EXAM 1 **TMath120** Fall 2012 Key

NAME:

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be non-zero real numbers.

T (F) $\frac{3}{x}+\frac{2}{x^2}=\frac{11}{x^2}$ $\frac{3}{x}\cdot\frac{x}{x}+\frac{2}{x^2}=\frac{3x+2}{x^2}$

(T) F $\frac{5}{\frac{3}{2}y}=\frac{10}{3y}$ $\frac{10}{3y}$

T (F) $f(x+1)=f(x)+1$ let $f(x)=x^2$ then $f(x+1)=(x+1)^2\neq x^2+1=f(x)+1$

T (F) $(2-i)-(7+\frac{2}{3}i)=-5-\frac{1}{3}i$ $2-i-7-\frac{2}{3}i=-5-\frac{5}{3}i$

T (F) $\frac{1-2i}{5+i}=\frac{7}{26}+\frac{11}{26}i$ $\frac{1-2i}{5+i}\cdot\frac{5-i}{5-i}=\frac{5-i-10i+2i^2}{25+5i-5i-i^2}=\frac{5-11i-2}{25-1}=\frac{3-11i}{26}$

T (F) All functions have an inverse. Example $f(x)=x^2$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] (§1.5 #19) Find any real or imaginary x such that:

$$\frac{1}{x+1}+\frac{1}{2}=\frac{1}{x+3}$$

Start (+.5)

$2\cdot(x+1)\left[\frac{1}{x+1}+\frac{1}{2}\right]=\frac{1}{x+3}\cdot 2\cdot(x+1)$ dom (+1) alg (+1)

$(x+3)\left[2+(x+1)\right]=\left[\frac{2(x+1)}{(x+3)}\right](x+3)$

$2x+6+(x+3)(x+1)=2x+2$

$2x+6+x^2+4x+3=2x+2$

$x^2+4x+9=2$

$-2\quad -2$

$x^2+4x+7=0$ +2

$x^2+4x+7=0$

$+4\quad +4$

$(x+2)^2+7=4$

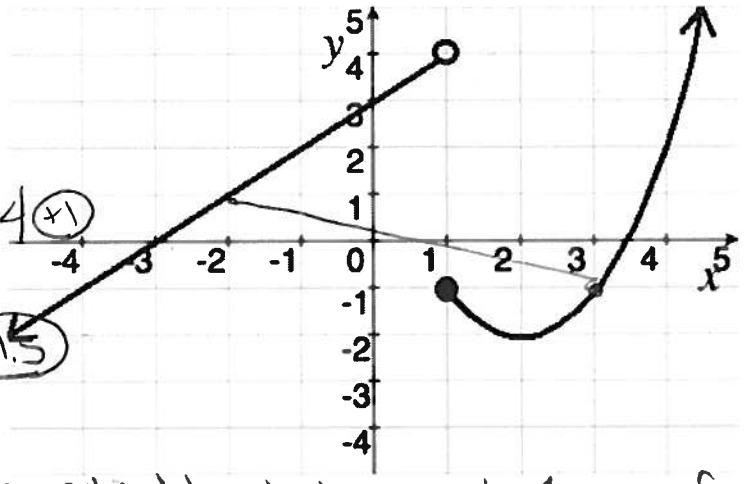
$-7\quad -7$

$(x+2)^2=-3$

$x+2=\pm\sqrt{-3}$

$x=-2\pm i\sqrt{3}$ +1.5

3. The graph of a piecewise defined function f is provided on the right. f consists of a linear part and a quadratic part.



(a) [4] (practice exam #3)
Evaluate the following:

$f(4) =$ 2 (+.5)

$f(1) =$ −1 (+1)

$-2f(0) + 2 = -2(3)+2 = 4$ (+1)

$(f \circ f)(-1) = f(f(-1))$
$= f(2) = -2$ (+1.5)

(b) [1] (WebHW2 #11)
If f a function? Why or why not?

yes b/c passes the vertical line test (+.5) (+.5) ie only 1 output for each input.

(c) [2] (WebHW4 #8) Determine the interval(s) on which f increasing.

$(-\infty, 1]$ and $[2, \infty)$ (+1) (+1)

(d) [2] (Quiz3 #2c) What is the average rate of change of f from $x = -2$ to 3?

(-2,1) and (3,-1) (+.5)

slope (+.5)

$\frac{-1-1}{3-(-2)} = \frac{-2}{5}$ (+1)

(e) [4] (practice exam #3) Find the formula for the function f in the indicated form.

$$f(x) = \begin{cases} x+3 & \text{if } x < 1 \\ (x-2)^2 - 2 & \text{if } x \geq 1 \end{cases}$$

(+.5) line form

line $y = mx + b$

(+.5) $m = 1$

(+.5) $b = 3$

$y = x + 3$ (+.5)

quadratic $y = a(x-h)^2 + k$
shifted right 2
and down 2
with vertex at (2,-2)

$y = (x-2)^2 - 2$ (+1) (+.5) quad form (+.5)

4. Let $f(x) = \dfrac{x-3}{3x+2}$.

(a) [2] (Quiz2 #2) What is the domain of f?

