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EXAM 2 **TMath 120** Fall 2011

NAME: Key

1. [8] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

   T (F) For all numbers x, $\sqrt{x^2} = x$ $\sqrt{(-2)^2} = \sqrt{4} = 2 \neq -2$

   (T) F For all numbers $x^{-2} = \frac{1}{x^2}$

   T (F) For all numbers x and b, $\sqrt{b+x} = \sqrt{b} + \sqrt{x}$ $\sqrt{1+1} = \sqrt{2} \neq \sqrt{1} + \sqrt{1} = 2$

   T (F) For all positive numbers x and y, $\log(x+y) = \log(x) \cdot \log(y)$ $\log(x \cdot y) = \log x + \log y$

   (T) F For all positive numbers b, $\log_b 1 = 0$ b/c $b^0 = 1$ for all $b > 0$

   T (F) For all positive numbers x, $\log 8x^2 = 2 \log 8x$ $2 \log 8x = \log(8x)^2$

   T (F) $\frac{\pi}{4}$ radians is equal to $\frac{9\pi}{4}$ one circles completely around before stopping

   (T) F $\frac{3\pi}{2}$ radians is the same angle as 270° 

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [4] (mini-quiz) Let t be a non-zero real number, solve for r and simplify given:

$$\frac{1}{\frac{1}{r} + \frac{1}{2}} = t$$

start +.5
clear den +1
frac add +1
order of op +.5
simplify +.5
r on one side +.5

$$1 = t\left(\frac{1}{r} + \frac{1}{2}\right)$$

$$1 = \frac{t}{r} + \frac{t}{2}$$

$$-\frac{t}{2} \qquad -\frac{t}{2}$$

$$r \cdot 1 - \frac{t}{2} = \frac{t}{r} \cdot r$$

$$\frac{r\left(1 - \frac{t}{2}\right)}{1 - \frac{t}{2}} = \frac{t}{1 - \frac{t}{2}}$$

$$r = \frac{t}{1 - \frac{t}{2}}$$

$$= \frac{t}{\frac{2}{2} - \frac{t}{2}}$$

$$= \frac{t}{\frac{2-t}{2}} \cdot \frac{2}{2}$$

$$= \frac{2t}{2-t}$$

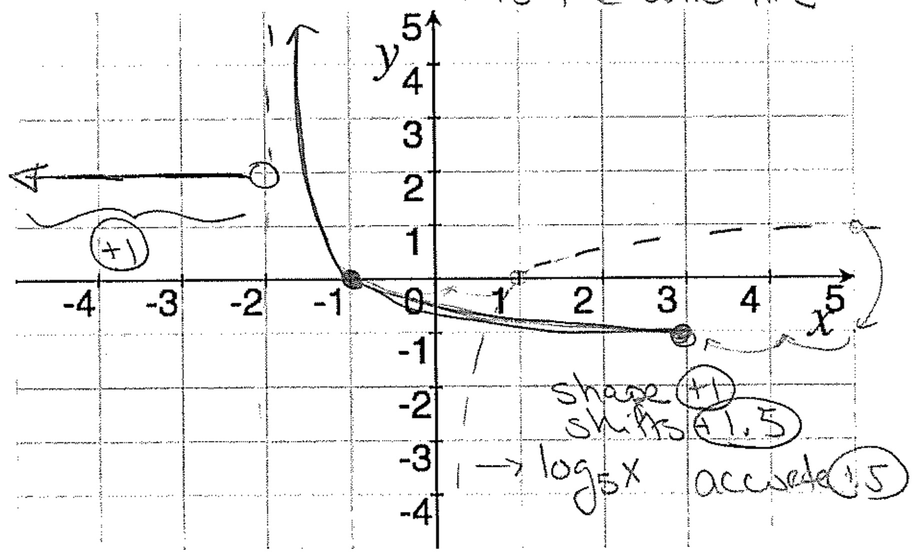
3. Let f be the *function* defined by:

$$f(x) = \begin{cases} 2 & x < -2 \\ -\log_5(x+2) & -2 < x \le 3 \end{cases}$$

(a) [4] (§4.2 #41) Graph f.

