

Key

NAME: This is a sample midterm to be used for practice. This is *not* a template for the midterm that will be given in class. Many of the questions on the Midterm will look quite different than those appearing here.

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

Let x and y be positive numbers.

T F $x^2 x^3 = x^6$

$$x^2 x^3 = (xx)(xxx) = x^5$$

T F $\log(x+y) = \log(x) \cdot \log(y)$

$$\log(xy) = \log x + \log y$$

T F $\log\left(\frac{x}{y}\right) = \log(x) - \log(xy)$

T F $\frac{\log x}{\log y} = \frac{x}{y}$

T F $\log_2 5x^7 = 7 \log_2 5x$

$$7 \log_2 5x = \log_2 (5x)^7$$

T F $\log(\log(10)) = 0$

$$\log(\log 10) = \log(1) = 0 \checkmark$$

T F For all numbers z , $\sqrt{z^2} = z$

$$\text{ex let } z = (-2)$$

LONG ANSWERS: Show all your work and circle your final answer. Correct answers will *not* get credit without supporting work.

2. Given $-x = \frac{2xy}{2y-1}$, solve for y .

$$(2y-1) \cdot -x = \frac{2xy}{2y-1} \cdot (2y-1)$$

$$\begin{array}{r} -2xy + x = 2xy \\ +2xy \qquad \qquad +2xy \\ \hline x = 4xy \end{array}$$

$$\frac{x}{4x} = \frac{4xy}{4x}$$

$$y = \frac{x}{4x} = \frac{1}{4} \quad \text{if } x \neq 0$$

3. [2] Define the rule of the function log.

$$\log_b a = x \text{ exactly when } b^x = a.$$

4. [4] Assume $b, x, y > 0$, simplify the following:

$$\frac{(b^x)^{x-1}}{b^{-x}}$$

rule $(b^a)^c = b^{ac}$

$$\frac{\sqrt[3]{x^2(y^2)^{\frac{3}{2}}}}{x^{\frac{2}{3}}y^2}$$

$$\begin{aligned} &= \frac{(x^2)^{\frac{1}{3}} (y^2)^{\frac{3}{2}}}{x^{\frac{2}{3}} y^2} \\ &= \frac{x^{\frac{2}{3}} y^3}{x^{\frac{2}{3}} y^2} \\ &= y^{3-2} = \textcircled{y} \end{aligned}$$

so

$$\begin{aligned} \frac{(b^x)^{x-1}}{b^{-x}} &= \frac{b^{x(x-1)}}{b^{-x}} \\ &= \frac{b^{x^2-x}}{b^{-x}} \\ &= b^{x^2-x-(-x)} \\ &= b^{x^2-x+x} \\ &= \textcircled{b^{x^2}} \end{aligned}$$

$$2 - \log_5(25z)$$

$$2 - [\log_5 25 + \log_5 z]$$

$$2 - \log_5 5^2 - \log_5 z$$

$$2 - 2 - \log_5 z$$

$$-\log_5 z \text{ or } \log_5 z^{-1}$$

$$\text{or } \log_5 \frac{1}{z}$$

$$\log_2 \frac{1}{4} + 2$$

$$= \log_2 4^{-1} + 2$$

$$= \log_2 ((2)^2)^{-1} + 2$$

$$= \log_2 2^{-2} + 2$$

$$= -2 + 2 = 0$$

5. [3] Find x in the following:

