EXAM 1 **TMath120** Fall 2011

NAME: Key

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be non-zero real numbers.

T F $\frac{1}{x}+\frac{1}{x+1}=\frac{3}{x+1}$ $\left(\frac{x+1}{x+1}\right)\frac{1}{x}+\frac{1}{x+1}\left(\frac{x}{x}\right) = \frac{x+1+x}{x(x+1)}$

T F $-2^4=-16$ $-2^4 = -2222 = -4 \cdot 4 = -16$

T F $2x+1$ is a polynomial.

Fix → T F $\frac{5-4i}{1-5i}=\frac{25}{26}+\frac{21i}{26}$ $\frac{5-4i}{1-5i}\cdot\frac{1+5i}{1+5i} = \frac{5+25i-4i+20}{1+25} = \frac{25}{26}\frac{21}{26}$

T F $(x+y)^2=x^2+y^2$ $(x+y)^2 = (x+y)(x+y) = x^2+2xy+y^2$

T F All functions pass the horizontal line test. ex $y=x^2$ ie parabola

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] (§1.5 #17) Solve for x given:

$$x\left[\frac{1}{x}\right]=\left[\frac{4}{3x+2}+1\right]x$$

$$3x+2\left[1\right]=\left[\frac{4x}{3x+2}+x\right]3x+2$$

$$3x+2 = 4x+3x^2+2x$$
$$-3x-2 \quad -3x \quad -2$$

$$\frac{0}{3} = \frac{3x^2+3x-2}{3}$$

$$0 = x^2+x-2/3$$

$$0 = x^2+x \quad +2/3$$
$$+(\tfrac{1}{2})^2 \quad +(\tfrac{1}{2})^2$$

$$\frac{1}{4} = (x+\tfrac{1}{2})^2 - 2/3$$
$$+2/3 \quad +2/3$$

$$\sqrt{\frac{3}{12}+\frac{8}{12}} = \sqrt{(x+\tfrac{1}{2})^2}$$

$$\pm\sqrt{\frac{11}{12}} = x+\tfrac{1}{2}$$

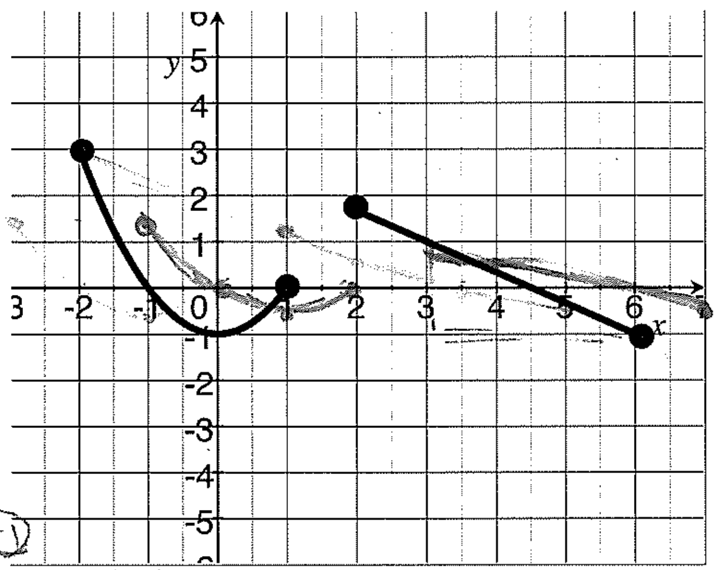
$$-\tfrac{1}{2}\pm\sqrt{11/12} = x$$

1.5 1.5 .5 1 1.5 1.5 1 8

or $\frac{-1\pm\sqrt{1^2-4(1)(-2/3)}}{2(1)}$ $\frac{-1\pm\sqrt{11/3}}{2}$

$= -\tfrac{1}{2}\pm\tfrac{1}{2}\sqrt{11/3} = -\tfrac{1}{2}\pm\sqrt{\tfrac{11}{12}}$

3. Let the following be the graph be a piece-wise defined graph of g comprised of a parabola shifted and a straight line.