$-\log_5(x+2)$ — vert. flip, partial +.5; horiz left 2 units, partial +1

f is the solid line



(b) [1] (§4.2 #53) What is the range of f?

$[-1, \infty)$

4. [5] (WebHW9 #14 & Practice Exam #4) Simplify each of the following:

(a) $\log_3\left(\frac{1}{27}\right) = \log_3(3^{-3}) = -3$

(+1)

cancels log & 3 (+.5)
got it (+.5)

(b) $\frac{\sqrt[3]{(x^2)}(y^2)^{\frac{3}{2}}}{x^{\frac{2}{3}}y^{-2}} = \frac{(x^2)^{1/3}(y^2)^{3/2}}{x^{2/3}\,y^{-2}} = \frac{x^{2/3}\,y^{3}}{x^{2/3}\,y^{-2}} = y^3y^2 = y^5$

Let x & y be positive #'s

(+.5)
(+1) mult exp
(+1) combine x's & y's
(+.5) alg/notation

5. Find all x that satisfy the following:

(a) [3] (WebHW10 #12) $4(1 + 10^{9x}) = 5$

order of op (+1) log (+.5) used correctly (+.5) alg (+1)

$1 + 10^{9x} = 5/4$

$10^{9x} = 5/4 - 1$

$10^{9x} = 1/4$

$\log 10^{9x} = \log 1/4$

$9x = \log .25$

$x = \frac{\log .25}{9}$

(b) [3] (§4.4 #49) $\ln(x - 5) - \ln(4) = \ln(5) - \ln(x + 3)$

log prop (+.5) exp found (+.5) used correctly (+.5) solve good (+.5) check answer (+.5)

$\ln(x-5) + \ln(x+3) = \ln 5 + \ln 4$

$\ln[(x-5)(x+3)] = \ln(5 \cdot 4)$

$e^{\ln[(x-5)(x+3)]} = e^{\ln(20)}$

$(x-5)(x+3) = 20$

$x^2 - 2x - 15 = 20$

$x^2 - 2x - 35 = 0$

$(x-7)(x+5) = 0$

$x - 7 = 0$ or $x + 5 = 0$

$x = 7$ or ~~$x = -5$~~

Check: $\ln(7-5) - \ln(4) = \ln 5 - \ln(7+3)$ ✓

6. [3] (Circle Wks #4) Find the point(s) ~~on~~ that are on ~~the unit~~ circle *and* on the line $y = x$.

points on unit circle satisfy $x^2 + y^2 = 1$ (+.5)

b/c also on the line $x = y$ (sub +1)

$\Rightarrow y^2 + y^2 = 1$

$\Rightarrow 2y^2 = 1$

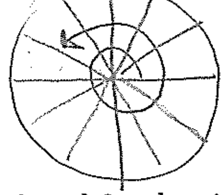
$y^2 = 1/2$

$y = \pm\sqrt{1/2}$ (alg +1) (both answers +.5)

so b/c $y = x$: $(\sqrt{1/2}, \sqrt{1/2})$ & $(-\sqrt{1/2}, -\sqrt{1/2})$

7. (§6.1 #43) Let $\theta = \dfrac{17\pi}{6}$ be and angle in standard position.

(a) [2] Sketch the angle θ.



note $\frac{17\pi}{6}$ rad $\cdot \frac{180°}{\pi \text{ rad}} = 17 \cdot 30 = 510°$

partial +1

start (+.5) div into $\pi/6$'s (+.5) once around (+.5) got it (+.5)

(b) [2] Find and angle between 0 and 2π that is coterminal with θ.

