Quiz 4 Key

Show *all* your work. No credit is given without reasonable supporting work. There are *two* sides to this quiz.

1. [1] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Assume a, b, and c are positive numbers.

   (T) F $\log_7 \sqrt[3]{49} = \frac{2}{3}$ $\log_7 49^{\frac{1}{3}} = \log_7 (7^2)^{1/3} = \log_7 7^{2/3} = 2/3$

   T (F) $\ln\left(\frac{a^7}{b\sqrt[3]{c}}\right) = 7\ln a - \ln b + \frac{1}{3}\ln c$ $\ln\left(\frac{a^7}{bc^{1/3}}\right) = \ln a^7 - \ln(bc^{1/3})$ $= 7\ln a - [\ln b + \ln c^{1/3}]$

   $(x^2)^3 = x^2 x^2 x^2 = x^6$

2. [3] (§1.2 #39) Simplify the expression.

$$\frac{(8x^2y^3)^{\frac{1}{3}}xy^{-3}}{2x^2y}$$

(+.5) $\frac{8^{1/3}(x^2)^{1/3}(y^3)^{1/3}\,x\,y^{-3}}{2x^2y} = \frac{2\,x^{2/3}\,y\,xy^{-3}}{2x^2y}$ (+.5)

same base mult (+1)

$= \frac{2\,x^{2/3+1}\,y^{1-3}}{2x^2y} = \frac{2\,x^{5/3}\,y^{-2}}{2x^2y} = x^{5/3-2}\,y^{-2-1}$

neg exp/den (+1)

$= x^{-1/3}\,y^{-3}$ or $\frac{1}{x^{1/3}y^3}$ or $\frac{1}{\sqrt[3]{x}\,y^3}$

3. [2] (10/15 Lecture) A sum of \$1000 dollars is invested at an interest rate of 8% per year. Approximate or calculate how long it will take this \$1000 to double if it is continuously compounded.

Rule of 70: (+.5) or

time it takes to double $\approx \frac{70}{8} = 8.75$ years (+1) (+.5)

\$ after t years $= 1000e^{.08t}$ (+.5)

find t so that $2000 = 1000e^{.08t}$ (+.5)

$\frac{2000}{1000} = \frac{1000e^{.08t}}{1000}$

$2 = e^{.08t}$

$\ln 2 = \ln e^{.08t}$

$\frac{\ln 2}{.08} = \frac{.08t}{.08}$

$t = \frac{\ln 2}{.08} \approx 8.66$ years (+1)

4. (WebHW9 #5) A radioactive substance decays in such a way that the amount of mass remaining after t days is given by the function

$$m(t) = 13e^{-0.015t}$$

where $m(t)$ is measured in kilograms.

(a) [1] Find the mass after 45 days.

$13e^{-0.015(45)} \approx 6.62$ days (+.5)

(b) [3] Find when there is 7.5 kilograms left (that is, half of it has decayed).

we want to find t so that

$\frac{7.5}{13} = \frac{13e^{-0.015t}}{13}$ (+1)

(+.5) solved for remaining e

$\frac{1}{2} = e^{-0.015t}$

(+.5) used ln correctly

exponential written as logarithmic

$\frac{\ln \frac{1}{2}}{-0.015} = \frac{-0.015t}{-0.015}$

$t = \frac{\ln \frac{1}{2}}{-0.015} \approx 46.2 \approx \frac{-.69}{-.015}$