(a) [1] (WebHW2 #11) Is g a function? Why or why not?

yes b/c it passes the vert line test. (+1)

(b) [2] (§2.2 #24b) What is the domain of g?

$[-2,1] \cup [2,6]$

end points (+1) #'s (+1)

(c) [1] (§2.2 #26) Estimate the value of $g(-2)$.

3

(d) [2] (WebHW3 #9) Estimate the value of $g \circ g(4.5)$.

$(g\circ g)(4.5) = g(g(4.5)) = g(0) = -1$ (+.5) (+.5) (+.5)

notation (+.5)

(e) [1] (WebHW4 #9) Estimate the average rate of change between -2 and 6.

$\frac{g(6)-g(-2)}{6--2} = \frac{-4}{8} = \frac{-1}{2}$ (+.5) (+.5)

(f) [4] (Quiz) Find a formula for g in the indicated form:

$$g(x) = \begin{cases} x^2-1 & \text{if } -2 \le x \le 1 \\ -2/3x+3 & \text{if } 2 \le x \le 6 \end{cases}$$

quad (+1) shift (+.5)

line (+1)

slope thru $(3,1)$, $(6,-1)$ $\Rightarrow -2/3$ (+.5)

pass thru $(3,1)$ so $1 = -2/3(3)+b \Rightarrow 1+2=b$ (+.5)

strategy (+.5)

(g) [2] (§2.4 #20) Sketch the graph of $\frac{1}{2}g(x-1)$ on the graph above.

↳ vert shrink by $1/2$ (+.5)

↳ horiz shift right 1 unit.

implemented (+.5)

(+.5) (+.5)

$\frac12 g(2-1) = \frac12(g(0)) = \frac12(-1) = -\frac12 \Rightarrow (1,-\frac12)$

4. Let $f(x) = 2x^2 - 16x + 35$.

(a) [3] (§2.5 #14) Complete the square to put f in vertex form.

factor (+.5)

(+.5) $\frac{f(x)}{2} = 2\,\frac{x^2 - 16x + 35}{2}$

$\frac{1}{2}f(x) = x^2 - 8x + \frac{35}{2}$

(+1) $+\left(\frac{-8}{2}\right)^2 \quad + \left(\frac{-8}{2}\right)^2$

$\frac{1}{2}f(x) + 16 = x^2 - 8x + 16 + \frac{35}{2}$

$\frac{1}{2}f(x) + 16 = (x-4)^2 + \frac{35}{2}$
$\quad -16 \qquad\qquad -16$

$2\left[\frac{1}{2}f(x)\right] = \left[(x-4)^2 + \frac{35}{2} - \frac{32}{2}\right]2$

$f(x) = 2(x-4)^2 + \frac{3}{2}\cdot 2$

$f(x) = 2(x-4)^2 + 3$ alg (+.5)

form (+.5)

(b) [2] (Quiz 3 #1b) Find all the roots of f (including complex ones).

roots are when $f(x) = 0$

(+.5) ie $0 = 2x^2 - 16x + 35$

alg (+.5) note from above we can also

recognize (+.5) write $0 = 2(x-4)^2 + 3$

$-3 = 2(x-4)^2$

$\pm\sqrt{\frac{-3}{2}} = \sqrt{(x-4)^2}$

$\pm\sqrt{\frac{-3}{2}} = x - 4$

$x = 4 \pm i\sqrt{\frac{3}{2}}$

get ans (+.5)

(c) [4] (WebHW7 #9) Let $m(x) = 2x^3 - 20x^2 + 67x - 70$ and f be the same as that defined above. Use long division to find $G(x)$ and $R(x)$ so that $\dfrac{m(x)}{f(x)} = G(x) + \dfrac{R(x)}{f(x)}$.

