Exam 1

# TMath 120

Practice

Key

NAME: This is a sample exam to be used for practice. This is *not* a template for the exam that will be given in class. Many of the questions on the exam will look quite different than those appearing here.

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let $f$ be a function, and $x$, $y$, and $z$ be real numbers with $z \neq 0$.

T F $\frac{3x+y}{3z} = \frac{x+y}{z}$ $\quad \frac{x+y}{z} = \frac{3(x+y)}{3 \cdot z} \neq \frac{3x+y}{3z}$

T F $(x+y)^2 = x^2 + y^2$ $\quad (x+y)^2 = (x+y)(x+y) = x^2 + xy + yx + y^2$

T F $|x| = x$ $\quad$ let $x = -1$, then $|x| = |-1| = 1 \neq x$

T F $\frac{3+5i}{1-2i} = -\frac{7}{5} + \frac{11}{5}i$ $\quad \frac{3+5i}{1-2i} \cdot \frac{(1+2i)}{(1+2i)} = \frac{3+6i+5i+10i^2}{1+\cancel{2i}-\cancel{2i}-4i^2} = \frac{3+11i-10}{1-4(-1)} = \frac{-7+11i}{5}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [3] Find any real or imaginary $x$ such that $3(7+x)^2 - 4 = 2$.

$3(7+x)^2 - 4 = 2$

$\quad\quad\quad +4 \quad +4$

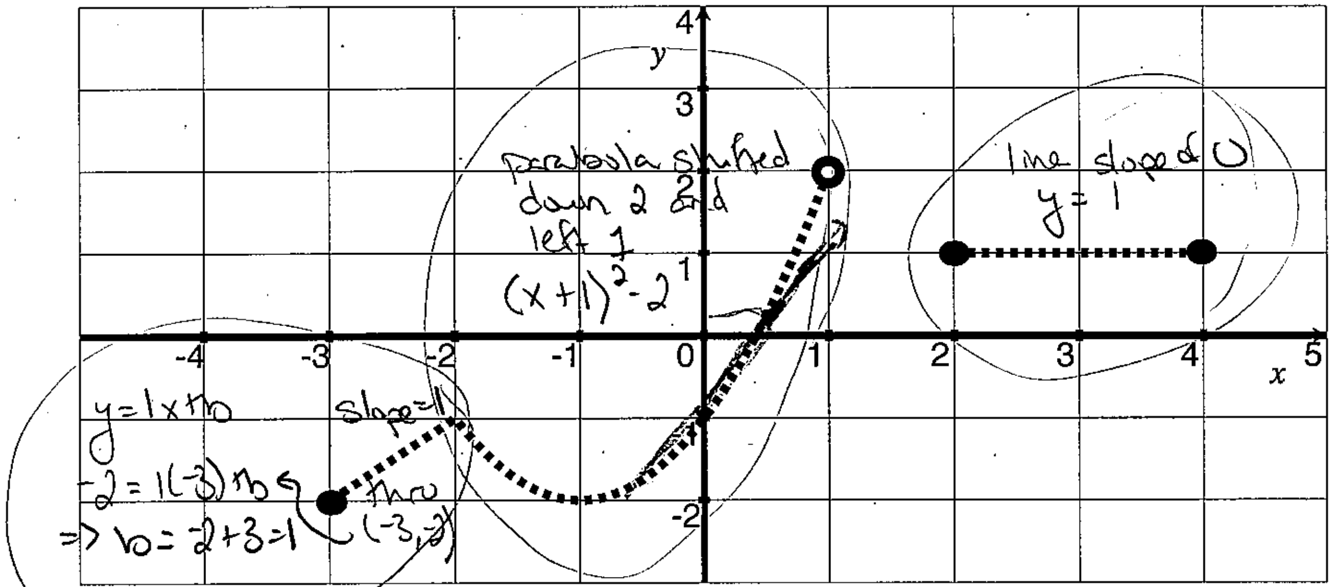
$\frac{3(7+x)^2}{3} = \frac{6}{3}$

$(7+x)^2 = 2$

$7 + x = \pm\sqrt{2}$

$x = -7 \pm \sqrt{2}$

3. [4] Let $f$ be the function comprised of two lines and a parabola that has only been shifted (not vertically stretched) and whose graph is below:




Estimate the following *if* possible:

