} x^{6--1}$ (+.5) (+1) alg/frac (+.5)

$= \frac{2}{5} z^{3/2} x^7$

Note: on students copy an extra curve appeared => not function?

4. [3] (WebHW6 #12) Let h be a 5^{th} degree polynomial that has $(x+2)^2$ as a factor (but $(x+2)^3$ is not a factor). Which of the following could be the graph of h? (Circle all that are possible.) x+2 is a factor => −2 is a root



5. [4] (WebHW6 #16) Let $f(x) = x^3 - x^2 - 7x + 3$. Use the fact that the polynomial $x^2 + 2x - 1$ is a factor of $f(x)$ to find all the real roots of $f(x)$.
Remember what a factor is!! For example, 4 is a factor of 12 because $12 = 4 * 3$!!
Note: using your calculator to find the roots is *not enough* to earn full marks here!!

(+.5) Recall a root/zero/x-intercept of f is when y=0 so we want to find x so that

$x^3 - x^2 - 7x + 3 = 0$

Since x^2+2x-1 is a factor we know

$(x^2+2x-1) \cdot \text{something} = x^3 - x^2 - 7x + 3$

$\Rightarrow \text{something} = \dfrac{x^3 - x^2 - 7x + 3}{x^2+2x-1}$ so we can use long division.

$x^2+2x-1 \overline{)\, x^3 - x^2 - 7x + 3}$ quotient $x - 3$, R 0
$-(x^3 + 2x^2 - x)$
$-3x^2 - 6x + 3$
$-(-3x^2 - 6x + 3)$

start long ÷ (+.5) algorithm (+1) got it (+.5)

So $(x^2+2x-1)(x-3) = x^3 - x^2 - 7x + 3$. Back to finding the roots.

Find x so that

$(x^2+2x-1)(x-3) = 0$

$\Rightarrow x^2+2x-1 = 0$ or $x - 3 = 0$

$(x^2+2x+1) - 1 - 1 = 0$ or $x = 3$ (+.5)

$(x+1)^2 - 2 = 0$

$x + 1 = \pm\sqrt{2}$

$x = \pm\sqrt{2} - 1$

(completed the □)

6. Find all x that satisfy the following:

(a) [3] (§3.4 #29) $3 \cdot 4^{2x-1} + 4 = 14$

(+1) order of operations
(+.5) used log
(+1) used log prop correctly
(+.5) alg

$-4 \quad -4$

$\frac{3 \cdot 4^{2x-1}}{3} = \frac{10}{3}$

$4^{2x-1} = 10/3$

$\log_4(10/3) = 2x - 1$

$\log_4(10/3) + 1 = 2x$

$x = \frac{\log_4(10/3)+1}{2}$ check ✓

(b) [3] (10/29 lecture) $\log(x+1) + \log(x-1) = 0$

(+.5) order of op
(+.5) use exp
(+1) used prop correctly
(+.5) alg
(+.5) checked answers

$\log[(x+1)(x-1)] = 0$

$10^0 = (x+1)(x-1)$

$1 = x^2 - 1$

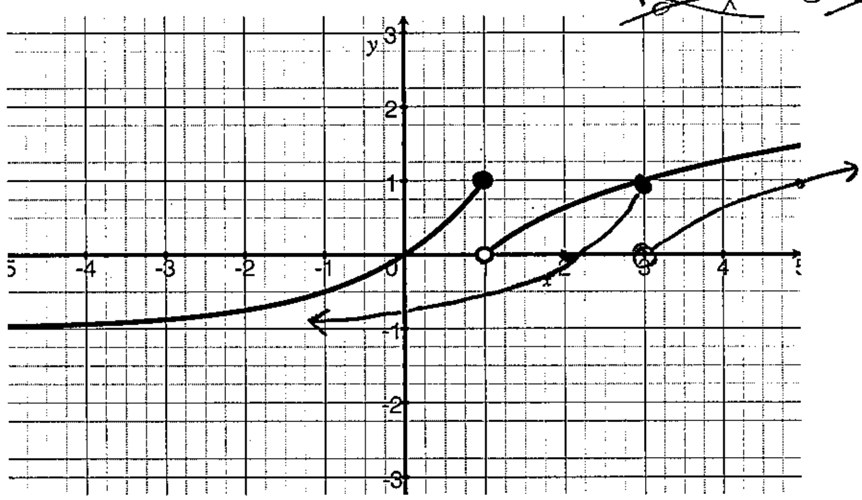
$+1 \quad +1$

$2 = x^2$

$\sqrt{2} = x$ check ✓

or ~~$-\sqrt{2} = x$ check~~

7. Let f be the function whose graph is given to the right. The function f is of the form $a^x + c$ when $x \le 1$ and $\log_b(x)$ when $1 < x$.



(a) [2] (WebHW7 #22) Sketch the graph of $f(x-2)$.

horizontal shift (+.5) to the right (+.5) two units (+.5) did it (+.5)

(b) [5] (§3.2 #62 &§3.1 #54) Find a formula for f in the indicated form. Recall! The function f is of the form $a^x + c$ when $x \le 1$ and $\log_b(x)$ when $1 < x$.

$$f(x) = \begin{cases} 2^x - 1 & x \le 1 \\ \log_3 x & 1 < x \end{cases}$$

passes thru (0,0) so
$a^0 + c = 0$
$\Rightarrow 1 + c = 0 \Rightarrow c = -1$
passes thru (1,1) so
$a^1 - 1 = 1$
$\Rightarrow a = 2$

form (+.5)
(+.5) graph reading
(+.5) arithmetic
c (+.5)
a (+.5)

form (+.5)
graph reading (+.5)
arithmetic (+.5)
log def/prop (+.5)
got it (+.5)

passes thru (3,1)
$\log_b 3 = 1$
$\Rightarrow b^1 = 3$
$\Rightarrow b = 3$

8. [3] Create a word problem that makes use of either an exponential or logarithmic function.