$$2^{4x-1} = 3^{1-x}$$

$$\ln 2^{4x-1} = \ln 3^{1-x}$$

$$(4x-1)\ln 2 = (1-x)\ln 3$$

$$4x\ln 2 - \ln 2 = \ln 3 - x\ln 3$$

$$4x\ln 2 + x\ln 3 = \ln 3 + \ln 2$$

$$x(4\ln 2 + \ln 3) = \ln 3 + \ln 2$$

$$x = \frac{\ln 3 + \ln 2}{4\ln 2 + \ln 3}$$

6. Find a formula for the inverse function f^{-1} of the indicated function f .

$$f(x) = 4x^{3/7} - 1$$

$$x = 4y^{3/7} - 1$$

$$x + 1 = 4y^{3/7}$$

$$\frac{x+1}{4} = y^{3/7}$$

$$\left(\frac{x+1}{4}\right)^{7/3} = y$$

7. Write the given expression as a single logarithm.

$$2\ln 2x - 3(\ln x^2 + \ln x)$$

$$2\ln 2x - 3(\ln x^2 \cdot x)$$

$$2\ln 2x - 3\ln x^3$$

$$\ln(2x)^2 - \ln(x^3)^3$$

$$\ln(4x^2) - \ln x^9$$

$$\ln \frac{4x^2}{x^9}$$

$$\ln \frac{4}{x^7}$$

$$5^x = 2$$

$$\ln 5^x = \ln 2$$

$$x \ln 5 = \ln 2$$

$$x = \frac{\ln 2}{\ln 5}$$

$$f(x) = 3 \cdot 2^x + 4$$

$$x = 3 \cdot 2^y + 4$$

$$x - 4 = 3 \cdot 2^y$$

$$\frac{x-4}{3} = 2^y$$

$$\log_2 \left(\frac{x-4}{3}\right) = y$$

$$3 - \log_6(36y)$$

$$\log_6 6^3 = 3 \text{ so}$$

$$3 - \log_6(36y)$$

$$= \log_6 6^3 - \log_6(36y)$$

$$= \log_6 \frac{6^3}{36y} = \log_6 \frac{6}{y}$$

8. [4] Solve for x :

$$\log(x-16) = 2 - \log(x-1)$$

$$\log(x-16) + \log(x-1) = 2$$

$$\log(x-16)(x-1) = 2$$

$$(x-16)(x-1) = 100$$

$$x^2 - 17x + 16 = 100$$

$$x^2 - 17x - 84 = 0$$

$$(x-21)(x+4) = 0$$

$$x-21 = 0 \text{ or } x+4 = 0$$

$$x = 21 \text{ or } x = -4$$

b/c Domain \Rightarrow

$$4^x - 3 \cdot 2^x = 10$$

$$(2^x)^2 - 3 \cdot 2^x = 10$$

$$\text{let } y = 2^x$$

$$y^2 - 3y - 10 = 0$$

$$(y-5)(y+2) = 0$$

$$y = 5 \text{ or } y = -2$$

9. [] Chani has \$20,000 in student loans at the end of her college education with 8% interest compounded quarterly. Chani chooses not to make payments after graduation since she has had trouble securing employment.

(a) How much money will Chani owe after 2 years assuming no extra fees will be incurred?

$$20,000 \left(1 + \frac{0.08}{4}\right)^{4 \cdot 2}$$

$2^x = 5$ or 2
 $\Rightarrow x = \log_2 5$
 or $\log_2 2$

(b) How long will it take for Chani's loan to double?

when will

$$40,000 = 20,000 \left(1 + \frac{0.08}{4}\right)^{4t}$$

$$2 = \left(1 + \frac{0.08}{4}\right)^{4t}$$

$$\ln 2 = \ln \left(1 + \frac{0.08}{4}\right)^{4t}$$

$$\ln 2 = 4t \ln \left(1 + \frac{0.08}{4}\right)$$

$$\frac{\ln 2}{\ln \left(1 + \frac{0.08}{4}\right)} = 4t$$

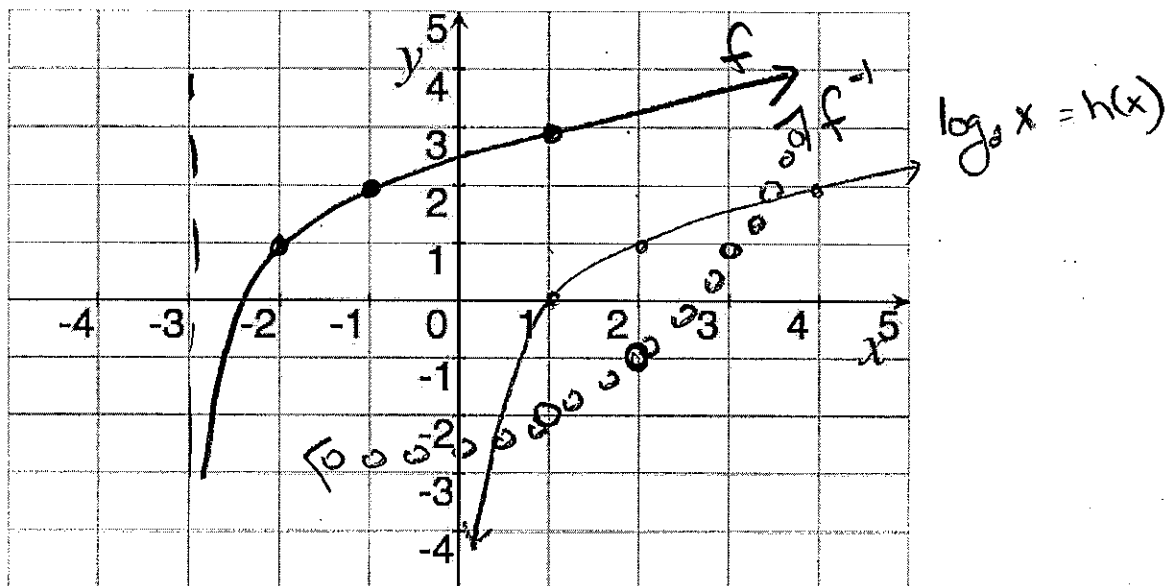
$$t = \frac{\ln 2}{4 \ln \left(1 + \frac{0.08}{4}\right)}$$