$f(-3)$ $\quad -2$

$\frac{f(-3)-1}{f(-1)} = \frac{-2-1}{-2} = \frac{-3}{-2} = 3/2$

$f(1)$

does not exist

$(f \circ f)(-2) = f(f(-2)) = f(-1) = -2$

$f(-1)f(2) = (-2)(1) = -2$

$f(0+.5) = f(.5) \approx .25$

The average rate of change of $f$ from $x = 0$ to $x = .5$

Slope of line drawn above

$\frac{\text{rise}}{\text{run}} \approx \frac{1.25}{.5} = \frac{5/4}{1/2} = 5/4 \cdot \frac{2}{1}$

$\approx 5/2$

The piece-wise defined rule of $f$:

$$f(x) = \begin{cases} x+1 & \text{if } -3 \le x < -2 \\ (x+1)^2 - 2 & \text{if } -2 \le x < 1 \\ 1 & \text{if } 2 \le x \le 4 \end{cases}$$

4. [4] Find the domain of $g$ where $g(x) = \frac{2-\sqrt{5-2x}}{x+10}$.

need

# under square roots $\geq 0$ AND denominator $\neq 0$

$5-2x \geq 0$
$-5 \quad -5$
$\frac{-2x}{-2} \geq \frac{-5}{-2}$
$x \leq 5/2$

$x+10 \neq 0$
$x \neq -10$

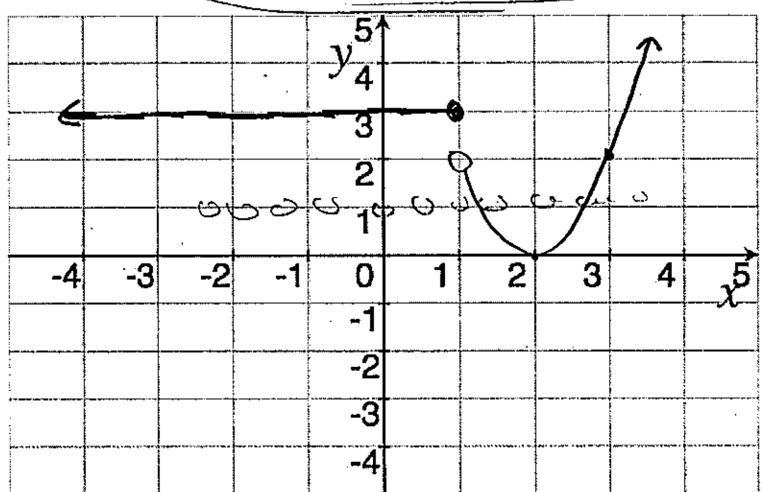
So $x \leq 5/2$ AND $x \neq -10$
or $(-\infty,-10) \cup (-10, 5/2]$

5. Let $h$ be the *function* defined by:

$$h(x) = \begin{cases} 3 & x \leq 1 \\ 2(x-2)^2 & 1 < x \end{cases}$$

(a) [3] Graph $h$. (Explaining graph transformations is worth partial credit.)

$2(x-2)^2$ vertical stretch by 2
shift right by 2




(b) [] What are the coordinates of the vertex on the piece of the graph above that is a parabola?

$(2,0)$

(c) Identify the $x$-intercept(s).

when $x=2$ or $(2,0)$

(d) [2] Find all possible input(s) so that $h(x) = 1$.

ie. find $x$ so that $1 = 2(x-2)^2$
$\Rightarrow 1/2 = (x-2)^2$
$\Rightarrow \pm\sqrt{1/2} = x-2$
$2 \pm \sqrt{1/2} = x$

(e) What is the range $h$?

$[0,\infty)$

6. [4] Given that $j(x) = -3x^2 + 6x - 2$. Write $j$ in vertex (standard) form.

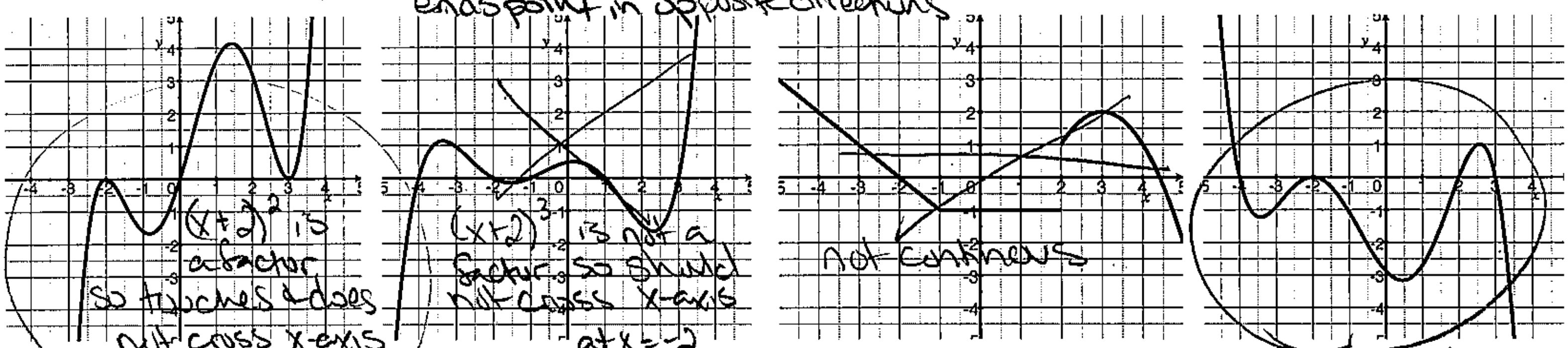
$$\frac{-3x^2+6x-2}{-3} = \frac{y}{-3}$$