(+1) { denominator $\neq 0$

$3x+2 \neq 0$ (+.5)

$3x \neq -2$

$x = -2/3$ (+.5)

dg

or $(-\infty, -2/3) \cup (-2/3, \infty)$

(b) [3] (§2.4 #38) $f(x)$ is a one-to-one function. Compute $f^{-1}(x)$.

$x = \dfrac{f^{-1}(x) - 3}{3f^{-1}(x) + 2}$ (+1) (+1)

$\Rightarrow x(3f^{-1}(x)+2) = f^{-1}(x) - 3$

$\Rightarrow 3x f^{-1}(x) + 2x = f^{-1}(x) - 3$

$\Rightarrow 3x f^{-1}(x) - f^{-1}(x) = -2x - 3$

$\Rightarrow f^{-1}(x)[3x-1] = -2x-3$

$f^{-1}(x) = \dfrac{-2x-3}{3x-1}$

algebra (+1)

5. Let $f(x) = 2x^2 - 12x + 35$.

(a) [3] (§2.5 #14) Complete the square to put f in vertex form.

(+.5) $\dfrac{2x^2 - 12x + 35}{2} = \dfrac{y}{2}$ (+.5)

$x^2 - 6x + \dfrac{35}{2} = \frac{1}{2}y$

(+1) { $+9 \quad +9$

(+.5) { $(x-3)^2 + \dfrac{35}{2} = \frac{1}{2}y + 9$

$(x-3)^2 + \dfrac{35}{2} - \dfrac{18}{2} = \frac{1}{2}y$

$2\left[(x-3)^2 + \frac{17}{2}\right] = y$

$2(x-3)^2 + 17 = y$ } (+.5)

35
-18
17

(b) [4] (WebHW7 #9) Let $m(x) = 2x^3 - 20x^2 + 67x - 70$ and f be the same as that defined above. Use long division to find $G(x)$ and $R(x)$ so that $\dfrac{m(x)}{f(x)} = G(x) + \dfrac{R(x)}{f(x)}$.

$2x^2 - 12x + 35 \;\overline{)\; 2x^3 - 20x^2 + 67x - 70}$

x (+.5) $\quad -4$ (+.5)

$-(2x^3 - 12x^2 + 35x)$

$-8x^2 + 32x - 70$

$-(-8x^2 + 48x - 140)$

$-16x + 70$ remainder } (+1)

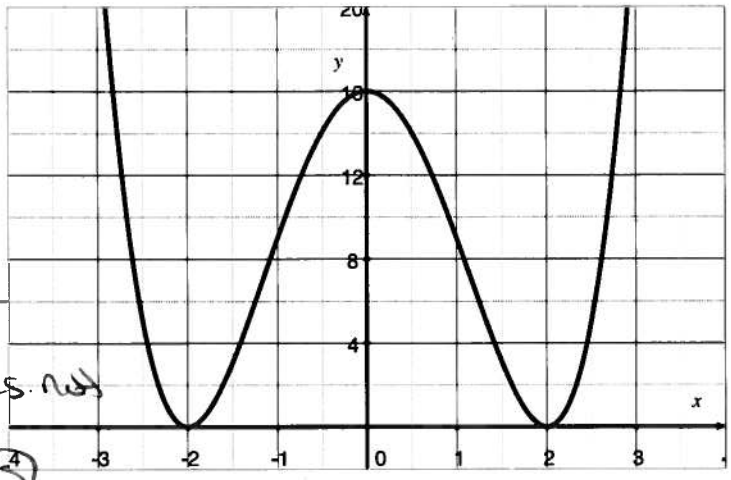
algorithm (+1)

subtraction (+1)

$\dfrac{2x^3 - 20x^2 + 167x - 70}{2x^2 - 12x + 35}$

$= x - 4 + \dfrac{-16x + 70}{2x^2 - 12x + 35}$

6. [3] (§3.2 #65) Find a degree 4 polynomial with the graph shown to the right.



(+.5) -2 is a root $\Rightarrow (x-2)$ is a factor

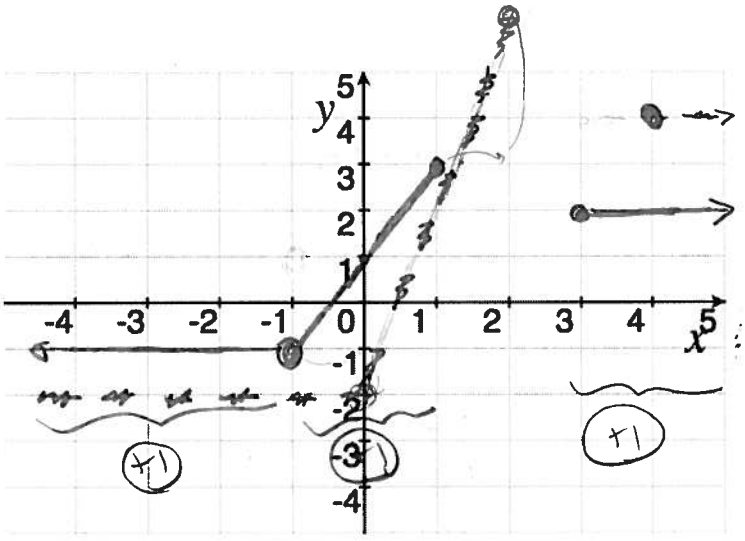
(+.5) 2 is a root $\Rightarrow (x-2)$ is a factor