$\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$ (+1)

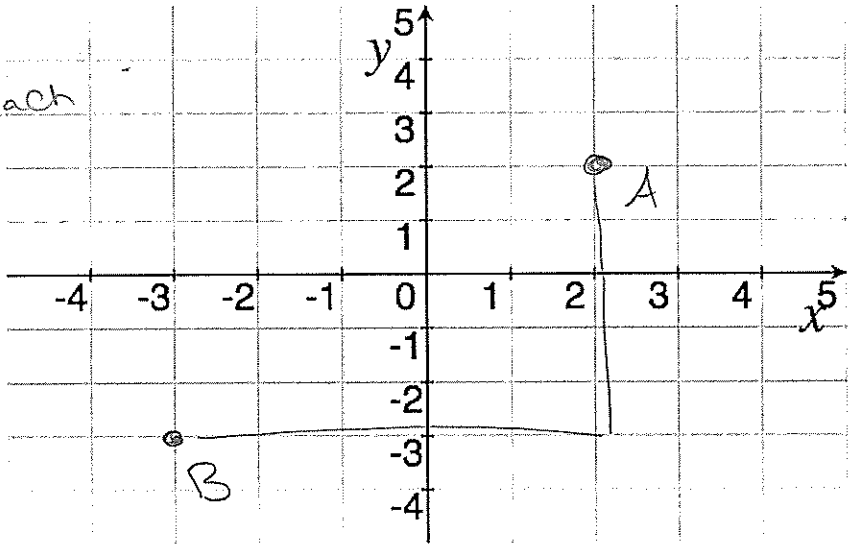
notation (+.5) got it (+.5)

note $\frac{5\pi}{6}$ rad $\cdot \frac{180°}{\pi \text{ rad}} = 5 \cdot 30 = 150°$ works too

8. Let points A and B have the coordinates $(2, 2)$ and $(-3, -3)$, respectively.

(a) [1] (§1.8 #1) Plot the points A and B on the axes provided. +.5 each

(b) [4] (§1.8 #84) Find an equation of the circle with the endpoints of a diameter A and B.



(+.5) $(x-h)^2 + (y-k)^2 = r^2$

note distance $\overline{AB}$ is $\sqrt{(2-(-3))^2 + (2-(-3))^2}$ (+1)

$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

$\Rightarrow$ radius is $\frac{5\sqrt{2}}{2}$ (+.5)

center is the midpoint of A & B so (+1)

$\left(\frac{2+(-3)}{2}, \frac{2+(-3)}{2}\right) = \left(-\frac{1}{2}, -\frac{1}{2}\right)$

so $\left(x+\frac{1}{2}\right)^2 + \left(y+\frac{1}{2}\right)^2 = \left(\frac{5}{2}\right)^2$

or $\left(x+\frac{1}{2}\right)^2 + \left(y+\frac{1}{2}\right)^2 = \frac{50}{4}$

plug in (+1)

9. A sound with intensity x has $10 \log \frac{x}{I_0}$ decibels, where $I_0 = 10^{-12}$ watts per square meter (W/m^2).

(a) [2] (§4.5 #39) France passed a law limiting iPods and other MP3 players to a maximum possible volume of 100 decibels. Find the maximum intensity (in W/m^2) an iPod is legally allowed to output in France.

(+.5) knows what looking for

(+.5) $\frac{100}{10} = \frac{10 \log \frac{x}{10^{-12}}}{10}$

alg (+1.5)

alg with exp (+1.5)

$10 = \log \frac{x}{10^{-12}}$

$10^{10} = 10^{\log \frac{x}{10^{-12}}}$

$10^{-12} \cdot 10^{10} = \frac{x}{10^{-12}} \cdot 10^{-12}$

$10^{-2} = x \Rightarrow x = \frac{1}{100}$

(b) [3] (§4.5 #40) Normal conversation has a sound level of about 65 decibels. How many more times intense than normal conversation is the sound an iPod operating at the French maximum of 100 decibels?