$$x^2-2x+\tfrac{2}{3} = -\tfrac{1}{3}y$$
$$+1 \qquad +1$$
$$\underbrace{x^2-2x+1}+\tfrac{2}{3} = -\tfrac{1}{3}y+1$$
$$(x-1)^2+\tfrac{2}{3} = -\tfrac{1}{3}y+1$$

$$(x-1)^2+\tfrac{2}{3} = -\tfrac{1}{3}y+1$$
$$-1 \qquad -1$$
$$-3\left[(x-1)^2-\tfrac{1}{3}\right] = \left[-\tfrac{1}{3}y\right]-3$$
$$-3(x-1)^2+1 = y$$

7. [2] Let $h$ be a $5^{\text{th}}$ degree polynomial that has $(x+2)^2$ as a factor (but $(x+2)^3$ is not a factor). Which of the following could be the graph of $h$? (Circle all that are possible.)

ends point in opposite directions



$(x+2)^2$ is a factor so touches & does not cross x-axis at $x=-2$

$(x+2)^3$ is not a factor so should not cross x-axis at $x=-2$

not continuous

8. [3] Find a cubic polynomial whose graph passes through the points $(-2, 0)$ and $(1, 0)$ and has a root at 6. Note: there are many correct answers possible here.

$-2$, $1$ and $6$ are roots

so $(x-2)$, $(x-1)$, and $(x-6)$ are factors

The polynomial $(x+2)(x-1)(x-6)$ works, but so does $2(x+2)(x-1)(x-6)$ or $-7.88(x+2)(x-1)(x-6)$ etc

9. Let $m(x) = x^3 + x^2 - \frac{39}{4}x + 9$ and $n(x) = x + 4$. Use long division to find $D(x)$ and $R(x)$ so that $\dfrac{m(x)}{n(x)} = D(x) + \dfrac{R(x)}{n(x)}$

Recall thru long ÷ we can write things of this form

ex $\frac{100}{9} = 11 + \frac{1}{9}$

$$
\begin{array}{r}
x^2 - 3x + 9/4 \quad R0 \\
x+4 \overline{\big)\, x^3+x^2+\frac{39}{4}x+9} \\
-(x^3+4x^2) \\
\hline
-3x^2+\frac{39}{4}x+9 \\
-(-3x^2-12x) \\
\hline
\frac{9}{4}x+9 \\
-(\frac{9}{4}x+9) \\
\hline
0
\end{array}
$$

So $\dfrac{x^3+x^2+\frac{39}{4}x+9}{x+4} = x^2-3x+\frac{9}{4} + \dfrac{0}{x+4}$

10 Let $p(x) = \dfrac{x-5}{7x+5} + 3$.

(a) Given that $p$ is one-to-one (ie has an inverse), find $p^{-1}$.

$$x = \frac{y-5}{7y+5} + 3$$

$$x - 3 = \frac{y-5}{7y+5}$$

$$(7y+5)(x-3) = y-5$$

$$7xy - 21y + 5x - 15 = y - 5$$

$$7xy - 21y - y = 15 - 5x - 5$$

$$y(7x - 21 - 1) = 15 - 5x - 5$$

$$y = \frac{10-5x}{7x-22}$$

(b) Write the expression $p(a+h)$ and simplify.

$$p(a+h) = \frac{a+h-5}{7(a+h)+5} + 3 = \frac{a+h-5}{7a+7h+5} + 3$$

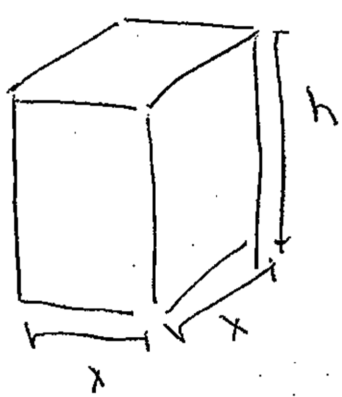
(c) Write the expression $\dfrac{p(a+h)-p(a)}{h}$ and simplify.

