EXAM 2

TMath 120

Practice

Key

NAME: This is a sample exam to be used for practice only. This is *not* a template for the exam that will be given in class. Many of the questions on the exam will look quite different than those appearing here.

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

Let x and y be positive numbers.

T F $x^2x^3 = x^6$ $\quad x^2x^3 = (xx)(xxx) = x^5$

T F $\log(x+y) = \log(x)\cdot\log(y)$ $\quad \log(x\cdot y) = \log x + \log y$ $\quad$ Let $x = 10 = y$ then $\log(x+y) = \log 20 \neq \log x + \log y = 1$

T F $\log(\frac{x}{y}) = \log(x) - \log(y)$

T F $\frac{\log x}{\log y} = \frac{x}{y}$

T F $\log_2 5x^7 = 7\log_2 5x$ $\quad \log_2 5x^7 = \log_2 5 + \log_2 x^7 = \log_2 5 + 7\log_2 x$

T F $\log(\log(10)) = 0.$ $\quad \log(\log 10) = \log(1) = 0$

T F For all numbers z, $\sqrt{z^2} = z$ $\quad$ Let $z = -2$ then $\sqrt{(-2)^2} = 2 \neq -2$

LONG ANSWERS: Show all your work and circle you final answer. Correct answers will *not* get credit without supporting work.

2. Given $-x = \dfrac{2xy}{2y-1}$, solve for y.

$2y-1 \cdot -x = \dfrac{2xy}{\cancel{2y-1}} \cdot \cancel{2y-1}$

$-\cancel{2xy} + x = 2xy$
$+\cancel{2xy} \qquad +2xy$

$\dfrac{x}{4x} = \dfrac{\cancel{4x}y}{\cancel{4x}}$

$y = \dfrac{x}{4x} = 1/4$ if $x \neq 0$

3. [2] Define the rule of the function log.

$\log_b a = x$ exactly when $b^x = a$.

4. [4] Assume b, x,$y > 0$, simplify the following:

$\frac{(b^x)^{x-1}}{b^{-x}}$ rule $(b^a)^c = b^{ac}$

so $\frac{(b^x)^{x-1}}{b^{-x}} = \frac{b^{x(x-1)}}{b^{-x}}$

$= \frac{b^{x^2-x}}{b^{-x}}$

$= b^{x^2-x-(-x)}$

$= b^{x^2-x+x}$

$= b^{x^2}$

$\frac{\sqrt[3]{x^2}(y^2)^{\frac{3}{2}}}{x^{\frac{2}{3}}y^2} = \frac{(x^2)^{1/3}(y^2)^{3/2}}{x^{2/3}\,y^2}$

$= \frac{\cancel{x^{2/3}}\,y^3}{\cancel{x^{2/3}}\,y^2}$

$= y^{3-2} = y$

$2 - \log_5(25z)$

$2 - [\log_5 25 + \log_5 z]$

$2 - \log_5 5^2 - \log_5 z$

$2 - 2 - \log_5 z$

$-\log_5 z$ or $\log_5 z^{-1}$

or $\log_5 \frac{1}{z}$

$\log_2 \frac{1}{4} + 2$

$= \log_2 4^{-1} + 2$

$= \log_2 ((2)^2)^{-1} + 2$

$= \cancel{\log_2 2^{-2}} + 2$

$= -2 + 2 = 0$

5. [3] Find x in the following:

$2^{4x-1} = 3^{1-x}$

$\ln 2^{4x-1} = \ln 3^{1-x}$

$(4x-1)\ln 2 = (1-x)\ln 3$

$4x\ln 2 - \ln 2 = \ln 3 - x\ln 3$

$4x\ln 2 + x\ln 3 = \ln 3 + \ln 2$

$x(4\ln 2 + \ln 3) = \ln 3 + \ln 2$

$x = \dfrac{\ln 3 + \ln 2}{4\ln 2 + \ln 3}$

$5^x = 2$

$\ln 5^x = \ln 2$

$x\ln 5 = \ln 2$

$x = \dfrac{\ln 2}{\ln 5}$

6. Find a formula for the inverse function f^{-1} of the indicated function f.

$f(x) = 4x^{\frac{3}{7}} - 1$

$x = 4y^{3/7} - 1$

$x + 1 = 4y^{3/7}$

$\dfrac{x+1}{4} = y^{3/7}$

$\left(\dfrac{x+1}{4}\right)^{7/3} = y$

$f(x) = 3 \cdot 2^x + 4$

$x = 3 \cdot 2^y + 4$

$x - 4 = 3 \cdot 2^y$

$\dfrac{x-4}{3} = 2^y$

$\log_2\left(\dfrac{x-4}{3}\right) = y$

7. Write the given expression as a single logarithm.

$2\ln 2x - 3(\ln x^2 + \ln x)$

$2\ln 2x - 3(\ln x^2 \cdot x)$

$2\ln 2x - 3\ln x^3$

$\ln (2x)^2 - \ln (x^3)^3$

$\ln (4x^2) - \ln x^9$

$\ln \dfrac{4x^2}{x^9}$

$\ln \dfrac{4}{x^7}$

$3 - \log_6(36y)$

$\log_6 6^3 = 3$ so

$3 - \log_6(36y)$

$= \log_6 6^3 - \log_6(36y)$

$= \log_6 \dfrac{6^3}{36y} = \log_6 \dfrac{6}{y}$

8. [4] Solve for x:

$\log(x-16) = 2 - \log(x-1)$

$+\log(x-1) \qquad +\log(x-1)$

$\log(x-16) + \log(x-1) = 2$

$\log[(x-16)(x-1)] = 2$

$10^{\log[(x-16)(x-1)]} = 10^2$

$(x-16)(x-1) = 100$

$x^2 - 17x + 16 = 100$

$x^2 - 17x - 84 = 0$

prop 1 $\Rightarrow (x-21)(x+4) = 0$

$\Rightarrow x - 21 = 0$ or $x + 4 = 0$

$\Rightarrow x = 21$ or $x = -4$

Check:

$\log(21-16) = 2 - \log(21-1)$ ✓

$\log(\cancel{-4-16}) \Rightarrow$ not defined

$4^x - 3 * 2^x = 10$

$(2^2)^x - 3 \cdot 2^x = 10$

let $y = 2^x$ then

$2^{2x} - 3 \cdot 2^x = 10$

$(2^x)^2 - 3 \cdot (2^x) = 10$

$y^2 - 3y = 10$

$y^2 - 3y - 10 = 0$

$(y-5)(y+2) = 0$

$\Rightarrow y = 5$ or $y = -2$

$\Rightarrow 2^x = 5$ or $2^x = -2$

$\Rightarrow x = \log_2 5$ or $\cancel{\log_2 -2}$

not defined

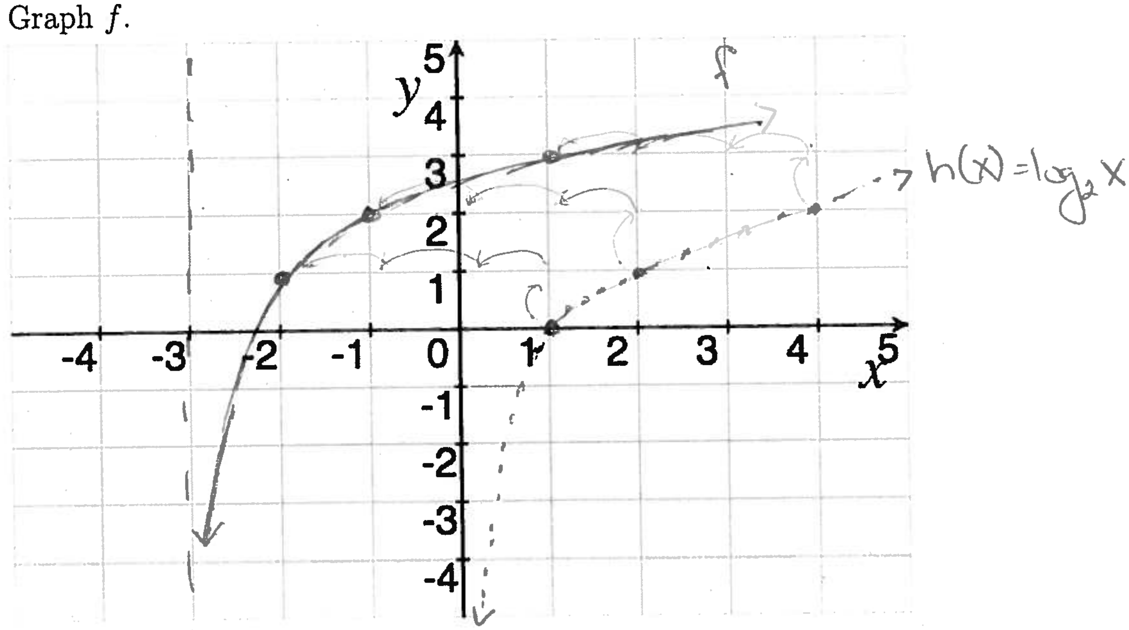
9. Let $h(x) = \log_2 x$ and $f(x) = \log_2(x+3) + 1$.

(a) List the transformations needed to transform the graph of h to the graph of f.

vertical transformation up 1 unit

horizontal shift left 3 units.

(b) Graph f.



(c) Find the inverse function to f.