9. [3] (§3.3 #13) Given that $\log x = 2$, $\log y = 3$, and $\log 2 \approx 0.3$, evaluate: $\log(2x^2y)$.

$\log[(2)(x^2 y)]$

$= \log 2 + \log(x^2 y)$

$= \log 2 + (\log x^2 + \log y)$

$\rightarrow \log 2 + \log x^2 + \log y$

$\log 2 + 2\log x + \log y$

$0.3 + 2 \cdot 2 + 3$

$.3 + 4 + 3 = 7.3$

start (+.5)
prop log (+.5)
prop correct (+1.5)
got it (+.5)

10. [4] (§3.1 #92) Fidelity Federal offers three type of investments: (i) 9.7% compounded annually, (ii) 9.6% compounded monthly, and (iii) 9.5% compounded continuously. Which investments is the best deal?

Let P_0 be the initial investment. Deal

(i) returns $P_0 1.097 = P_0(1+.097)^1$ after 1 year. formula (+.5) correct formula (+.5)

best → (ii) returns $P_0 1.1003 \approx P_0\left(1+\frac{.096}{12}\right)^{12}$ after 1 year. formula (+.5) correct formula (+.5)

(iii) returns $\approx P_0 e^{.095 \cdot 1}$ after 1 year. $P_0 1.0997$ formula (+.5) correct formula (+.5)

(+1) To determine which is a better deal we need to compute which underlined value is the biggest. (note: you could choose to let P_0 be \$100 or something and compare values that way too.)

11. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want considered for credit.

(a) (§3.2 #106) In a lake, $\frac{1}{4}$ of the water is replaced by clean water every year. Sixteen thousand cubic meters of soluable toxic chemical spill takes place in the lake.

start (+.5)
variables (+.5)

i. How much toxin will be left after 12 years?

ii. When will 80% of the toxin be eliminated?

(b) (Word Problem Wks #10) Recall from class that pH is measured on a logarithmic scale and that the pH level of a substance can be computed by $pH = -\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions measured in moles per liter (M). Assume that the white vinegar in this problem has a pH level of 2.5 and your stomach acid has a pH level of 1.6.

i. How many times stronger is stomach acid than the white vinegar?

ii. If you found a substance X whose pH level is 1.5 more than the pH value of vinegar. How are the concentration of hydrogen ions in X and vinegar related?

(a)

time (years)	amount of toxins in lake
0	$16{,}000\ m^3$
1	$16{,}000 - \frac{1}{4}\cdot 16{,}000$ (quarter of toxin in the lake water that was replaced.) $16000(1-\frac{1}{4}) = 16000\cdot\frac{3}{4}$
2	$16000(\frac{3}{4}) - \frac{1}{4}[16000\cdot\frac{3}{4}]$ $16000(\frac{3}{4})(1-\frac{1}{4}) = 16000\cdot\frac{3}{4}^2$
⇒ n	$16000(\frac{3}{4})^n$ (+.5)

(+2) +.5 exp

i) when $n = 12$ (+.5) there will be approx $16000(\frac{3}{4})^{12}\ m^3 \approx$ (+.5) alg/order of op (+.5)

ii) find n so that $.8\cdot 16000 = 16000(\frac{3}{4})^n$

$\Rightarrow .8 = (\frac{3}{4})^n \Rightarrow \ln .8 = n\ln(\frac{3}{4})$

$\Rightarrow n = \frac{\ln .8}{\ln .75} \approx$ (+.5) use logs

alg/order of op (+1)

(b) let $[H^+]_s$ be the concentration of hydrogen ions in stomach acid

$[H^+]_v$ be the concentration for vinegar.

i) (+1) $? \cdot [H^+]_v = [H^+]_s$ or $? = \frac{[H^+]_s}{[H^+]_v}$

(+1.5) Since $1.6 = -\log[H^+]_s \Rightarrow -1.6 = \log[H^+]_s$
$\Rightarrow 10^{-1.6} = [H^+]_s$

Since $2.5 = -\log[H^+]_v \Rightarrow -2.5 = \log[H^+]_v$
$\Rightarrow 10^{-2.5} = [H^+]_v$

(+.5) So $[H^+]_s / [H^+]_v = \frac{10^{-1.6}}{10^{-2.5}} = 10^{.9} \approx 7.9$

ii) let $[H^+]_x$ (+1) be the consentation for X

(+1) log prop $pH_x = pH_v + 1.5 \Rightarrow -\log[H^+]_x = -\log[H^+]_v$ ...

$\Rightarrow \log[H^+]_v - \log[H^+]_x = 1.5 \Rightarrow \log\frac{[H]_v}{[H]_x} = 1.5$

$\Rightarrow 10^{1.5} = \frac{[H]_v}{[H]_x}$ about $10^{1.5}$ times less hydrogen ions ...