EXAM 2 **TQS 120** Fall 2010

NAME: Key

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let x and y be non-zero real numbers.

(T) F $\dfrac{1}{\frac{1}{2}a} = \dfrac{2}{a}$

T (F) $x^{-2} = x^{\frac{1}{2}}$ $\quad x^{-2} = \dfrac{1}{x^2}$ $\quad x^{\frac{1}{2}} = \sqrt{x}$

(T) F $\log_3 x = \dfrac{\log x}{\log 3}$

T (F) $(x+3)^2 = x^2 + 9$ $\quad (x+3)^2 = (x+3)(x+3) = x^2 + 6x + 9$

(T) F $\log(\log(10)) = 0$ $\quad \log(1) = 0$

(T) F The graph of $\log_3 x$ is increasing from $(0, \infty)$

(T) F $\log(100x) = 2 + \log x$ $\quad \log 100 + \log x = \log 10^2 + \log x = 2 + \log x$

(T) F $\ln(\frac{x}{y}) = \ln x - \ln y$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [3] (Exponent Wks #4) Find all x so that:

$3x^{-2} + 1 = 6$

$\dfrac{3}{x^2} + 1 = 6$

$\dfrac{3}{x^2} = 5$

$3 = 5x^2$

$\dfrac{3}{5} = x^2$

$x = \pm\sqrt{\frac{3}{5}}$

$\approx \pm .774597\ldots$

started +.5
neg exp +.5
alg/order of op +1
both answers +.5
square root +.5

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3^x vert flipped

3. Let the piece-wise defined function f be defined by: $f(x) = \begin{cases} -3^x & \text{if } x < 1 \\ x - 5 & \text{if } 1 \le x < 3 \end{cases}$

(a) [4] (§2.2 #49 & 4.1 #13) Graph the function f.

(b) [2] (§4.1 #25)What is the domain of f?

$(-\infty, 3)$

endpts #15 (+1) (+1)

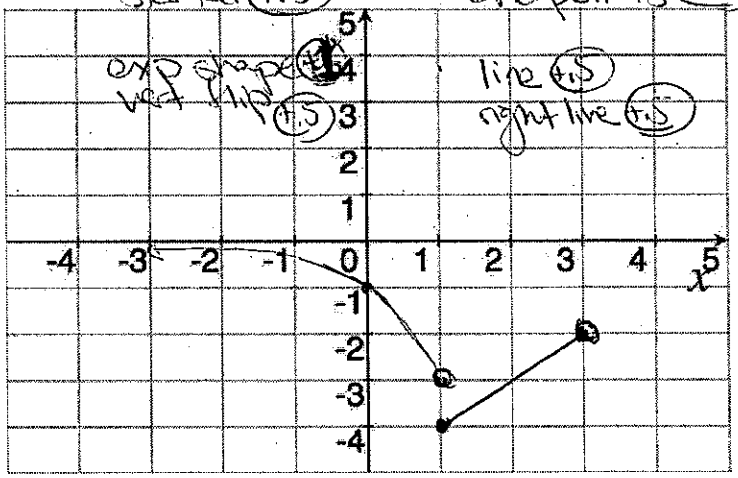
(c) [2] Does f have an inverse? Why or why not?

(+.5) nope: f fails the horiz line test. (+1)

started (+.5)

started (+.5) endpoints (+1.0)

exp shape (+1) vert flip (+.5) line (+.5) right line (+.5)



4. [3] (§4.1 #43) Let $g(x) = 2^x$. Find the *difference quotient* of g at 1. Do *not* simplify. That is find:

$$\frac{g(1+h) - g(1)}{h}.$$

(and do *not* simplify!)

started (+.5)

$g(1+h) = 2^{1+h}$ (+1)

$g(1) = 2^1 = 2$ (+.5)

So

$$\frac{g(1+h) - g(1)}{h} = \frac{2^{1+h} - 2}{h}$$ (+.5)

~~started~~ simplified wrong (-.5)

5. [3 each] Find x in the following:

(§4.4 #28) $x^2 3^x = 3^x$

started +.5
order op/alg +1
factoring +.5
checked/cases +.5
both +.5

$x^2 3^x - 3^x = 0$

$3^x(x^2-1) = 0$

$3^x(x+1)(x-1) = 0$

$3^x = 0$ or $x+1=0$ or $x-1=0$

$\Rightarrow$ $x = -1$ or 1

(§4.4 #19) $5^x = 4^{x+1}$

started +.5
used log +.5
prop log +1
order of op/alg +1

$\ln 5^x = \ln 4^{x+1}$

$x \ln 5 = (x+1)\ln 4$

$x \ln 5 = x \ln 4 + \ln 4$

$x \ln 5 - x \ln 4 = \ln 4$

$x(\ln 5 - \ln 4) = \ln 4$

$x = \dfrac{\ln 4}{\ln 5 - \ln 4}$

(§4.2 #31) $\log_x 16 = 4$

started +.5
used exp +.5
exp prop +.5
alg/order of op +1
get x +.5

equivalent to

$x^4 = 16$

$\Rightarrow$ $x = 16^{1/4}$

$= \pm 2$

but +2 b/c def of logs can't have a neg base

(Quiz 4 #3) $\log_5 x + \log_5(x-1) = \log_5(4x)$

started +.5
used exp +.5
log prop +.5
order of op/alg +1
checked answer

$5^{\log_5 x(x-1)} = 5^{\log_5(4x)}$

$x^2 - x = 4x$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0$ or $x - 5 = 0$

$x \neq 0$ or $x = 5$

b/c domain

6. [3 each] Use the Laws of Logarithms to combine the expressions and simplify.

