Quiz 6 Key

Show *all* your work. No credit is given without reasonable supporting work. There are *two* sides to this quiz.

1. [2] (WebHW15 #4 &§7.2 #4) TRUE/FALSE: Circle T in each of the following cases if the statement is true for all θ and ϕ in the domain. Otherwise, circle F.

   T (F) $\dfrac{\sec\theta - \cos\theta}{\tan\theta} = \csc\theta$

   $\hookrightarrow \dfrac{\frac{1}{\cos\theta} - \cos\theta}{\frac{\sin\theta}{\cos\theta}}\left(\dfrac{\cos\theta}{\cos\theta}\right) = \dfrac{\cos\theta - \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1-\cos^2\theta}{\sin\theta} = \dfrac{\sin^2\theta}{\sin\theta} = \sin\theta$

   (T) F $\sin(\phi+\theta) = \cos\phi\sin\theta + \sin\phi\cos\theta$

   additive identity of sin

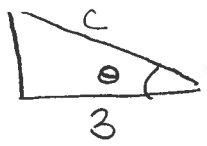
2. [3] (§7.3 #2) Given $\frac{\pi}{2} \le \theta \le \pi$ and $\tan\theta = \frac{-4}{3}$. Find the exact value of $\sin 2\theta$.

   $\sin 2\theta = \sin(\theta+\theta) = \cos\theta\sin\theta + \sin\theta\cos\theta$ (+.5)
   $= \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$ (+.5)
   $= \frac{-12}{25} + \frac{-12}{25} = \frac{-24}{25}$

   finding $\cos\theta$ and $\sin\theta$:

   Method 1)

   (.5) $\tan\theta = -\frac{4}{3}$

   $\Rightarrow 3^2 + 4^2 = c^2 \Rightarrow c = 5$

   (.5) So $\sin\theta = \frac{4}{5}$

   (.5) $\cos\theta = -\frac{3}{5}$

   sign (+.5)

   Method 2)

   $\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \Rightarrow \tan^2\theta + 1 = \sec^2\theta$ (+.5)

   So $\sec^2\theta = \left(-\frac{4}{3}\right)^2 + 1 = \frac{16}{9} + 1 = \frac{25}{9}$

   $\Rightarrow \sec\theta = \pm 5/3 \Rightarrow \cos\theta = \pm 3/5$ (+.5)

   b/c ~~$\cos\theta = 3/5$~~ $\cos\theta = -3/5$ sign (+.5)

   Recall $\sin^2\theta + \cos^2\theta = 1$

   So $\sin^2\theta = 1 - \left(-\frac{3}{5}\right)^2 = \frac{16}{25}$

   $\Rightarrow \sin\theta = \pm\frac{4}{5}$ b/c ~~$\sin\theta = -4/5$~~ (+.5)

   $\sin\theta = 4/5$

3. [3] (WebHW14a #6) Write an equation that represents the curve below in the form:

$$y = a\cos k(x - b)$$

Answer 1)

amplitude = $\frac{1}{8} = a$

period = $2\pi \Rightarrow K = 1$

phase shift = $-\frac{\pi}{2}$

$\frac{1}{8}\cos\left(x + \frac{\pi}{2}\right)$

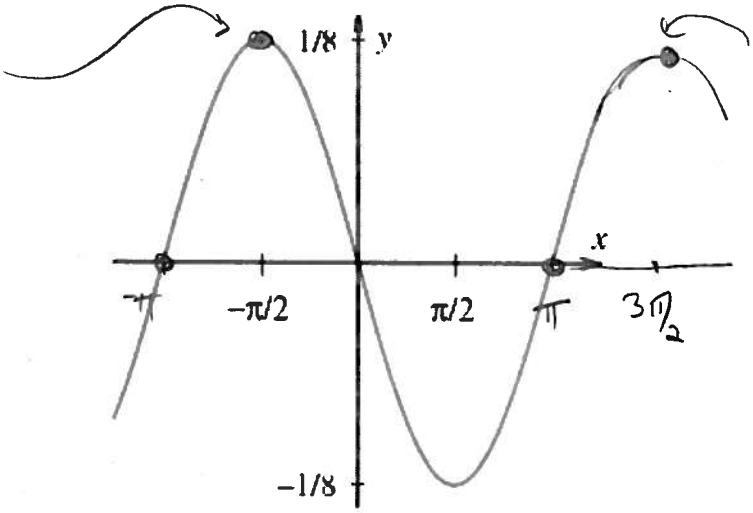
Answer 2)

amplitude = $\frac{1}{8} = a$

period = $2\pi \Rightarrow K = 1$

phase shift = $\frac{3\pi}{2}$

$\frac{1}{8}\cos\left(x - \frac{3\pi}{2}\right)$



4. [2] (pg571 #70) Evaluate $\sin(\cos^{-1}\left(\frac{1}{4}\right))$ exactly:

Method 1)

$\cos^{-1}(\frac{1}{4}) = ?$ when $\cos ? = \frac{1}{4}$

$\Rightarrow a^2 + 1^2 = 4^2$

$\Rightarrow a^2 = 16 - 1 = 15$

$\Rightarrow a = \sqrt{15}$

and $0 \le ? \le \pi$

$\sin(\cos^{-1}(\frac{1}{4})) = \sin(?)$

$= \frac{\sqrt{15}}{4}$ (+.5)

Method 2)

Let $\cos^{-1}(\frac{1}{4}) = ?$ Then $\cos ? = \frac{1}{4}$ (+.5)

(+.5) We want to find $\sin(\cos^{-1}(\frac{1}{4})) = \sin(?)$

Recall $\sin^2 ? + \cos^2 ? = 1$ (+.5)

$\Rightarrow \sin^2 ? + (\frac{1}{4})^2 = 1$

$\Rightarrow \sin^2 ? = 1 - \frac{1}{16} = \frac{15}{16}$

(+.5) $\Rightarrow \sin ? = \frac{+\sqrt{15}}{4}$ b/c $0 \le ? \le \pi$