

Key

NAME:

1. [7] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T F $\frac{3}{x} + \frac{2}{x^2} = \frac{11}{x^2}$

$\frac{3x}{x^2} + \frac{2}{x^2} = \frac{3x+2}{x^2}$

T F $\sqrt{(x^2)} = x$ for all x .

let $x = -2$ note $\sqrt{(-2)^2} = 2 \neq -2$

→ T F $6^{\log_6(x)} = x$ for all x .

log's have rec'd inverse... for all x that make sense

T F $\log_5(x) - \log_5(y) = \frac{\log_5(x)}{\log_5(y)}$ for x and y greater than zero.

$\log_5 x - \log_5 y = \log_5 \frac{x}{y}$

T F The domain of $\log_3(x+3)$ is $(-3, \infty)$.



shift left 3 units

T F $2 * 8^x = 16^x$.

PEMDAS

T F $(b^x)^{x-1} = b^{x^2-x}$ for $b > 0$.

$(x^2)^3 = x^2 x^2 x^2 = x x x x x x = x^6$
multiplied

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] (mini-quiz) Solve for y and simplify as much as possible given:

start (1)

alg (2)

solve for y (1)

$(xy-6) - x = \frac{3y}{xy-6}$

$-x^2y + 6x = 3y$
 $+ x^2y$ $+ x^2y$

$6x = 3y + x^2y$

$6x = (3 + x^2)y$

$\frac{6x}{3+x^2} = y$

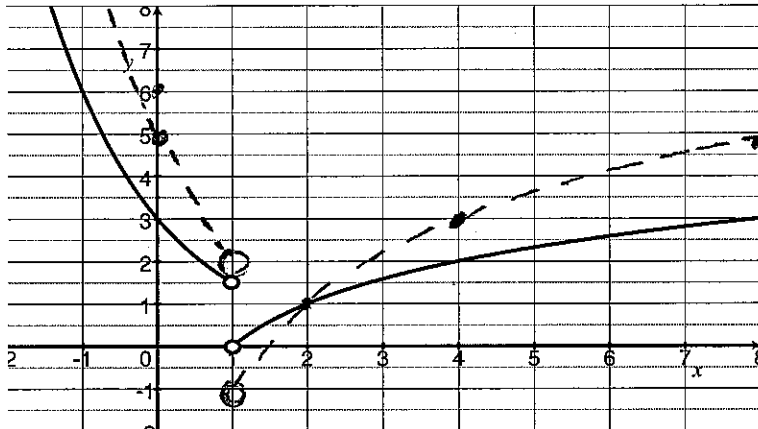
3. Let f be the function whose graph is given below.

- (a) [1] (§4.2 #41)
Find the range of f .

$(0, \infty)$

- (b) [4] (§4.2 #55)
Sketch the graph of $2f(x) - 1$

(+) { vert stretch by 2
vert shift down by 1
order (+)



- (c) [5] (WebHW #4 & §4.2 #38) The function f is of the form Ca^x when $x < 1$ and of the form $\log_b(x)$ when $1 < x$. Find a formula for f in this indicated form.

$$f(x) = \begin{cases} 3\left(\frac{1}{2}\right)^x & x < 1 \\ \log_2(x) & 1 < x \end{cases}$$

formulation (+.5)
SKA (+.5)

(+) { $y = Ca^x$
passes thru $(0, 3)$ & $(-1, 6)$

(+.5) { $3 = Ca^0 = C \cdot 1$
 $\Rightarrow C = 3$

(+) { $6 = 3a^{-1}$
 $2 = \frac{1}{a} \Rightarrow a = \frac{1}{2}$

(+) { passes thru $(2, 1)$
 $\log_b(x)$

(+.5) { so $\log_b 2 = 1$
 $\Rightarrow b^1 = 2 \Rightarrow b = 2$

4. Let $g(x) = \log_3(x+2)$

- (a) [2] (§4.2 #42) Sketch a graph of g

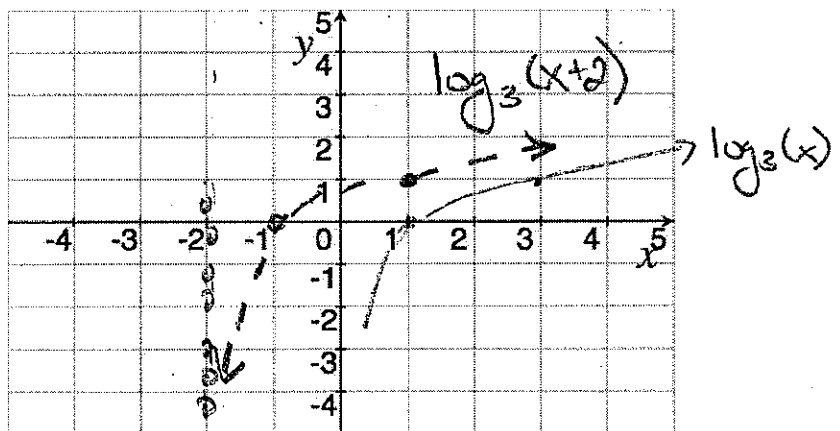
(+) { shift left horiz by 2
get + (+)

- (b) [2] (WebHW9 #16)
Find the domain of g .