(+.5) (+.5)

$x - 2 \quad R0$

$2x^2 - 16x + 35 \,\big)\, 2x^3 - 20x^2 + 67x - 70$
$\quad -(2x^3 - 16x^2 + 35x)$
$\qquad -4x^2 + 32x - 70$
$\quad -(-4x^2 + 32x - 70)$

set up (+.5)

subtract (+1)

alg (+.5)

algorithm (+1)

$\dfrac{2x^3 - 20x^2 + 67x - 70}{2x^2 - 16x + 35} = x - 2 + \dfrac{0}{2x^2 - 16x + 35}$

5. Let $\alpha(x) = \dfrac{x-1}{-2x+7}$ and $\beta(x) = \sqrt{7-3x}$.

(a) [2] (§2.1 #50) What is the domain of β?

$7 - 3x \geq 0$ (+1)

$\Rightarrow -3x \geq -7$ alg (.5)

$\Rightarrow x \leq 7/3$ flip (.5) or $(-\infty, 7/3]$

(+.5)

(b) [2] (WebHW3 #10) Find $(\alpha \circ \beta)(x)$. Do *not* simplify.

$(\alpha \circ \beta)(x) = \alpha(\beta(x)) = \alpha\left(\sqrt{7-3x}\right) = \dfrac{\sqrt{7-3x} - 1}{-2\sqrt{7-3x} + 7}$

comp (+.5)

notation (+.5)

(+.5)

(c) [3] (§2.8 #38) Given that α has an inverse, find α^{-1}.

(+1) $x = \dfrac{y-1}{-2y+7}$

$x(-2y+7) = y-1$

$-2xy + 7x = y - 1$

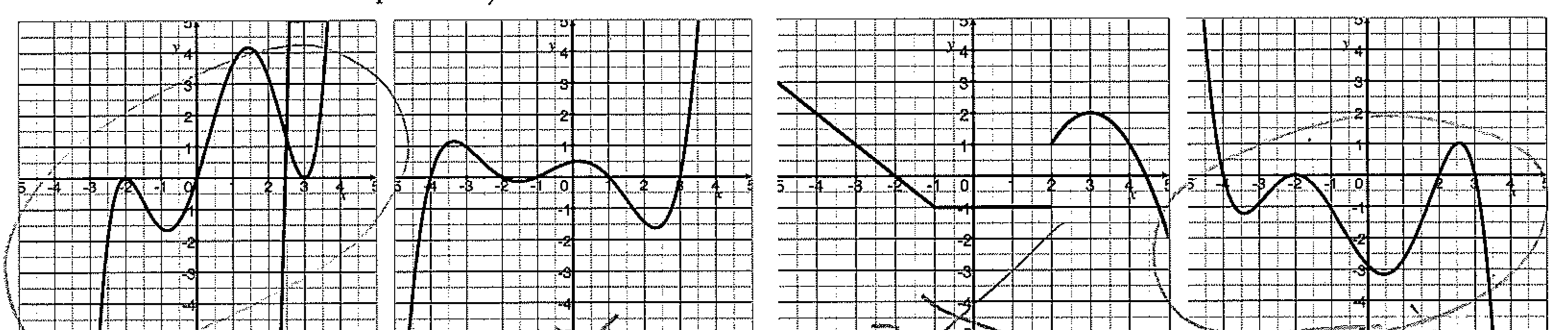
$7x + 1 = y + 2xy$

$7x + 1 = y(1+2x)$

$\dfrac{7x+1}{1+2x} = y$

alg (+.5)
factor (+.5)
legal op (+.5)
y on 1 side (+.5)

6. [2] (WebHW7 #2) Let h be a 5^{th} degree polynomial that has $(x+2)^2$ as a factor (but $(x+2)^3$ is not a factor). Which of the following could be the graph of h? (Circle all that are possible.)



(+.5)

b/c not continuous.