(lecture) $\log_6 4 + \log_6 9$

started +.5
log prop +.5
alg/order op +1
exp/reduce +1

$\log_6(4 \cdot 9) = \log_6 36$

$= \log_6 6^2$

$= 2$

(Practice Exam #7) $2 - \log_6(36y)$

started +.5
log prop +.5
alg/order op +1
exp/reduce +1

$2 - \log_6(36y) = 2 - [\log_6 36 + \log_6 y]$

$= 2 - \log_6 6^2 - \log_6 y$

$= 2 - 2 - \log_6 y$

$= -\log_6 y$

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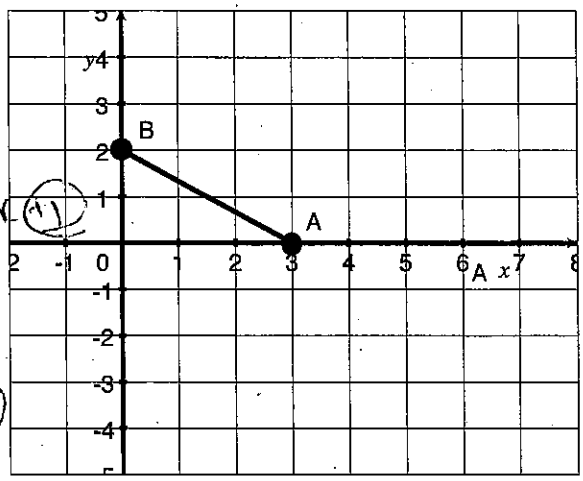
7. Consider the points A and B on the graph.



started +.5

(a) [2] (§1.8 #3) Find the length of the line segment that connects A and B.

$2^2 + 3^2 = 4 + 9 = 13$

so $\sqrt{13}$ by Pyth. got it +.5

work +1

started +.5

(b) [3] (§1.8 #83) Find the equation of the circle centered at A that passes through B.

$(x-h)^2 + (y-k)^2 = r^2$ +.5

$(x-3)^2 + (y-0)^2 = 13$ +.5 +1

Sign +.5

8. (Lecture) On Monday two people were infected with the Tyrant virus and became zombies. The next day six people where infected.

(a) [2] Assuming the growth continues exponentially, how many people are infected by Friday?

days since Mon	infected
0	2
(Tue) 1	6
(Fri) 4	?

need formula to describe # of infected.

Pe^{rt} +.5

note: $P = 2$ +.5

and $2 \cdot e^{r \cdot 1} = 6 \Rightarrow e^r = 3 \Rightarrow r = \ln 3$ +.5

so we have $2e^{(\ln 3)t}$ or $2 \cdot 3^t$

So on friday there are $2e^{(\ln 3) \cdot 4}$ or $2 \cdot 3^4$ got it +.5

160 people

(b) [3] Assuming the growth continues exponentially, how long until the continental U.S. is overrun? (Note the population of the U.S. is approximately 280,000,000.)

started +.5

find t so that +.5

$280{,}000{,}000 = 2e^{(\ln 3) \cdot t}$ +.5

used log +.5

order of op/alg +1

$140{,}000{,}000 = e^{\ln(3) \cdot t}$

$140{,}000{,}000 = 3^t$

$t = \dfrac{\ln 140{,}000{,}000}{\ln 3} \approx 17$ days

9. (§4.1 #83) Suppose you are offered a job that lasts one month. You are to be very well paid but the wages are given to you in a nonstandard manner. On the first day of the month you make two cents. On the second day of the month you make 4 cents. On the third day you are given 8 cents. In general, the wages on your x^{th} day are twice what you received on the previous day.

(a) [2] Find a function that describes how much you get paid on the x^{th} day.

looks like exponential growth.

Pe^{rx}

I don't know P but I know.

$Pe^{1r} = 2$ and $Pe^{2r} = 4$

$\Rightarrow e^{1r} = \frac{2}{P}$

$r = \ln(\frac{2}{P})$ sub into $Pe^{2\ln(2/P)} = 4$

$P(\frac{2}{P})^2 = 4$

$\frac{4}{P} = 4$

$P = 1$ and $r = \ln(\frac{2}{1}) = \ln(2)$

$\$1 \cdot e^{x \ln 2}$ or $\$2^x$

x	?
1	2
2	4
3	8

work (+1) get it (+1)

(b) [1] If you took this job at the start of November and worked everyday of the month, how much would you be given for your last day of work? (There are 30 days in November.)

plug in 30 (+.5) correctly (+.5) so on the last day of work you make

$\$2^{30} = \$107,374,1824$

So $10,737,418.24

(c) [1] Would you rather have the above pay system or be given a million dollars at the end of the month? Justify your answer.

(+.5) note: we make more than 10 million dollars on the last day of work.

That's way better than just 1 million dollars

(+.5) Take the above pay system.