$f(x) = \log_2(x+3) + 1$

step 1: swap x with y

$f(y) = \log_2(y+3) + 1$

step 2 set equal to x

$x = \log_2(y+3) + 1$

step 3: solve for y

$x = \log_2(y+3) + \cancel{1}$

$-1 \qquad\qquad -\cancel{1}$

$x - 1 = \log_2(y+3)$

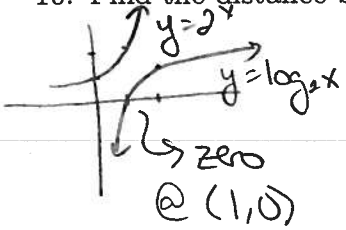
$2^{x+1} = \cancel{2}^{\log_2}(y+3)$

$2^{x+1} = y + \cancel{3}$

$-3 \qquad\quad -\cancel{3}$

$y = 2^{x+1} - 3$

10. Find the distance between the zero of $\log_2 x$ and the point $(-2,3)$.



distance between $(1,0)$ and $(-2,3)$

$\sqrt{(3-0)^2+(-2-1)^2}$
$=\sqrt{9+9}=\sqrt{18}$
$=\sqrt{9}\sqrt{2}=3\sqrt{2}$

11. A sound with intensity x has $10\log\dfrac{x}{I_0}$ decibels, where $I_0 = 10^{-12}$ watts per square meter (W/m^2).

(a) [2] (§4.5 #39) France passed a law limiting iPods and other MP3 players to a maximum possible volume of 100 decibels. Find the maximum intensity (in W/m^2) an iPod is legally allowed to output in France.

$$\frac{100}{10}=\frac{\cancel{10}\log\frac{x}{10^{-12}}}{\cancel{10}}$$

$$10=\log\frac{x}{10^{-12}}$$

$$10^{10}=\cancel{10}^{\cancel{\log}\frac{x}{10^{-12}}}$$

$$10^{-12}\cdot 10^{10}=\frac{x}{\cancel{10^{-12}}}\cancel{\cdot 10^{-12}}$$

$$10^{-2}=x \Rightarrow x=1/100$$

(b) [3] (§4.5 #40) Normal conversation has a sound level of about 65 decibels. How many more times intense than normal conversation is the sound an iPod operating at the French maximum of 100 decibels?

Let I_c be the intensity of a conversation and I_p be the intensity of an iPod at max level.

we want to find K so that

$KI_c = I_p$ or $K = I_p/I_c$

Part (a) implies $I_p = 10^{-2}$

implies $I_c = 10^{-5.5}$

to find I_c: $65 = 10\log I_c/10^{-12}$

$6.5=\log I_c/10^{-12}$

$10^{6.5}=I_c/10^{-12}$

$\Rightarrow I_c = 10^{6.5}\cdot 10^{-12}=10^{-5.5}$

So $\dfrac{I_p}{I_c}=\dfrac{10^{-2}}{10^{-5.5}}=10^{3.5}$

≈ 3162 times more intense?

12. At current growth rates, the Earth's population is doubling about every 69 years. If this growth rate were to continue, about how many years will it take for the Earth's population to become one-fourth larger than the current level?

Use Pa^x or Pe^{rt}

With Pe^{rt} we need to find r.

Given when t=69, pop doubles so

$$\frac{2P_0}{P_0} = \frac{P_0 e^{r \cdot 69}}{P_0}$$ solve for r..

...e this ...on §3.5 #17 or 80

$$2 = e^{69r} \Rightarrow \ln 2 = 69r$$

$$\Rightarrow r = \frac{\ln 2}{69} \approx .0100456$$

When is pop $P_0 + \frac{1}{4}P_0 = \frac{5}{4}P_0$

So find t when

$$\frac{\frac{5}{4}P_0}{P_0} = \frac{P_0 e^{.0100456t}}{P_0}$$

$$\ln \frac{5}{4} = .0100456t \Rightarrow t = 22.2$$

22 years

13. Pay Day Loans can give you a 10% loan on \$250 for up to 45 days (the actual rate is 15% and doesn't jump down to 10% until \$500, but for the purposes of this problem, assume that you know someone at Pay Day Loans and they are given you a "deal" with 10%) . At the end of that 45 days you will have to pay off both the principal and the 10% interest on the principal. If you are unable to pay this amount at the end of the 45 days one option is to take out another loan to cover the new amount of money that you owe.

typo → giving

(a) If you need a loan for \$250 on Jan. 1st of 2010 but don't have access to cash until Jan 1st of 2011, you might choose to go to Pay Day Loans and "renew" the loan every 45 days until the end of the year. If you choose to do this, how much money will you owe on Jan 1st 2011?

If you 'renew' your loan everytime it is due you will have to 'renew' it 9 times (8 doesn't quite make it to the end of the year).

So you'll owe $250(1+.10)^9 \approx \$589.49$

(assuming you can cover any additional fees they throw at you).

(b) What is the effective annual interest rate of the plan above?

We want to know r so that

$$250(1+r)^1 = 589.49$$

$$1+r = 2.35996$$

$$r = 1.35996$$

$\Rightarrow$ the rate is 135.996% or 136%!