(+1) at $x=2$ & -2, graph does not cross x-axis

(+1.5) $(x+2)^2(x-2)^2$ deg 4 (+.5)

7. Consider the function defined by

$$f(x) = \begin{cases} -1 & \text{if } x < -1 \\ 2x+1 & \text{if } -1 \le x \le 1 \\ 2 & \text{if } 3 \le x \end{cases}$$



(a) [3] (§2.2 #44) Graph the function f

(b) [1] Find the x-intercept.

when $y=0$ (+.5)

$0 = 2x+1$

$-1 = 2x$

$-\frac{1}{2} = x$ (+.5)

(c) [3] (transformation wks #5) Given $g(x) = 2f(x-1)$, draw the graph of g.

↓ vertical stretch by 2 (+1)

→ shift to the right by 1 unit (+1)

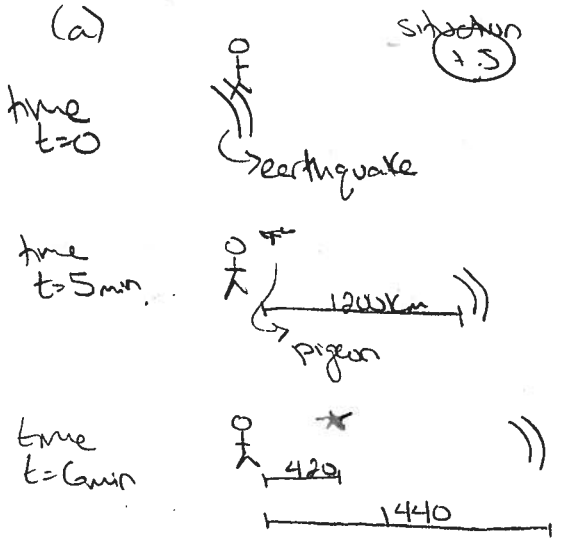
did it.

8. [5] (Story Problem Worksheet) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit. No, doing both questions will not earn you extra credit.

   (a) (story wks #3) Seismic waves travel at about 4km/s but Megan has (a really fast!) carrier pigeon that travels 7km/s. Assume that Megan's first instinct when feeling a quake is to "tweet" the experience and that process (recognizing it's an earthquake, finding her carrier pigeon, attaching a message to the bird's leg, and the message being broadcast) takes 5 minutes. How far does a Megan follower have to be from Megan to know there is an earthquake before feeling it? (inspired by http://xkcd.com/723)

   (b) (practice exam #12) A manufacturer of soft drinks advertises their orange soda as "naturally flavored", although it contains only 6% orange juice. A new federal regulation stipulates that to be called "natural" a drink must contain at least 15% fruit juice. The manufacturer mixes their juices in closed 900 gallon containers (to avoid contamination). How much juice must they remove from the 900 gallon container and replace with pure orange juice to conform to the new regulation?

(a)



r_w = rate of seismic wave
$= 4\,km/s \cdot \frac{60\,sec}{1\,min} = 240\,km/min$

r_p = rate of pigeon
$= 7\,km/s \cdot \frac{60\,sec}{1\,min} = 420\,km/min$ } +1

d_w = distance wave has traveled
d_p = distance pigeon has traveled
t_w = time wave has been moving
t_p = time pigeon has been moving

when does the pigeon catch up?
ie when $d_w = d_p$
$r_w t_w = r_p t_p$ } +.5

$240\,t_w = 420\,t_p$

notice $t_w = t_p + 5$ b/c } +1
the wave had a 5min head start

So
$240(t_p + 5) = 420\,t_p$ } +.5 sub 5

$240t_p + 1200 = 420t_p$
$-240t_p \qquad -240t_p$
$1200 = 180t_p$
$\frac{20}{3} = \frac{60}{9} = \frac{1200}{180} = t_p$ } +1 alg

distance is then
$420 \cdot \frac{20}{3} = \frac{8400}{3}\,km$ } +.5
$\approx 2800\,km$

(b)

situation (+.5)

s → amount of original mixture keep (gal)
f → amount of pure fruit juice added (gal) } (+1)

use { s + f = 900 gal } (+.5)

{ orange juice want = orange juice from original + pure orange juice add } (+1)

$900 \cdot .15 = s \cdot .06 + f \quad (*)$

since $s + f = 900 \Rightarrow f = 900 - s$

(+.5) { sub into (*) } $\Rightarrow 900.15 = s \cdot .06 + (900 - s)$

alg (+1)

$135 = .06s + 900 - s$

$-900 \qquad -900$

$\frac{-765}{-.94} = \frac{-.94s}{-.94}$

$s = 813.8 \approx 814$ gal

$\Rightarrow f = 900 - 814$

(+.5) { = 86 gal replaced w/ pure juice