10. Let $h(x) = \log_2 x$ and $f(x) = \log_2(x+3) + 1$.

(a) List the transformations needed to transform the graph of h to the graph of f .

shift up 1 unit
shift left 3 units.

(b) Graph f .



(c) Find the inverse function to f .

note f passes the horiz. line test
so f has an inverse

$$x = \log_2(y+3) + 1$$

$$x - 1 = \log_2(y+3)$$

$$2^{x-1} = y+3 \Rightarrow 2^{x-1} - 3 = y$$

(d) Graph the inverse function to f .

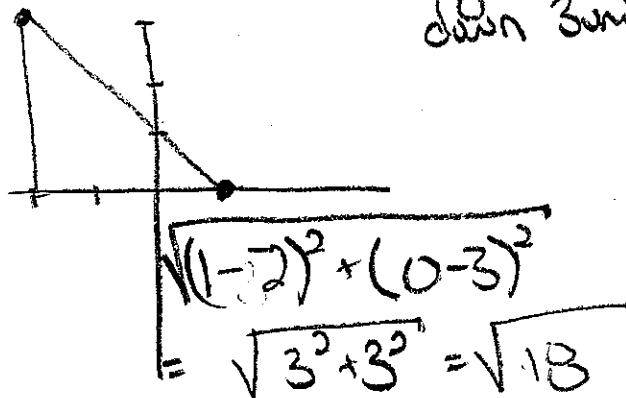
points on f : $(-2, 1)$
 $(-1, 2)$ \Rightarrow points on f^{-1} : $(1, -2)$
 $(2, -1)$

or the graph of 2^x
shifted
right 1 unit
down 3 units

11. Find the distance between the zero of $\log_2 x$ and the point $(-2, 3)$.

note the zero of $\log_2 x$
is when $\log_2 x = 0$
 $\Rightarrow x = 1$

So we want the distance
between $(1, 0)$ + $(-2, 3)$



12. At current growth rates, the Earth's population is doubling about every 69 years. If this growth rate were to continue, about how many years will it take for the Earth's population to become one-fourth larger than the current level?

Use $P_0 e^{rt}$ or $P_0 e^{rt}$

With $P_0 e^{rt}$ we need to find r .

Given when $t=69$, pop doubles so

$$\frac{2P_0}{P_0} = \frac{P_0 e^{r \cdot 69}}{P_0} \quad \text{solve for } r$$

$$2 = e^{69r} \Rightarrow \ln 2 = 69r$$

$$r = \frac{\ln 2}{69} \approx .0100456$$

When is pop $P_0 + \frac{1}{4}P_0 = \frac{5}{4}P_0$

So find t when

$$\frac{5/4 P_0}{P_0} = \frac{P_0 e^{.0100456 t}}{P_0}$$

$$\ln \frac{5}{4} = .0100456 t \Rightarrow t = 22 \text{ years}$$

13. Pay Day Loans can give you a 10% loan on \$250 for up to 45 days (the actual rate is 15% and doesn't jump down to 10% until \$500, but for the purposes of this problem, assume that you know someone at Pay Day Loans and they are giving you a "deal" with 10%). At the end of that 45 days you will have to pay off both the principal and the 10% interest on the principal. If you are unable to pay this amount at the end of the 45 days one option is to take out another loan to cover the new amount of money that you owe.

- (a) If you need a loan for \$250 on Jan. 1st of 2010 but don't have access to cash until Jan 1st of 2011, you might choose to go to Pay Day Loans and "renew" the loan every 45 days until the end of the year. If you choose to do this, how much money will you owe on Jan 1st 2011?

If you 'renew' your loan everytime it is due you will have to 'renew' it 9 times (8 doesn't quite make it to the end of the year).

So you'll owe $250(1+.10)^9 \approx \$589.49$

(assuming you can't cover any additional fees they throw at you).

- (b) What is the effective annual interest rate of the plan above?

We want to know r so that

$$250(1+r)^1 = 589.49$$

$$1+r = 2.35996$$

$$r = 1.35996$$

\Rightarrow the rate is 135.996% or 136% ∇