(+.5) start (+1.5)

(+.5) know what looking for

Let I_c be the intensity of a conversation & I_p be the intensity of an iPod at max

we want to find K so that

$KI_c = I_p$ or $K = I_p/I_c$

(+.5) Part a $\Rightarrow I_p = 10^{-2}$

$I_c = 10^{-5.5}$

(+1) to find I_c: $65 = 10 \log \frac{I_c}{10^{-12}}$

$6.5 = \log \frac{I_c}{10^{-12}}$

$10^{6.5} = I_c/10^{-12}$

(+.5) so $I_p/I_c = \frac{10^{-2}}{10^{-5.5}} = 10^{3.5} \approx 3162$ times more $\Rightarrow I_c = 10^{6.5} \cdot 10^{-12} = 10^{-5.5}$

10. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

(a) (WebHW11 #7) The University of Washington has been raising its tuition rates a fixed percentage each year for the past 4 years. The '07 to '08 school year had a tuition of approximately \$6000. Last year ('10 to '11 school year) the tuition was approximately \$8,700.

i. What percentage rate is the University of Washington raising its tuition every year?

ii. If the University of Washington continues increasing tuition rates at the above fixed percentage, when will the cost of tuition exceed \$100,000?

(b) (Word Problem Wks #3) You took out a ten-year subsidized loan each of the four years you attended college to pay for tuition, books, and miscellaneous academic fees. Below is a table of the loans taken and their respective effective annual interest rates (AIR):

loan (\$)	7,500	8,000	9,000	10,000
AIR (%)	3.51	4.22	5.01	6.31

Start (1.5) After graduation you are given the option of consolidating (that is take out one loan to pay off *all* the balances on your current loans). Assume all loans are continuously compounded once interest start accruing. You do not have a job lined up yet so you doubt you will be able be to make any payments for the three years, what rate would you need to consolidate your loans at to be in a better position three years from now?

a)

t	year	tuition
0	07-08	6000
1	08-09	
2	09-10	
3	10-11	8700

raising tuition a fixed percentage each year is exponential growth, I'll use Pe^{rt} (+1.5)

and let t be years since the 07-08 year (+1.5) consistent with time (+1.5)

Then P=6000, we need to find r

used to find r (+1.5)

because (3, 8700) is on the exp function

$\frac{8700}{6000} = \frac{6000 \cdot e^{r \cdot 3}}{6000}$

$\frac{87}{60} = e^{r3}$

$\ln \frac{87}{60} = \ln e^{r3}$ (+1)

$\frac{\ln \frac{87}{60}}{3} = \frac{3r}{3}$

$r = \frac{\ln(87/60)}{3} \approx .12385$

so 12.4%

Note: it could be argued that $P(1+r)^t$ is a better model b/c inc happens

b) find t so that

$\frac{100000}{6000} = \frac{6000e^{.124t}}{6000}$ (+1)

$16.\overline{6} = e^{.124t}$ alg (+1)

$\ln(50/3) = .124t$

$t = \frac{\ln(50/3)}{.124} = 22.7$ yrs

so 2030

b) if I don't consolidate in 3 years I'll owe

$$7500 \cdot e^{.0351 \cdot 3} + \underbrace{8000 e^{.0422 \cdot 3}}_{2^{nd}\text{ loan}} + \underbrace{9000 e^{.0501 \cdot 3}}_{3^{rd}\text{ loan}} + \underbrace{10000 e^{3.0(}}_{4^{th}\text{ loan}}$$

$\underbrace{\qquad}_{1^{st}\text{ loan}}$

$= \$39,976.21$

start (+.5)

Fe^{rt} (+.5)

let $t = 3$ (+.5)

compute (+1)

if I consolidate I need a principal of

$7500 + 8000 + 9000 + 10,000 = 34,500$ (+.5)

to do better than the loans I already have we need a rate r so that

$$\frac{39,976.21}{34500} > \frac{34500 e^{r \cdot 3}}{34500} \quad \} (+1)$$

$$\frac{39976.21}{34500} > e^{3r} \qquad \text{alg } (+1)$$

$$\Rightarrow \ln\left(\frac{39976.21}{34500}\right) > 3r$$

$$.049108 \approx \frac{\ln\left(\frac{39976.21}{34500}\right)}{3} > r$$

so r needs to be less than 4.9%