$(-2, \infty)$

(+.5)

new domain is (+)



5. [3] (exp wks #2) Simplify: $\frac{(6st^3)^2}{2t^{-1}\sqrt{s}}$

$$\frac{(6st^3)^2}{2t^{-1}\sqrt{s}} = \frac{36s^2t^6}{2t^{-1}\sqrt{s}} = \frac{36s^2t^6t^1}{2s^{1/2}} = 18s^{3/2}t^7$$

6. Find all x that satisfy the following:

(a) [3] (WebHW10 #14) $1 + \frac{e^{-x}}{1 + e^{-x}} = 7 (1 + e^{-x})$

start (1.5)
used ln (1)
order of ops (1)
got it (1.5)

$$80 = 7 + 7e^{-x}$$

$$73 = 7e^{-x}$$

$$\ln \frac{73}{7} = -x$$

$$x = -\ln \left(\frac{73}{7} \right) \approx -2.34$$

(b) [3] (10/30 lecture) $\log_6(x+1) = 1 - \log_6(x+2)$

start (1.5)
used log prop (1)
used exp (1.5)
alg (1.5)
check answer (1.5)

$$\log_6(x+1) + \log_6(x+2) = 1$$

$$\log_6(x+1)(x+2) = 1$$

$$x^2 + 3x + 2 = 6$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$\Rightarrow x = -4 \text{ or } 1$$

7. [3] (quiz4 #3) A sum is invested at an interest rate of 8% per year. Use exponential growth models to approximate how long it will take for the investment to triple.

note: A was not specified how many times a year the A was compounded \Rightarrow many right approximations

set (1.5)
exponent

continuously

$$3P = Pe^{.08t}$$

$$3 = e^{.08t}$$

$$\ln 3 = .08t$$

$$t = \frac{\ln 3}{.08} \approx 13.7$$

annually

$$3P = P(1 + .08)^t$$

$$3 = 1.08^t$$

$$\ln 3 = t \ln 1.08$$

$$t = \frac{\ln 3}{\ln 1.08} \approx 14.3$$

monthly

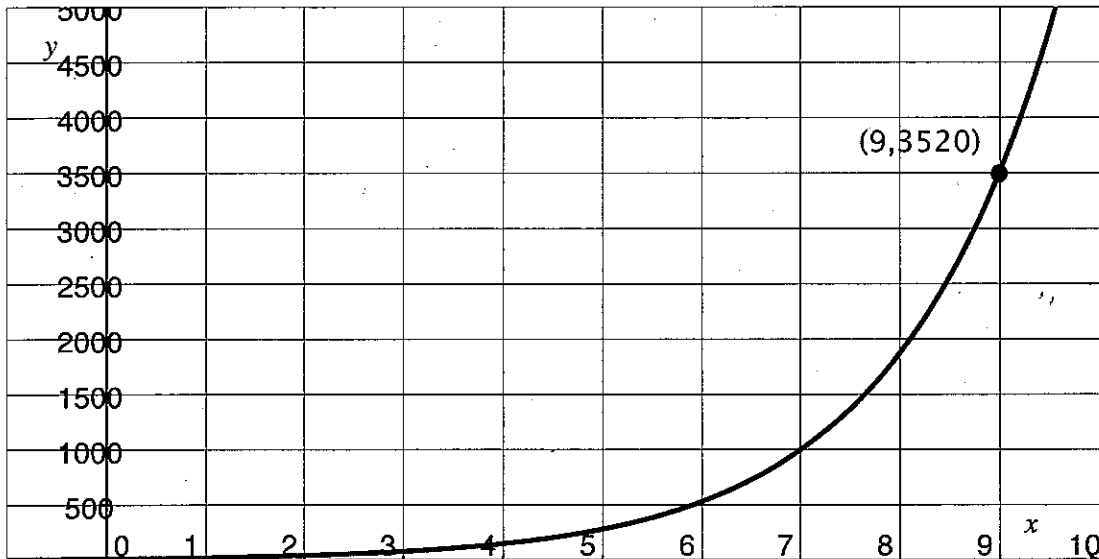
$$3P = P \left(1 + \frac{.08}{12} \right)^{12t}$$

$$3 = \left(1 + \frac{.08}{12} \right)^{12t}$$

$$\ln 3 = 12t \ln \left(1 + \frac{.08}{12} \right)$$

$$t = \frac{\ln 3}{12 \ln \left(1 + \frac{.08}{12} \right)} \approx 13.8$$

8. Cyrano Jones gives one tribble to Uhura in the hopes to boost his sales. It turns out that tribbles are born pregnant and once fed they give birth. Dr. Spock noticed this phenomenon and began to track the population of tribbles when Uhura showed him her new family of twelve tribbles at 0800 (8am). The data is plotted below where the horizontal axis is recording the number of hours since 0800 and the vertical axis is recording the number of tribbles on the Enterprise.