$$\frac{p(a+h)-p(a)}{h} = \left(\left[\frac{a+h-5}{7a+7h+5}+3\right] - \left[\frac{a-5}{7a+5}+3\right]\right) \div h = \left(\frac{a+h-5}{7a+7h+5} \cancel{+3} - \frac{a-5}{7a+5} \cancel{-3}\right) \div$$

$$= \frac{(a+h-5)(7a+5) - (a-5)(7a+7h+5)}{(7a+7h+5)(7a+5)} \cdot \frac{1}{h} = \frac{(\cancel{7a^2} + \cancel{5a} + \cancel{7ah} + 5h - \cancel{35a} - \cancel{25}) - (\cancel{7a^2} + \cancel{7ah} + \cancel{5a} - \cancel{35a} - 35h}{(7a+7h+5)(7a+5)h}$$

$$= \frac{5h - 35h}{(7a+7h+5)(7a+5)h} = \frac{-30\cancel{h}}{\cancel{h}(7a+7h+5)(7a+5)} = \frac{-30}{(7a+7h+5)(7a+5)}$$

11. A rectangular box with a volume of 60 ft$^3$ has a square base. Find a function that models its surface area $S$ in terms of the length $x$ of one side of its base.




$$x \cdot x \cdot h = 60 ft^3 \Rightarrow h = \frac{60 ft^3}{x^2}$$

$$S = xh + xh + xh + xh + x^2 + x^2 = 4xh + x^2$$

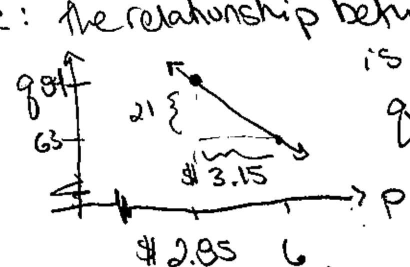
$$= 4x\left(\frac{60 ft^3}{x^2}\right) + x^2 = \frac{240}{x} + x^2$$

12. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) You would like to set the price for a UWT fund-raising raffle. You did a similar thing last year and when you set the price to \$6 about 63 people bought tickets. The stats class did some research for you and reported that if ticket prices reduced by \$3.15, sales would increase by about 21 tickets. What price should you set the tickets so as to maximize income from ticket sales (to the nearest penny)?

(b) A manufacturer of soft drinks advertises their orange soda as "naturally flavored", although it contains only 5% orange juice. A new federal regulation stipulates that to be called "natural" a drink must contain at least 10% fruit juice. The manufacturer mixes their juices in closed 900 gallon containers (to avoid contamination). How much juice must they remove from the 900 gallon container and replace with pure orange juice to conform to the new regulation?

a) Let p be the price of the tickets and q be the # of tickets sold.
we want to maximize income
ie Income = p·q
note: the relationship between p & q is a line




$q = mp + b$

$m = \frac{-21}{3.15} = \frac{-20}{3}$

passes thru (6,63) so $63 = \frac{-420}{63}(6) + b \Rightarrow b = 103$

$q = \frac{-20}{3}p + 103$

So income
$p \cdot q = p(-\frac{20}{3}p + 103)$
$= -\frac{20}{3}p^2 + 103p$
which is a parabola opening down
$\Rightarrow$ the max is at the vertex

$\text{income} = \frac{-20}{3}p^2 + 103p$

$-\frac{3}{20}\,\text{income} = p^2 - \frac{306}{20}p$

$-\frac{3}{20}\,\text{income} + \left(\frac{306}{40}\right)^2 = \left(p - \frac{306}{40}\right)^2$

So max when $p = \frac{306}{40} \approx \$7.73$

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b) Let x be the amount of pure orange juice used and y be the amount of original mixture kept.

Note:

full tank = amount of mixture kept + amount of pure orange juice added $\Big\}$ $900 = y + x$ $\Rightarrow$ $y = 900 - x$ (1)

orange juice to mix we want = pure orange juice from original + pure orange juice added $\Big\}$ $.10 \cdot 900 = .05y + x$ $\Rightarrow$ $90 = .05y + x$ (2)

System of two equations can be solved for the 2 unknowns
Sub (1) into (2) $\Rightarrow 90 = .05(900 - x) + x$
$\Rightarrow 90 = 45 - .05x + x$
$\Rightarrow 45 = .95x$
$x = 45/.95 \approx 47.37$ gal
[illegible] 47.37 gal) and replace w/ ora[illegible]