(+.5)

b/c $(x+2)^2$ is a factor -2 is a root & the graph touches but does not cross

no more (+.5)
(+.5) if get 1

7. [4] (§1.2 #69) Simplify the following as much as possible (remember to show your work):

$$\frac{(9st)^{\frac{3}{2}}}{(27s^3t^{-4})^{\frac{2}{3}}}$$

(+.5) (+.5) $\dfrac{9^{3/2} s^{3/2} t^{3/2}}{27^{2/3} (s^3)^{2/3} (t^{-4})^{2/3}} = \dfrac{9^{3/2} s^{3/2} t^{3/2}}{27^{2/3} s^{2} t^{-8/3}}$ (+.5) $= \dfrac{27 s^{3/2-2} t^{3/2+8/3}}{9}$ (+1)

alg (+.1)

coef (+.5) $= 3 s^{-1/2} t^{25/6}$

8. [5] (Story Problem Worksheet) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit. No, doing both questions will not earn you extra credit.

(a) You would like to set the price for a UWT fund-raising raffle. You did a similar thing last year and when you set the price to \$6 about 63 people bought tickets. The stats class did some research for you and reported that if ticket prices reduced by \$3.15, sales would increase by about 21 tickets. What price should you set the tickets so as to maximize income from ticket sales (to the nearest penny)?

(b) A manufacturer of soft drinks advertises their orange soda as "naturally flavored", although it contains only 5% orange juice. A new federal regulation stipulates that to be called "natural" a drink must contain at least 10% fruit juice. The manufacturer mixes their juices in closed 900 gallon containers (to avoid contamination). How much juice must they remove from the 900 gallon container and replace with pure orange juice to conform to the new regulation?

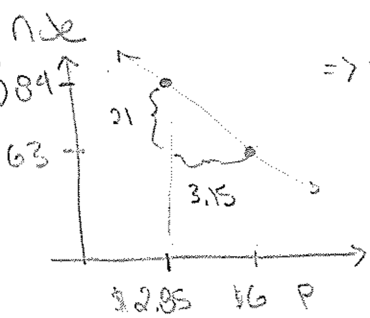
Start (+.5)

a) Let p be the price of the tickets and q be the # of tickets sold. variables (+.5) matching (+.5)

We want to maximize income

ie Income = p·q product (+.5)

note



(+1)

⇒ relation between p & q is a straight line

$q = mp + b$

$m = \dfrac{-21}{3.15} = \dfrac{-2100}{315} = \dfrac{-420}{63}$

passes thru (6, 63) so $63 = \dfrac{-420}{63} 6 + b$

$\Rightarrow b = 63 + \dfrac{420}{63} \cdot 6 = 63 + \dfrac{420 \cdot 2}{21} = \dfrac{1323 + 840}{21}$

= 483 ... -420 = 2163 = -20 ≈ 103

So

Income = p·q

$= p\left(-\dfrac{20}{3} p + 103\right)$

(+.5) $= -\dfrac{20}{3} p^2 + 103p$

which is a parabola opening down. The max happens @ the vertex (+.5) which is when

$p = \dfrac{-103}{2\left(-\frac{20}{3}\right)} = \dfrac{-103 \cdot 3}{40} \approx \$7.725$

find it (+.5)

So \$7.73 is the price that will maximize incom

Start (+.5)

Let x be the amount of pure orange juice used and y be the amount of original mixture kept. } variables (+1) matching (+.5)

Note

full tank = amount of mixture kept + amount of pure orange juice added

$$\underbrace{900 = y + x}_{(+.5)}$$

% of orange juice wanted = pure orange juice from original + pure orange juice added

$$\underbrace{.10 \cdot 900}_{\text{product } (+.5)} = \underbrace{.05y + x}_{(+1)}$$

$\Rightarrow$ $y = 900 - x$

sub into $90 = .05y + x$ to get

$90 = .05(900 - x) + x$ & solve for x } (+1)

$\Rightarrow 90 = 45 - .05x + x$

$\Rightarrow 90 = \cancel{45} + .95x$

$-45 \quad -\cancel{45}$

$45 = .95x$

$\frac{45}{.95} = x$

$\Rightarrow x = 47.37$ gal

so remove 47.4 gal & replace with pure orange juice