- (a) [2] (§2.2 23) Use the graph to *estimate* when there were 2000 tribbles on the Enterprise.

after 8 hours
so after 1600, or 4pm

- (b) [1] (§2.2 #55) Is the above a graph of a function? Why or why not?

yes it passes the vert. line test.

- (c) [1] Reread the above description. What is the initial population that Dr. Spock recorded?

12 tribbles

- (d) [4] (§4.1 #39) Given that the population of tribbles f is well approximated by an exponential function in the number of hours x since 0800, use the data point (9, 3520) on the graph to find the rule to describe the population at time x .

$P = Pe^{rt}$
 $P = 12$
 $P(t) = 12e^{rt}$
 question (9, 3520)
 $3520 = 12e^{r \cdot 9}$
 $\frac{3520}{12} = \frac{12e^{r \cdot 9}}{12}$
 $\frac{3520}{12} = e^{r \cdot 9}$
 $\ln \frac{3520}{12} = \ln e^{r \cdot 9}$
 $\ln \frac{3520}{12} = 9r$
 $r = \frac{\ln \frac{3520}{12}}{9}$
 used ln
 alg
 $P(t) = 12e^{t \cdot \frac{\ln \frac{3520}{12}}{9}}$

9. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

(a) (Fall11 Exam2#10) Legislation passed last year to let the University of Washington raise its own tuition, but previous to that the state set the maximum percentage increase that tuition could be raised, call it r . Between '07 and '10, the University raised tuition by as much as the state would allow for each year. The '07 to '08 school year had a tuition of approximately \$6000. In the '10 school year the tuition was approximately \$8,700.

- What percentage rate is the University of Washington raising its tuition every year?
- If the University of Washington continues increasing tuition rates at the above fixed percentage, when will the cost of tuition exceed \$100,000?

(b) (Word Problem Wks #10) Recall from class that pH is measured on a logarithmic scale and that the pH level of a substance can be computed by $\text{pH} = -\log[\text{H}^+]$, where $[\text{H}^+]$ is usually the concentration of hydrogen ions measured in moles per liter (M).

Assume that the white vinegar in this problem has a pH level of 2.5 and your stomach acid has a pH level of 1.6.

- How many times stronger is stomach acid than the white vinegar?
- If you found a substance X whose $[\text{H}^+]$ was six times as intense as vinegar, what would its pH measure be?

Start (+.5)

t = years since '07	school yr	tuition
0	07-08	6,000
1	08-09	
2	09-10	
3	10-11	8,700

raising tuition a fixed percentage each year is exponential growth? I'll use $Pert$ (+.5)

Initial cost = $P = 6000$ (+.5)

(+.5) Since (3, 8700) is on the exp function

$$\frac{87}{60} = e^{3r}$$

$$\ln \frac{87}{60} = \frac{3r}{3}$$

$$r = \frac{1}{3} \ln \left(\frac{87}{60} \right)$$

$$\approx .12385$$

b) find t so that

$$100,000 = 6000 e^{.124t}$$

$$\frac{50}{3} = e^{.124t}$$

$$\ln \left(\frac{50}{3} \right) = .124t$$

$$t = \frac{\ln \left(\frac{50}{3} \right)}{.124} \approx 22.7 \text{ years.}$$

note: we could have used $P(1+r)^t$ instead

definet (+.5)

als (+.5)

alg (+.1)

Started (1.5)

b) Let $[H_s^+]$ be the hydrogen ions/mole in stomach acid } (1)
and $[H_w^+]$ " " " white vinegar
and $[H_x^+]$ " " " substance X

i) $2.5 = -\log [H_w^+]$

$1.6 = -\log [H_s^+]$

$\Rightarrow 10^{-2.5} = [H_w^+]$ } (1)
 $10^{-1.6} = [H_s^+]$ }

we want to know K so that

$K [H_w^+] = [H_s^+]$

or $\frac{[H_s^+]}{[H_w^+]}$ } (1.5)

so $\frac{[H_s^+]}{[H_w^+]} = \frac{10^{-1.6}}{10^{-2.5}} = 10^{0.9}$ } (1.5)
 $\approx 7.9 \text{ times}$

ii) note $[H_x^+] = 6 [H_w^+]$ } (1.5)

we want the pH level so

$-\log (6 [H_w^+]) = -\log (6 \cdot 10^{-2.5}) = 1.72$
(1.5) (1.5)