

TMATH 120 Algebra Review Sheet

Knowledge of the following topics is a prerequisite for TMATH 120. During the class I will assume proficiency at these skills. The “You Try” problems suggested are to be used only as a guideline-if you have trouble with the ones listed, *attempt more problems until you feel confident*.

Basic Simplification

Apply order of operations to numerical and algebraic expressions; recall the rules for exponents.

Ex 1. Simplify: $-3^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{1}{6}\right)^{-1} + (-2)^3$.

Solution:

$$-3^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{1}{6}\right)^{-1} + (-2)^3 = -9 + \frac{9}{4} + \frac{6}{1} + -8 = -17 + \frac{9}{4} + \frac{6}{1} = \frac{-68}{4} + \frac{9}{4} + \frac{24}{4} = \frac{-35}{4}$$

(Another answer is -8.75 . $-8\frac{3}{4}$ is also technically correct, but we avoid mixed fractions because of potential confusion: Taken out of context, $-8\frac{3}{4}$ could mean *either* $-(8 + \frac{3}{4})$ *or* $(-8) \cdot \frac{3}{4}$.

You Try: §1.2#9, 11, 23

Algebraic Manipulation

Add, subtract, multiply, and divide algebraic expressions; combine algebraic terms that are alike; apply the distributive property to algebraic expressions (the term “FOIL” *only* refers to distribution between two binomials; ask me to show you why FOIL works if you don’t remember).

Ex 2 Simplify by combining like terms: $(9x^2 + 4xy - 7y^2) - (5xy - 6x^2 - 1)$.

Solution:

$$(9x^2 + 4xy - 7y^2) - (5xy - 6x^2 - 1) = 9x^2 + 4xy - 7y^2 - 5xy + 6x^2 + 1 = 15x^2 - xy - 7y^2 + 1$$

Ex 3 Expand and simplify completely: $3(4v - 2a)^2$.

Solution:

$$\begin{aligned} 3(4v - 2a)^2 &= 3(4v - 2a) \cdot (4v - 2a) = 3[(4v)(4v) - (4v)(2a) - (2a)(4v) + (2a)(2a)] \\ &= 3[16v^2 - 16av + 4a^2] = 48v^2 - 48av + 12a^2 \end{aligned}$$

You Try: §1.3#7, 13, 15, 19, 21, 25, 39

Factoring

Factor binomials of the form $a^2 - b^2$; factor trinomials of the form $ax^2 + bx + c$ using factoring by grouping or trial and error; recognize and factor out the greatest common factor of an expression; check answers by using distribution.

Ex 5 Factor completely: $16w^4 - 81$.

Solution:

$$16w^4 - 81 = (4w^2 + 9)(4w^2 - 9) = (4w^2 + 9)(2w + 3)(2w - 3)$$

(Note that $4x^2 + 9$ does not factor any further since the *sum* of squares is prime.)

Ex 6 Factor completely: $12t^2 + 14t - 6$.

Solution:

$$12t^2 + 14t - 6 = 2(6t^2 + 7t - 3) = 2(2t + 3)(3t - 1)$$

You Try: §1.3#43, 45, 47, 49, 71, 73,

Equations of Lines

Find the slope of the line defined by two points; write the equation of a line in slope-intercept form.

Ex 4. Find the slope of the line passing through the points $(-2, 7)$ and $(4, -7)$.

$$\text{Solution: slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 7}{4 - (-2)} = \frac{-14}{6} = \frac{-7}{3}$$

You Try: §1.10 #3,9,11,15,17,19

Rational Expressions

Simplify fractions by finding factors in common; add/subtract fractions by first rewriting with the least common denominator; multiply/divide rational expressions; simplify complex fractions.

Ex 7 Reduce to lowest terms: $\frac{10x - 20}{2x^2 - 8}$.

$$\text{Solution: } \frac{10x - 20}{2x^2 - 8} = \frac{10(x - 2)}{2(x^2 - 4)} = \frac{10(x - 2)}{2(x + 2)(x - 2)} = \frac{2(x - 2)}{2(x - 2)} \cdot \frac{5}{x + 2} = \frac{5}{x + 2}$$

Ex 8 Perform the indicated operations and simplify: $\frac{2}{3x + 2} - \frac{1}{3x + 1}$.

$$\begin{aligned} \text{Solution: } \frac{2}{3x + 2} - \frac{1}{3x + 1} &= \frac{3x + 1}{3x + 1} \cdot \frac{2}{3x + 2} - \frac{3x + 2}{3x + 2} \cdot \frac{1}{3x + 1} \\ &= \frac{2(3x + 1) - (3x + 2)(1)}{(3x + 1)(3x + 2)} = \frac{6x + 2 - 3x - 2}{(3x + 1)(3x + 2)} = \frac{3x}{(3x + 1)(3x + 2)} \end{aligned}$$

(There is no further simplification here, $3x$ has *no* factors in common with $3x + 1$ or $3x + 2$.)

Ex 9 Perform the indicated operations and simplify: $\frac{2z + 6}{12z} \div \frac{z^2 - 9}{9z^3 + 18z^2}$.

$$\begin{aligned} \text{Solution: } \frac{2z + 6}{12z} \div \frac{z^2 - 9}{9z^3 + 18z^2} &= \frac{2z + 6}{12z} \cdot \frac{9z^3 + 18z^2}{z^2 - 9} = \frac{2(z + 3)}{12z} \cdot \frac{9z^2(z + 2)}{(z + 3)(z - 3)} \\ &= \frac{2 \cdot 3 \cdot z(z + 3)}{2 \cdot 3 \cdot z(z + 3)} \cdot \frac{3z(z + 2)}{2(z - 3)} = \frac{3z(z + 2)}{2(z - 3)} \end{aligned}$$

You Try: §1.4 #7,9,13,17,19,29,31,35,41

Solving Equations

Solve linear equations; solve quadratic equations by employing factoring or the quadratic formula.

Ex 10 Solve for m : $3(m + 4) + 2m = 4 - 3m$.

Solution

$$3(m + 4) + 2m = 4 - 3m \Rightarrow 3m + 12 + 2m = 4 - 3m \Rightarrow 8m = -8 \Rightarrow m = -1$$

Ex 11 Solve for q : $2q^2 + 7 = 9q$.

Solution

$$2q^2 + 7 = 9q \Rightarrow 2q^2 - 9q + 7 = 0 \Rightarrow (2q - 7)(q - 1) = 0 \Rightarrow q = \frac{7}{2} \text{ or } q = 1$$

You Try: §1.5 #3,7,9,11,17,19,39,55